

3A5 THERMODYNAMICS & POWER GENERATION  
2021 SOLUTIONS

(Examiners' comments on last page)

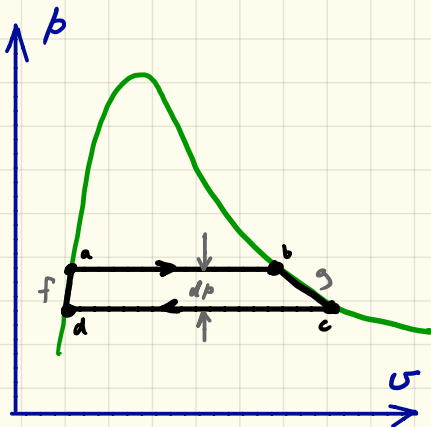
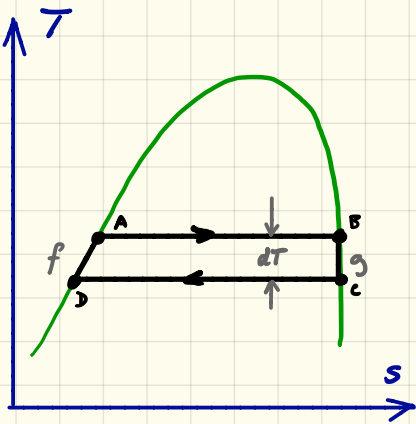
AJW

RSC

18 May 2021

Q1

(a)



(i) Cycle is reversible  $\therefore dq = Tds$  &  $dwo = pdv$

$$\text{First Law: } \oint dq - \oint dwo = 0$$

$$\therefore \oint Tds = \oint pdv \quad \Rightarrow \quad \underline{\text{Area ABCD} = \text{Area abcd}}$$

$$\text{For small } dT \text{ \& } dp: \quad ABCD = s_{fg} dT; \quad abcd = v_{fg} dp$$

Also, for evaporation at constant p:

$$Tds = dh - vdp \Rightarrow T s_{fg} = h_{fg}$$

$$\text{Equating areas: } \frac{h_{fg}}{T} dT = v_{fg} dp \quad \text{on saturation line}$$

$$\therefore \underline{\left(\frac{dT}{dp}\right)_{\text{SAT}}} = \frac{T v_{fg}}{h_{fg}} \quad [6]$$

(ii) Assume:  $v_f \ll v_g$ ; ideal gas;  $h_{fg} = L \sim \text{const}$

$$\therefore \left(\frac{dT}{dp}\right)_{\text{SAT}} \approx \frac{T(RT/p)}{L}$$

$$\text{INTEGRATE: } \frac{1}{T_{s1}} - \frac{1}{T_{s2}} = \frac{\bar{R}}{ML} \ln\left(\frac{p_2}{p_1}\right)$$

$$\therefore \frac{1}{T_{s2}} = \frac{1}{230.7} - \frac{8.314}{44 \times 425.9} \times \ln 2$$

$$\Rightarrow \underline{T_{s2} = 248.3 \text{ K} = 248 \text{ K} \approx 88^\circ\text{F}}$$

NOTE "REFPROP" gives  
 $T_{s2} = 247.7 \text{ K}$

[5]

$$(b) (i) \text{ At saturation, } \bar{\mu}_i^v(T, p_{si}) = \bar{\mu}_i^l(T, p_{si})$$

$$\text{Also assume } \bar{v}_f \ll \bar{v}_g (= \bar{R}T/p)$$

$$\therefore \bar{\mu}_i^v(T, p_{si}) + \int_{p_{si}}^p \frac{\bar{R}T}{p} dp + \bar{R}T \ln X_i^v = \bar{\mu}_i^l(T, p_{si}) + \int_{p_{si}}^p \bar{v}_g dp + \bar{R}T \ln X_i^l$$

*equal*

$$\Rightarrow \bar{R}T \ln \left( \frac{p}{p_{si}} \right) + \bar{R}T \ln X_i^v = \bar{R}T \ln X_i^l$$

$$\Rightarrow \underline{p X_i^v = p_{si} X_i^l} \quad (1)$$

[4]

(ii) Let  $x$  be mole fraction of butane in liquid phase and  $y$  be mole fraction of butane in vapour phase

Summing (1) over both components, at the bubble point:

$$\begin{aligned} p = p_{\text{BUB.}} &= (1-x) p_{\text{sp}} + x p_{\text{sb}} \\ &= 0.5 \times 1.0 + 0.5 \times 0.147 = \underline{\underline{0.5735 \text{ bar}}} \end{aligned}$$

Applying eq. (1) for butane at the bubble point:

