2023 Q1 convection

Monday, 22 May 2023 17:40

(a) $$\underline{u} \cdot \nabla (\rho c_p T) = \nabla \cdot (\lambda \nabla T)$$

$$\underline{u} = u \, \underline{e}_x = u_c \left(1 - \left(\frac{y}{\delta}\right)^2 \right) \underline{e}_x$$

$$\underline{u} \cdot \nabla = u_x \frac{\partial}{\partial x}$$

$$\nabla \cdot (\lambda \nabla T) = \lambda \left(\underbrace{\frac{\partial^2 T}{\partial x^2}}_{\text{neglect}} + \frac{\partial^2 T}{\partial y^2} \right)$$

$$\therefore \boxed{u_c \left(1 - \left(\frac{y}{\delta}\right)^2 \right) \frac{\partial T}{\partial x} = \underbrace{\frac{\lambda}{\rho c_p}}_{\alpha} \frac{\partial^2 T}{\partial y^2}} \quad \text{QED}$$

(b) $$\frac{\partial^2 T}{\partial y^2} = \frac{u_c}{\alpha} \left(1 - \left(\frac{y}{\delta}\right)^2 \right) \frac{\partial T}{\partial x}$$

$$\frac{\partial T}{\partial y} = \frac{u_c}{\alpha} \frac{\partial T}{\partial x} \left(y - \frac{1}{\delta^2} \frac{y^3}{3} \right) + c_1(x)$$

$$\left. \frac{\partial T}{\partial y} \right|_{y=+\delta} = \frac{2}{3} \frac{u_c \delta}{\alpha} \frac{\partial T}{\partial x} + c_1(x)$$

$$\left. \frac{\partial T}{\partial y} \right|_{y=-\delta} = -\frac{2}{3} \frac{u_c \delta}{\alpha} \frac{\partial T}{\partial x} + c_1(x)$$

$$q_+ = -\lambda \left. \frac{\partial T}{\partial y} \right|_{y=+\delta} = -\lambda \left[\frac{2}{3} \frac{u_c \delta}{\alpha} \frac{\partial T}{\partial x} + c_1 \right]$$

$$q_- = -\lambda \left. \frac{\partial T}{\partial y} \right|_{y=-\delta} = -\lambda \left[-\frac{2}{3} \frac{u_c \delta}{\alpha} \frac{\partial T}{\partial x} + c_1 \right]$$

$$c_1 = -\frac{(q_+ + q_-)}{2\lambda}$$

$$\boxed{\frac{\partial T}{\partial y} = \frac{u_c}{\alpha}\frac{\partial T}{\partial x}\, y\left(1 - \frac{1}{3}\left(\frac{y}{\delta}\right)^2\right) - \left(\frac{q_+ + q_-}{2\lambda}\right)}$$

(e) $$T = \frac{u_c}{\alpha}\frac{\partial T}{\partial x}\left[\frac{y^2}{2} - \frac{1}{3}\frac{1}{\delta^2}\frac{y^4}{4}\right] - \left(\frac{q_+ + q_-}{2\lambda}\right) y + C_2(x)$$

$$T = \frac{u_c}{\alpha}\frac{\partial T}{\partial x}\underbrace{\frac{y^2}{2}\left[1 - \frac{1}{6}\left(\frac{y}{\delta}\right)^2\right]}_{\text{SYMMETRIC ABOUT } Y=0} - \underbrace{\left(\frac{q_+ + q_-}{2\lambda}\right) y}_{\text{LINEAR}} + C_2(x)$$

OR

$$T(\delta) = \frac{u_c}{\alpha}\frac{\partial T}{\partial x}\frac{\delta^2}{2}\frac{5}{6} - \left(\frac{q_+ + q_-}{2\lambda}\right)\delta + C_2(x)$$

$$T(-\delta) = \frac{u_c}{\alpha}\frac{\partial T}{\partial x}\frac{\delta^2}{2}\frac{5}{6} - \left(\frac{q_+ + q_-}{2\lambda}\right)(-\delta) + C_2(x)$$

$$\boxed{\Delta T = T(\delta) - T(-\delta) = -(q_+ + q_-)\frac{\delta}{\lambda}}$$

