3A5 THERMODYNAMICS AND POWER GENERATION

2023 - SOLUTIONS

(Examiners' comments on final page)

May 2023

Q1 (a) $b = h - T_0 s$

h = specific enthalpy
s = specific entropy
T_0 = environment temperature

$\dot{E}_Q = \int_1^2 \left(1 - \frac{T_0}{T_s}\right) d\dot{Q}_s$

$\dot{Q}_s$ = rate of heat transfer from source
T_s = temperature of source.

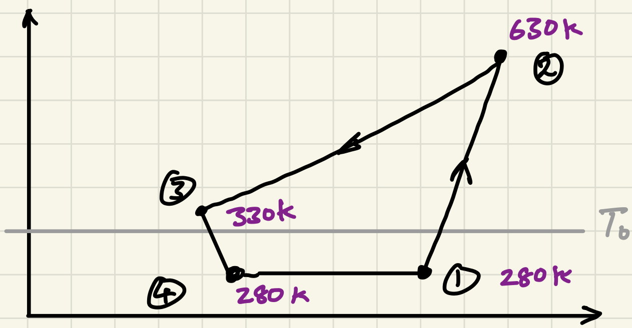
$\dot{W}_{L,Q} = \int_1^2 \left(1 - \frac{T_0}{T_L}\right) d\dot{Q}_L$

$\dot{Q}_L$ = rate of HX $\underline{\underline{\text{to}}}$ surroundings
T_L = temp. at which heat leaves CV

$\dot{W}_{L,irr} = T_0 \dot{S}_{irr}$

$\dot{S}_{irr}$ = entropy generation rate due to internal irreversibility. [4]

(b) (i)



[2]

(ii) $\rho_E = \rho \Delta b = \rho \Delta (h - T_0 s)$

$= \rho c_p \left(T_{20} - T_{30} - T_0 \ln\left(\frac{T_{20}}{T_{30}}\right)\right) = 800 \times 2000 \times \left(300 - 300 \ln\left(\frac{\cancel{630}\ 620}{\cancel{330}\ 320}\right)\right)$

$= \underline{\underline{162.5\ \text{MJ/m}^3}}$

For pumped hydro, $\rho_E = \rho g \Delta H$ = (say) $1000 \times 10 \times 500$

$= 5\ \text{MJ/m}^3$ (i.e. 32.5 bigger)

[4]

(iii) $\dot{W}_{L,irr} = T_0 \left(\dot{m}_o (s_{2o} - s_{3o}) + \dot{m}_N (s_{3N} - s_{2N}) \right)$

Note $(\dot{m}c_p)_{oil} = (\dot{m}c_p)_{N2}$ because ΔT is constant

$$\therefore \xi = \frac{T_0 \dot{S}_{IRR}}{\dot{m}\Delta b_{N2}} = \frac{\dot{m}c_p 300 \left\{ \ln\left(\frac{620}{320}\right) + \ln\left(\frac{330}{630}\right) \right\}}{\dot{m}c_p \left(300 - 300 \ln\left(\frac{630}{330}\right) \right)}$$

$$= \frac{\ln\left(\frac{31}{16} \times \frac{11}{21}\right)}{\left(1 - \ln\left(\frac{21}{11}\right)\right)} = \underline{\underline{4.18\ \%}}$$

Loss is due to HX across a finite temperature difference ΔT. [4]

(iv) $$\eta = \frac{\dot{m}_{OIL} \Delta b_{OIL}}{\dot{W}_C - \dot{W}_{E1} - \dot{W}_{E2}}$$

NOTE THAT DURING expansion in E_2, heat must be absorbed from atmosphere.