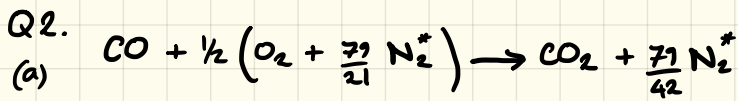
$$y p_{\text{BUB.}} = x p_{\text{sb}}$$

$$\therefore y = \frac{0.5 \times 0.147}{0.5735} = 0.12816$$

$\Rightarrow$  Bubbles are  $\sim$  13% butane, 87% propane vapor basis

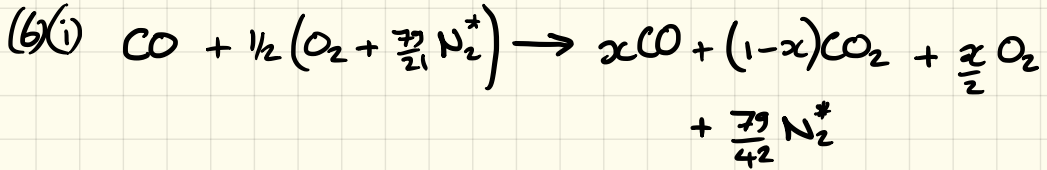
Note the higher fraction of more volatile component, as expected

[5]



$$AFR = \frac{\frac{1}{2} \left( 32 + \frac{79}{21} \times 28 \right)}{28} = \underline{\underline{2.452}}$$

[4] ~~[5]~~



$$n_o = 1 + \frac{1}{2} + \frac{79}{42}$$

$$\underline{n_o = \frac{71}{21}}$$

$$n^{Text} = \alpha + 1 - \alpha + \frac{\alpha}{2} + \frac{79}{42}$$

$$\underline{n = \frac{121}{42} + \frac{\alpha}{2}}$$

Equilibrium condition:

$$\frac{(P_{CO_2}/P_o)}{(P_{CO}/P_o) (P_{O_2}/P_o)^{1/2}} = \frac{\left( \frac{1-\alpha}{n} \right) \left( \frac{P}{P_o} \right)}{\frac{\alpha}{n} \left( \frac{P}{P_o} \right) \left[ \frac{\alpha}{2n} \frac{P}{P_o} \right]^{1/2}} = K_p(T)$$

$$\therefore \frac{1-\alpha}{\alpha^{3/2}} = \left( \frac{P}{2P_o n} \right)^{1/2} K_p(T)$$

Ideal gas:  $P_o V = n_o \bar{R} T_o$  ;  $P V = n \bar{R} T$

$$\therefore \frac{P}{n P_o} = \frac{1}{n_o} \left( \frac{T}{T_o} \right) \Rightarrow \frac{1-\alpha}{\alpha^{3/2}} = \left( \frac{1}{2n_o} \right)^{1/2} \left( \frac{T}{T_o} \right)^{1/2} K_p(T)$$

$$\Rightarrow \alpha = 3/2 ; \gamma = 1/2 ; \beta = \left( \frac{1}{2n_o} \right)^{1/2} = 0.38456$$

[8] ~~[7]~~

$$(ii) \text{ When } T = 2600 \text{ K, } \frac{1-x}{x^{3/2}} = 0.38456 \times \left( \frac{2600}{298.15} \right)^{1/2} \times e^{2.800}$$

$$\therefore x^{3/2} = \frac{1-x}{18.675} = x = \left( \frac{1-x}{18.675} \right)^{2/3}$$

We expect  $x$  to be small, so guess  $x=0$  on RHS

$$\Rightarrow x_1 = 0.142; x_2 = 0.128; x_3 = 0.1296; \underline{x = 0.1295}$$

$$\left( \frac{P}{P_0} \right) = \frac{n}{n_0} \frac{T}{T_0} = \frac{2.9457}{3.38095} \times \frac{2600}{298.15} = 7.60 \Rightarrow \underline{p = 7.6 \text{ bar}} \quad [5]$$

(iii) Even at elevated temperatures,  $K_p$  becomes very large as  $T$  is reduced (e.g.,  $K_p = e^{32}$  at 800 K), so at  $T=T_0$  there will be (almost) no CO [ $x \approx 0$ ]

$$\text{FIRST LAW:- } Q - \cancel{W} = \Delta U = (H_2 - P_2V) - (H_1 - P_1V)$$

$$\therefore Q = (H_2 - H_1) - (n_2 - n_0) \bar{R} T_0$$

$$= +\Delta \bar{H}_{298}^\circ + \frac{1}{2} \bar{R} T_0 \quad (\text{note that } n \text{ reduces by } \frac{1}{2} \text{ kmol. for complete reaction})$$

$$= -283.0 + 0.5 \times 8.3145 \times 298.15 / 1000$$

$$= \underline{\underline{\cancel{281.76} \text{ MJ}}} - 281.76 \text{ MJ} \quad [5]$$

$$\text{i.e., } \underline{\underline{|Q| = 282 \text{ MJ}}} \quad \text{to 3SF} \quad [3]$$