(d) ONE CAN INTEGRATE THE EQUATIONS FOR u AND T TO OBTAIN THE MEAN, BUT A SIMPLER WAY TO SHOW HOW $\bar{T}$ VARIES COMES FROM USING THE ORIGINAL 1D CONSERVATION EQUATION:

$$u\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial y^2}$$

$$\int_{-\delta}^{+\delta} u\frac{\partial T}{\partial x}\,dy = \frac{d}{dx}\int_{-\delta}^{+\delta} uT\,dy = \int_{-\delta}^{+\delta}\alpha\frac{\partial^2 T}{\partial y^2}\,dy =$$

$$= \alpha\left[\left.\frac{\partial T}{\partial y}\right|_{\delta} - \left.\frac{\partial T}{\partial y}\right|_{-\delta}\right]$$

$$= \frac{\alpha}{\lambda}\left[-q_+ - (-q_-)\right] = -\frac{\alpha}{\lambda}\left[q^+ - q^-\right]$$

$$2\delta\bar{u}\,\frac{d\bar{T}}{dx} = \frac{\alpha}{\lambda}\left(q_- - q_+\right) = \frac{\lambda}{\rho c_p}\left(q_- - q_+\right)$$

$$\dot{m}\, c_p \frac{d\bar{T}}{dx} = q_- - q_+$$

SO WE SHOW BOTH THAT THE MEAN TEMPERATURE RISE IS CONSTANT, AND equal TO THE NET HEAT FLUX, AS IT SHOULD BE.

2ai)

Conduction in r via diffusion

$$-\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)=0 \quad \text{where } \theta=\frac{T-T_\infty}{T_s-T_\infty}$$

$$\int d\left(r\frac{d\theta}{dr}\right)=\int 0\,dr \Rightarrow r\frac{d\theta}{dr}=a$$

$$\int d\theta=\int\frac{a}{r}dr \qquad \boxed{\theta=a\ln r+b}$$

2aii)

Dimensional

B.C.1 $T(r=r_1)=T_s$

B.C.2 $-\lambda\left.\frac{dT}{dr}\right|_{r=r_2}=h(T(r_2)-T_\infty)$

Non-D

$\theta(r_1)=1=a\ln r_1+b$

$$\left.\frac{d\theta}{dr}\right|_{r_2}=\frac{a}{r_2}=-\frac{h}{\lambda}\underbrace{(a\ln r_2+b)}_{\theta(r)}$$

B.C. 2 $\quad \frac{\lambda}{h}\frac{a}{r_2}=-a\ln r_2-\cancel{b}$

B.C. 1 $\quad + \quad 1=a\ln r_1+\cancel{b}$

$$\frac{\lambda}{h}\frac{a}{r_2}+1=a\ln r_1/r_2$$

$$1=a\left(\ln r_1/r_2-\frac{\lambda}{h}\frac{1}{r_2}\right)$$

$$\boxed{a=\left(\ln r_1/r_2-\frac{\lambda}{hr_2}\right)^{-1}}$$

$$b=1-\overbrace{\left(\ln r_1/r_2-\frac{\lambda}{h}\frac{1}{r_2}\right)^{-1}}^{a}\ln r_1$$

$$=\frac{\ln(\cancel{r_1}/r_2)-\frac{\lambda}{h}\frac{1}{r_2}-\cancel{\ln r_1}}{\ln r_1/r_2-\frac{\lambda}{h}\,{}^1/r_2}$$

$$\boxed{b=-\frac{\lambda+hr_2\ln r_2}{-\lambda+hr_2\ln r_1/r_2}}$$

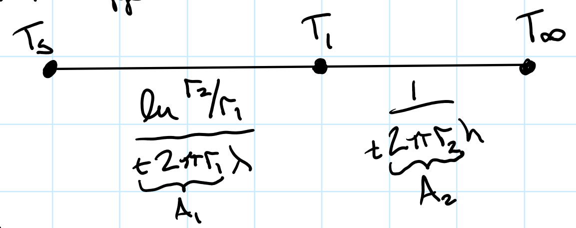
$$Q_w=-\lambda\left.\frac{\partial T}{\partial r}\right|_{r=r_1}2\pi r_1 t$$

$$=-\lambda(T_s-T_\infty)\left.\frac{d\theta}{dr}\right|_{r=r_1}2\pi r_1 t$$

$$=-\lambda(T_s-T_\infty)\left[\left(\ln r_1/r_2-\frac{\lambda}{hr_2}\right)\cancel{r_1}\right]^{-1}2\pi\cancel{r_1}t$$

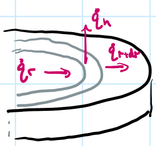
$$\boxed{Q=\frac{2\pi\lambda t(T_s-T_\infty)}{\frac{\lambda}{hr_2}+\ln(r_2/r_1)}}$$