$$\dot{W}_C = \dot{m}c_p \Delta T_C = \dot{m}c_p \times 350$$

$$\dot{W}_{E1} = \dot{m}c_p \Delta T_{E1} = \dot{m}c_p \times 50$$

$$\dot{W}_{E2} = \dot{Q}_{E2} = \dot{m} T_4 R \ln\left(\frac{P_4}{P_1}\right) = \dot{m}RT_4 \ln\left(\frac{P_4}{P_3} \cdot \frac{P_2}{P_1}\right)$$

$$= \dot{m}RT_4 \left(\frac{\gamma}{\gamma-1}\right) \left\{ \frac{1}{\eta_p} \ln\left(\frac{T_4}{T_3}\right) + \eta_p \ln\left(\frac{T_2}{T_1}\right) \right]$$

$$= \dot{m}c_p \times 280 \times \left(\frac{1}{0.9} \times \ln\left(\frac{280}{330}\right) + 0.9 \times \ln\left(\frac{630}{280}\right) \right)$$

$$= \dot{m}c_p\ 153.24$$

$$\therefore \eta = \frac{\cancel{\dot{m}c_p} \times 300 \left(1 - \ln\left(\frac{620}{320}\right)\right)}{\cancel{\dot{m}c_p} \left(350 - 50 - 153.24\right)} = \underline{\underline{69.2\ \%}}$$

- Aerodynamic internal irreversibility loss in C & E_1
- $\dot{W}_{L,Q}$ loss because HX from environment to E_2 across $\Delta T = 20K$ [6]

Q2 (a) (i) NH_3 yield increases. Fewer moles on RHS so increasing p favours forward reaction [2]

(ii) NH_3 yield decreases. Reaction is exothermic so increasing T must favour backward reaction. [2]

(iii) NH_3 yield unchanged. Partial pressures of all reacting components are unchanged. [2]

(b) $0.1 N_2 + 0.3 H_2 \longrightarrow a N_2 + b H_2 + c NH_3$

$$\left.\begin{array}{ll} N: & 2a + c = 0.2 \\ H: & 2b + 3c = 0.6 \end{array}\right\} \Rightarrow \begin{array}{l} c = 2(0.1 - a) \\ b = 3a \end{array}$$

EQ^m: $$\frac{\frac{c}{n_2}\left(\frac{p}{p_0}\right)}{\left(\frac{a}{n_2}\frac{p_2}{p_0}\right)^{1/2}\left(\frac{b}{n_2}\frac{p_2}{p_0}\right)^{3/2}} = K_p \Rightarrow \frac{c}{a^{1/2}(3a)^{3/2}} = K_p \frac{p_2}{n_2 p_0}$$

$$\therefore \frac{2(0.1-a)}{a^2} = 3^{1.5} \times \frac{P_2}{n P_0}$$

PG $$\left.\begin{array}{l} p_1 V = n_1 \bar{R} T_1 \\ p_2 V = n_2 \bar{R} T_2 \end{array}\right\} \therefore \frac{p_2}{p_1} = \frac{n_2}{n_1}\frac{T_2}{T_1} \Rightarrow \frac{P_2}{n_2 P_0} = \frac{P_1}{n_1 P_0}\frac{T_2}{T_1}$$

Thus $$\frac{2(0.1-a)}{a^2} = 3^{1.5} \times \frac{P_1}{n_1 P_0} \times \frac{T_2}{T_1} K_p = 3^{1.5} \times \frac{10}{0.4} \times \frac{600}{400} \times e^{-3.191}$$

$\uparrow$ R9. DATABK.

$$= 8.015$$

$\therefore 8.015a^2 + 2a - 0.2 = 0 \Rightarrow a = 0.0765$ [-ve sol^n not poss.]

$b = 0.2296$; $c = 0.04694$; $n_2 = a + b + c = 0.3531$

$x_{N_2} = \frac{a}{n_2} = 0.217$ $\quad x_{H_2} = b/n_2 = 0.650$ $\quad x_{NH_3} = \frac{c}{n_2} = 0.133$

$$p_2 = p_1 \times \frac{n_2}{n_1} \times \frac{T_2}{T_1} = \underline{\underline{13.24 \text{ bar}}}$$

[7]