$$Q3 (a) (i) \quad r_T = \beta^{\gamma-1/\gamma} = 16^{2/7} \quad (\text{isotropic } T \text{ ratio})$$

$$T_2 = T_1 + \frac{T_1 (r_T - 1)}{\eta_c} = 290 \left( 1 + \frac{(16^{2/7} - 1)}{0.9} \right) = \underline{\underline{728 \text{ K}}} \quad [2]$$

$$(ii) \quad \frac{T_2}{T_1} = \left( 1 + \frac{(\sqrt{r_T} - 1)}{\eta_c} \right)^2 \Rightarrow T_2 = 290 \times \left( 1 + \frac{(4^{2/7} - 1)}{0.9} \right)^2 = \underline{\underline{749.4 \text{ K}}} \quad [3]$$

(iii) Polytropic efficiency gives exit temperature that is independent of number of "stages" - essential for cycles with (e.g.) intercooling.

$$\left( \frac{T_2}{T_1} \right) = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma \eta_p}} \Rightarrow \eta_p = \frac{\gamma-1}{\gamma} \frac{\ln(P_2/P_1)}{\ln(T_2/T_1)} = \frac{2}{7} \frac{\ln 16}{\ln(728/290)} = \underline{\underline{0.961}} \quad [3]$$

$$(b) \quad T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma \eta_c}} = 280 \times 20^{(2/7 \times 0.99)} = 732.5$$

$$T_4 = T_3 \left( \frac{P_4}{P_3} \right)^{\eta_t(\gamma-1)/\gamma} = 1450 \times \left( \frac{1.1}{1.9} \right)^{0.961 \times 0.36 / 1.36} = 758 \text{ K.}$$

SFEE for combustion:

$$(i) \quad H_p - H_{p0} = H_k - H_{n0} + H_{n0} - H_{p0}$$

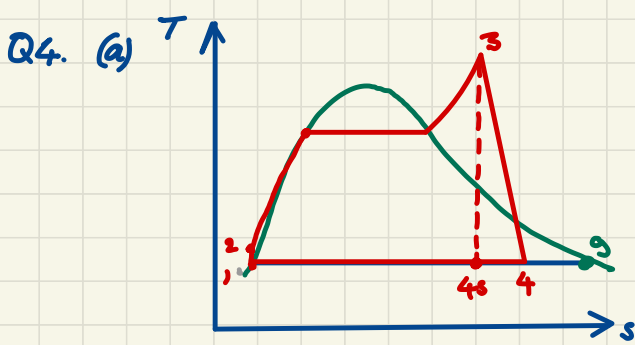
$$\therefore (1+f) C_{p,x} (T_3 - T_0) = C_{p,e} (T_2 - T_0) + f \cdot LCV$$

$$\Rightarrow f = \frac{C_{p,x} (T_3 - T_0) - C_{p,e} (T_2 - T_0)}{LCV - C_{p,x} (T_3 - T_0)}$$

$$f = \frac{1.1 \times (1450 - 298) - 1.005 (732.5 - 298)}{49000 - 1.1 \times (1450 - 298)} = \underline{\underline{0.0740}} \quad [6]$$

$$\begin{aligned} \text{(ii)} \quad \omega &= (1+f) c_{p,x} (T_3 - T_4) - c_{p,a} (T_2 - T_1) \\ &= 1.0174 \times 1.1 \times (1450 - 750) - 1.005 \times (732.5 - 280) \\ &= \underline{\underline{319.6 \text{ kJ/kg comp. air}}} \end{aligned} \quad [5]$$

$$\begin{aligned} \text{(iii)} \quad \eta &= \frac{\omega}{\dot{q}_{\text{in}}} = \frac{319.6}{0.0174 \times 49000} \\ \eta &= \underline{\underline{37.5\%}} \end{aligned} \quad [1]$$



$$h_1 = h_f(0.04) = 121.4 \text{ kJ/kg}$$

$$s_1 = 0.422 \text{ kJ/kgK}$$

$$h_3 = 3450.4 \text{ kJ/kg}$$

$$s_3 = 6.823 \text{ kJ/kgK}$$

$$s_5 = 8.473$$

$$h_5 = 2553.7$$

[1]

(ii) Using chart:  $h_{4s} = 1964.4$

$$\Rightarrow w_s = h_3 - h_{4s} = 1486 \text{ kJ/kg (isentropic spec. work)}$$

For estimate it is ok to neglect f.p. work.

$$\eta_{cr} \approx \frac{\dot{m}_s w_s}{\dot{q}_{in}} \Rightarrow \eta_{is} = \frac{\eta_{cr} \times \dot{q}_{in}}{w_s} = \frac{\eta_{cr} (h_3 - h_1)}{w_s}$$

$$= 0.36 \times (3450.4 - 121.4) / 1486$$

$$= \underline{\underline{0.807}}$$

[3]