Alt. Approach



$$Q=\frac{T_s-T_\infty}{\frac{\ln r_2/r_1}{2\pi r_1 t\lambda}+\frac{1}{2\pi r_2 th}}$$

2bi) Conservation of Energy Cylinder



ODE for dimensional $\dot{q}$

Conduction $\dot{q}_r=-\lambda\frac{dT}{dr}$ Convection $\dot{q}_c=h(T-T_\infty)$

$$\underbrace{\cancel{\dot{q}_r}}_{}\,2\pi r t-\Big(\underbrace{\dot{q}_r+\frac{d\dot{q}_r}{dr}\delta r}_{\text{conduction}}\Big)2\pi(r+\delta r)t-\underbrace{\dot{q}_c}_{\text{convection}}2\pi r\delta r=0$$

$$-\dot{q}_r\cancel{2\pi}\cancel{\delta r}t-\frac{d\dot{q}_r}{dr}\cancel{2\pi r\delta r}t-2\dot{q}_c\cancel{2\pi r\delta r}=0$$

$$\theta=\frac{T-T_\infty}{T_s-T_\infty}$$

$$\therefore\ T=(T_s-T_\infty)\theta+T_\infty$$

$$\frac{dT}{dr}=(T_s-T_\infty)\frac{d\theta}{dr} \qquad \frac{d^2T}{dr^2}=(T_s-T_\infty)\frac{d^2\theta}{dr^2}$$

Collecting terms

$$\frac{t\lambda}{r}\frac{d\theta}{dr}\cancel{(T_s-T_\infty)}+t\lambda\frac{d^2\theta}{dr^2}\cancel{(T_s-T_\infty)}-2h\theta\cancel{(T_s-T_\infty)}=0$$

$$=\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)-\frac{2h\theta}{\lambda t}=0$$

$$\boxed{\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)-\frac{\theta}{\delta^2}=0} \qquad \boxed{\delta^2=\frac{\lambda t}{2h}}$$

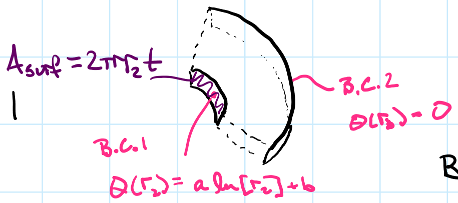
The non-dimensional temperature function in part b is not the same as in part a, as evidenced by the different function given in the problem. Thus, the non-dimension denominator can be defined in terms of T_2 rather than T_s.

the problem. Thus, the non-dimension denominator can be defined in terms of T_2 rather than T_s.

$$\boxed{\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right) - \frac{\theta}{\delta^2} = 0} \qquad \boxed{\delta^2 = \frac{\lambda t}{2h}}$$

2bii)

B.C. 1



$$\Theta = \frac{T - T_\infty}{T_2 - T_\infty}$$

$$T_2 = \Theta(r_2)(T_{in} - T_\infty)$$

$$T_2 = (a \ln r_2 + b)(T_{in} - T_\infty)$$

BC1 $\Theta(r = r_2) = c e^{r_2/\delta} + d e^{-r_2/\delta} = \Theta(r_2) = \Theta_2$

BC2 $\Theta(r = r_3) = c e^{r_3/\delta} + d e^{-r_3/\delta} = 0$

From BC2 $c = -d e^{-r_3/\delta} e^{-r_3/\delta} = \underline{-d e^{-2r_3/\delta}}$

into BC1 $-d e^{-2r_3/\delta} e^{r_2/\delta} + d e^{-r_2/\delta} = \Theta(r_2)$

$$\boxed{d = \Theta(r_2)\left\{e^{-r_2/\delta} - e^{(r_2 - 2r_3)/\delta}\right\}^{-1}}$$

$$c = \overbrace{-\Theta(r_2)\left\{e^{-r_2/\delta} - e^{(r_2-2r_3)/\delta}\right\}^{-1}}^{d} e^{-2r_3/\delta}$$

$$\boxed{c = \Theta(r_2)\left\{e^{r_2/\delta} - e^{(2r_3 - r_2)/\delta}\right\}^{-1}}$$