(ii) $$Q - W = \Delta U = \Delta(H - PV) = \Delta H - \Delta(n\bar{R}T)$$

$$\Delta H = H_{P2} - H_{R1} = (H_{P2} - H_{P1}) + (H_{P1} - H_{R1})$$

$$(H_{P2} - H_{P1}) = a\left[\bar{h}_{N_2}(T_2) - \bar{h}_{N_2}(T_1)\right] + b\left[\bar{h}_{H_2}(T_2) - \bar{h}_{H_2}(T_1)\right] + c \times \bar{c}_{p_{NH_3}}(T_2 - T_1)$$

$$= 0.0765 \times (17.56 - 11.64) + 0.2296 \times (17.27 - 11.42)$$

$$+ 0.0469 \times \frac{42 \times 200}{1000} = 2.1904 \text{ MJ}$$

$$(H_{P1} - H_{R1}) = c \times \Delta \bar{H}^{0}_{T1} = 0.0469 \times (-48.04) = -2.253 \text{ MJ}$$

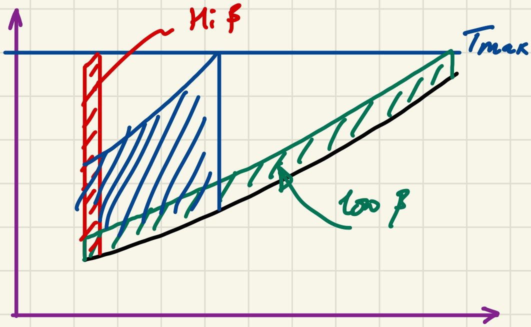
$$\Delta(n\bar{R}T) = (0.353 \times 600 - 0.4 \times 400) \times \frac{8.314}{1000} = +0.431 \text{ MJ}$$

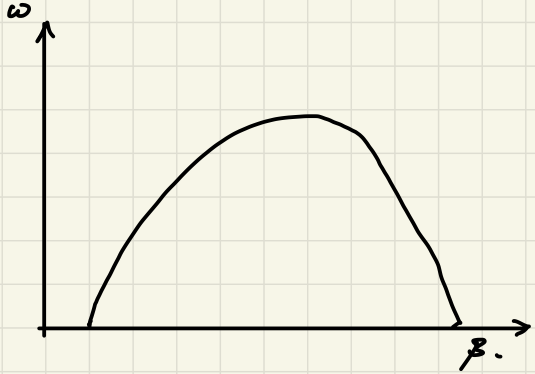
$$\therefore Q = 2.1904 - 2.253 - 0.431 = \underline{\underline{-0.494 \text{ MJ}}}$$

Heat is transferred from gas to surroundings.

[7]

Q3 (a)





For a given maximum T, high and low pressure ratio lea to very "skinny" cycles with low area and hence low sp. wk. At some intermediate P.R. the work is a maximum. [3]

(b) · Assume changes in GPE & KE can be neglected;

SFEE: $dq - dw_x = dh$; 2ND LAW: $dq = Tds - T\delta s_{irr}$

$= dh - vdp - T\delta s_{irr}$

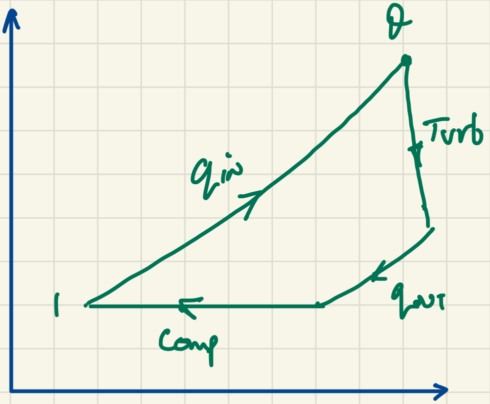
$\therefore \quad -dw_x = \cancel{dh} - (\cancel{dh} - vdp - T\delta s_{irr})$

$\Rightarrow \quad -dw_x = vdp + T\delta s_{irr} \Rightarrow -dw_x \geq vdp$.

$$-w_x = \int_1^2 \frac{dp}{\rho}$$

Undercooling increases ρ hence reduces w_c and increases w_{net} for a GT. [3]

(c)



η will be highest when all processes reversible.