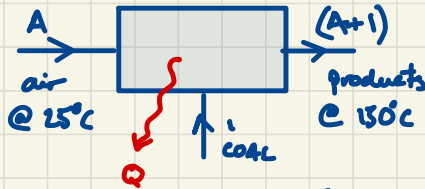
(iii)  $h_4 = h_3 - \eta_{is} w_s = 3450.4 - 0.807 \times 1486$

$$= 2252 \text{ kJ/kg}$$

$$x_4 = \frac{h_4 - h_1}{h_5 - h_1} = \frac{2252 - 121.4}{2553.7 - 121.4} = \underline{\underline{0.876}}$$

[1]

(b) SFEE for combustion:



$$\eta_B = \frac{Q}{LCV}; \quad Q = H_R - H_P = (H_R - H_{R0}) - (H_P - H_{P0}) + (H_{R0} - H_{R0})$$

$$= LCV - (A+1)C_p (T_P - T_0)$$

$$\therefore \eta_B = 1 - \frac{(A+1)C_p (T_P - T_0)}{LCV} = 1 - \frac{21 \times 1.1 \times 105}{23000} \approx 0.8945$$

$$\therefore \eta_{cr} = \eta_B \times \eta_{cr}$$

$$= 0.8945 \times 0.36 = \underline{\underline{32.2\%}}$$

[5]



(b) cost.  $P = \dot{m}_c LCV \times \eta_{ov}$

$$\therefore \dot{m}_c = \frac{P}{LCV \times \eta_{ov}} = \frac{500}{23 \times 0.32} = 67.5 \text{ kg s}^{-1}$$

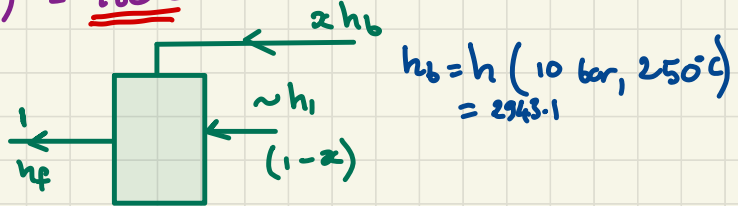
$$= \underline{\underline{5833 \text{ tonne/day}}}$$

$$\dot{m}_{CO_2} = \dot{m}_c \times 0.75 \times \frac{44}{12} = \underline{\underline{16041 \text{ tonne/day}}} \quad [2]$$

(c) Assume feedwater is saturated & ignore f.p. work

$$T_{fw} = T_{SAT}(10 \text{ bar}) \approx \underline{\underline{180^\circ\text{C}}} \quad [1]$$

SFEE for FH



$$h_{fw} = x h_b + (1-x) h_1 \Rightarrow x = \frac{h_{fw} - h_1}{h_b - h_1} = \frac{762.5 - 121.4}{2943.1 - 121.4}$$

$$= \underline{\underline{0.227 \text{ (bleed fraction)}}} \quad [2]$$

$$\eta_{ov} = \frac{\dot{Q}}{\dot{Q}'} = \frac{(h_3 - h_4) - x(h_b - h_4)}{h_3 - h_{fw}}$$

$$= \frac{(3450.2 - 2252.) - 0.227 \times (2943.1 - 2252.)}{3450.2 - 762.5} = \underline{\underline{38.7\%}} \quad [2]$$

(d) Higher FW temperature means stack temperature will increase thereby reducing the boiler efficiency. We need an air preheater (APH) to recovery heat from the flue gas.

Other cycle improvements:

- single / double reheat (inc. mean T of heat addition)
- Additional FH stages ( " )
- inc. boiler pressure ( " [supercritical?] )

[3]

## Examiners' comments

Q1. Most candidates correctly derived the C-C equation by equating the Gibbs' free energy between wet and dry saturated, but  $p$ - $v$  diagrams were often drawn incorrectly. Many were able to simplify the thermodynamic equilibrium criteria in part (b) to obtain Raoult's Law, but few managed to apply it successfully to obtain the bubble-point pressure and vapour.

Q2. Most did OK on part (a) but few got it completely right. Many candidates got into trouble in part (b) and did not see how to combine the equilibrium criterion with ideal gas relations at constant  $V$ , despite the similarity to an examples paper question.

Q3. This question on gas turbine cycles was tackled by all candidates. Most were familiar with polytropic efficiency but many found it harder to work through the combustion analysis and complete the cycle calculation.

Q4. Most were able to complete the turbine calculation, but there were surprisingly few correct answers for carbon dioxide emissions, although there seemed to be a slightly better understanding of the feed heater. Many candidates appeared to run out of time.

Dr A.J. White and Prof. R.S. Cant

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