(i) $w_T = c_p T_1 \theta \left(1 - 1/r_T\right)$

$$w_c = \int RT_1 \frac{dp}{p} = RT_1 \ln\left(\frac{p_2}{p_1}\right) = c_p T_1 \ln r_T .$$

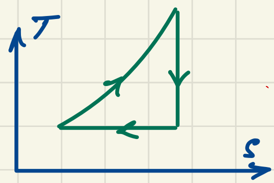
$$\therefore \quad \eta = \frac{w_T - w_c}{q_{in}} = \frac{c_p T_1\left[\theta\left(1 - 1/r_T\right) - \ln r_T\right]}{c_p T_1 (\theta - 1)}$$

$$\eta = \frac{\theta\left(1 - 1/r_T\right) - \ln r_T}{\theta - 1}$$

[4]

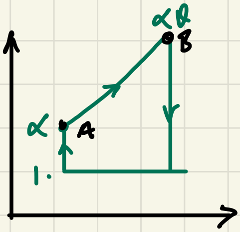
(ii) $$\frac{d\eta}{dr_T} = \frac{\frac{\theta}{r_T^2} - \frac{1}{r_T}}{\theta - 1} = 0 \quad @ \text{ max } \eta$$

$$\Rightarrow \theta = r_T \Rightarrow r_p = \theta^{\gamma/\gamma-1}$$



When $r_T = \theta$ the cycle looks like this and it is clear that all heat is being rejected at the lowest cycle temperature. Since all processes are reversible, this clearly gives the highest η. [4]

(c) The maximum efficiency occurs when the cycle looks like this



$$\eta_{max} = \frac{\text{Max work from A to B}}{\text{Heat input from A to B.}}$$

$$= \frac{\alpha\theta - \alpha - \ln\theta}{\alpha\theta - \alpha} = 1 - \frac{\ln\theta}{\alpha(\theta - 1)}$$

[6]

[Note: can also get this result from:

$\eta_{max} = \eta + (1-\eta)\underline{\left(1 - \frac{1}{\alpha}\right)}$ efficiency of bottoming cycle]

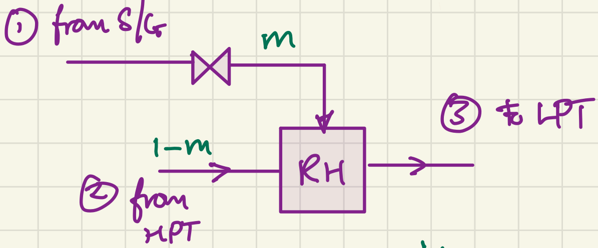
Q4 (a) + Increased $\bar{T}$ of heat addition ⇒ higher efficiency

+ reduced turbine wetness ⇒ higher η_T & less erosion.

− more complicated steam paths and additional pipework

• Use here driven by LPT wetness considerations. [2]

(b)



SFEE: $h_3 = m h_1 + (1-m) h_2$

$$\therefore \quad m = (h_3 - h_2)/(h_1 - h_2) = \frac{2875 - 2822}{3032 - 2822} = \underline{\underline{0.252}}$$

Loss due to irreversibility per kg flow through S/G $= T_0 \{ m(s_3 - s_1) + (1-m)(s_3 - s_2) \}$

$$= 289\big(0.252(6.262 - 6.116) + (1 - 0.252)(6.262 - 6.161)\big)$$

$$= \underline{\underline{32.4 \text{ kJ/kg}}}$$ [3]

(c) $\dot{W}_{net} = \dot{m}\left((1-m) w_{HPT} + w_{LPT} - w_{FP}\right)$

$$\dot{m} = 100 \times 10^3 / \left((1 - 0.252) \times (3032 - 2822) + (2875 - 2101) - (152 - 138) \right)$$

$$= \underline{\underline{109 \text{ kg/s}}}$$

$$\eta_{TH} = \frac{\dot{W}_{OUT}}{\dot{m}\, \Delta h_{s/g}} = \frac{100 \times 10^3}{109\left([3032 - 152]\right)} = \underline{\underline{31.8\ \%}}$$ [4]

(d) Feed pump $\Delta h_{is} = \frac{\Delta p}{\rho} \approx \frac{100\times10^5}{1000} = 10$ kJ/kg

quite accurate because $\rho \approx$ const.

$\therefore \eta_{FP} = \frac{\Delta h_s}{\Delta h} = \frac{10}{14} = \underline{71.4\,\%}$

HPT & LPT :

we use $T\Delta s = \Delta h$ @ exit to estimate efficiencies. This is exact for LPT

LEAST ACCURATE FOR HPT.

$$\therefore \eta_T = \frac{\Delta h_T}{\Delta h_{is}} = \frac{\Delta h_T}{\Delta h_T + T_x \Delta s_T}$$

$$\eta_{LPT} = \frac{945}{945 + 306 \times (7.817 - 6.772)} = \underline{\underline{0.85}}$$

$$\eta_{HPT} = \frac{210}{210 + (248+273) \times (6.161 - 6.116)} = \underline{\underline{90\,\%}}$$

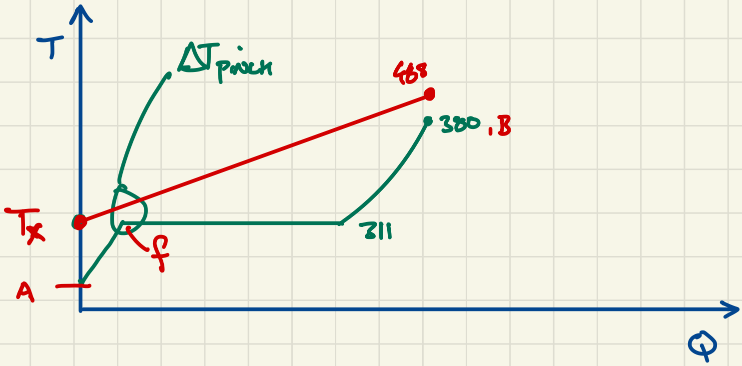
Accuracy : η_{LPT} better than η_{FP}, better than η_{HPT}

η_{LPT} clearly has greatest impact as Δh_{LPT} is greatest.

[5]

(e)

(i)



[2]

(ii) $$\frac{468 - T_x}{468 - (311 + 10)} = \frac{h_B - h_A}{h_B - h_f} = \frac{3032 - 152}{3032 - 1408}$$

[2]

$$\Rightarrow \quad T_x = 207.3\,^\circ C$$

(iii) Discussion might include:

- Effect of pressure on mean T of heat addition and hence η_{cc}.
- Effect of pressure on wetness at turbine exit. (Higher P ⇒ higher wetness)
- Effect of pressure on S/G efficiency (via T_x)
- Materials cost for boiler.

[2]

EXAMINERS' COMMENTS

Q1. *Exergy and energy storage*. Most that attempted this question provided correct expressions for the various terms in the exergy equation, and the *T-s* diagram for the energy storage cycle was also tackled quite well. A significant fraction of students made good headway in calculating the storage density, but few correctly calculated the exergetic loss for the heat exchange. Few attempted the last part on charge efficiency.

Q2. *Chemical equilibrium.* Candidates showed a good, qualitative understanding of how pressure and temperature changes affect chemical equilibrium. Most had a good idea of how to apply equilibrium relations, though only a minority correctly combined this with ideal gas equations to account for pressure changes, despite several past questions of this form. The most common error for the last (energy equation) section was to equate heat transfer to enthalpy rather than internal energy change, as required for a constant volume process.

Q3. *Closed cycle gas turbine.* Although the students had studied the effects of pressure ratio on work output in the coursework in some detail, it was disappointing that many failed to describe this properly in the first part of the question. Most students understood the purpose of intercooling in reducing compressor work. Analysis of the combined-cycle was poorly done with only a very small number of complete or near-complete answers.

Q4. *Steam plant and CSP.* Students in general understood the pros and cons of reheat and many spotted why it might be used to reduce wetness loss in this case. There were good answers in most cases when applying SFEE across the reheater and finding overall performance (efficiency, steam mass flow). Many students were able to determine the pump efficiency but struggled with computing turbine efficiencies. Analysis of the steam generator was also poorly done.

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