2biii)

$$\eta = \frac{\dot{Q}_{fin}}{h A_{fin}(T(r_2) - T_\infty)}$$

$$\dot{Q}_{fin} = -\underbrace{2\pi r_2 t}_{A_{surf}} \lambda \left.\frac{dT}{dr}\right|_{r=r_2}$$

$$\Theta = \frac{T - T_\infty}{T_2 - T_\infty}$$

$$\frac{d\Theta}{dr} = (T_2 - T_\infty)^{-1}\frac{dT}{dr}$$

$$\therefore \frac{dT}{dr} = (T_2 - T_\infty)\frac{d\Theta}{dr} = (T_2 - T_\infty)\left(\frac{c}{\delta} e^{r/\delta} - \frac{d}{\delta} e^{-r/\delta}\right)$$

$$\eta = -\frac{2 \cdot 2\cancel{\pi} r_2 \overset{\delta^2}{t\lambda} \cancel{(T_2 - T_\infty)} \left.\frac{d\Theta}{dr}\right|_{r_2}}{2h(2\cancel{\pi}(r_3^2 - r_2^2))\cancel{(T_2 - T_\infty)}} = \frac{-2\delta^2}{(r_3^2 - r_2^2)} r_2 \left.\frac{d\Theta}{dr}\right|_{r_2}$$

fin top & bottom

$$\eta = \frac{-2\cancel{\delta^2}}{100\cancel{\delta^2} - 25\cancel{\delta^2}} r_2 \left.\left(\frac{c}{\delta} e^{r/\delta} - \frac{d}{\delta} e^{-r/\delta}\right)\right|_{r_2 = 5\delta}$$

$r_2 = 5\delta$ $\qquad$ $e^{-10} \ll e^0$

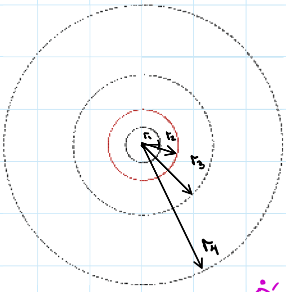
$$= -\frac{2 \cdot 5\cancel{\delta}}{75}\left(\frac{c}{\cancel{\delta}} e^5 - \frac{d}{\cancel{\delta}} e^{-5}\right) = -\frac{10}{75}\left(\cancel{-e^{-15} e^5} - e^5 e^{-5}\right)$$

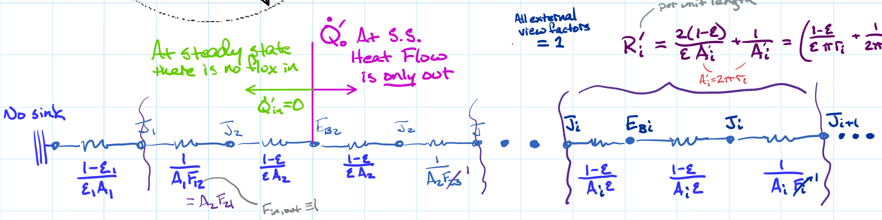
$$\boxed{\eta = \frac{10}{75} = 13\%}$$

Q3 Radiation 2023

04 January 2023 22:02

3ai)





per unit length

$$R'_i = \frac{2(1-\varepsilon)}{\varepsilon A_i} + \frac{1}{A_i} = \left(\frac{1-\varepsilon}{\varepsilon\pi r_i} + \frac{1}{2\pi r_i}\right) = \frac{2(1-\varepsilon)+\varepsilon}{2\pi r_i\varepsilon}$$

$A'_i = 2\pi r_i$

$$= \boxed{\frac{2-\varepsilon}{2\pi r_i\varepsilon} = R'_i}$$

$$R'_{eq} = \frac{1-\varepsilon}{\varepsilon 2\pi r_2} + \frac{1\cdot\varepsilon}{2\pi r_2\cdot\varepsilon} + \sum_{i=3}^{N}\frac{2-\varepsilon}{2\pi r_i\varepsilon}$$

$$\boxed{R'_{eq} = \frac{1}{\varepsilon 2\pi r_2} + \frac{2-\varepsilon}{2\pi\varepsilon}\sum_{i=3}^{N}\frac{1}{r_i}}$$

3aii)

$$R'_{N+2} = \frac{1}{\varepsilon\, 2\pi r_2} + \frac{2-\varepsilon}{2\pi\varepsilon}\sum_{i=3}^{N+2}(2r_{i-1})^{-1} = \frac{1}{2\pi\varepsilon r_2} + \frac{2-\varepsilon}{2\pi\varepsilon}\left(\underbrace{\frac{1}{2r_2}}_{r_3} + \underbrace{\frac{1}{4r_2}}_{r_4} + \underbrace{\frac{1}{8r_2}}_{r_5} \cdots \frac{1}{2^i r_2} + \cdots \frac{1}{2^{N+2-2}r_2}\right)$$

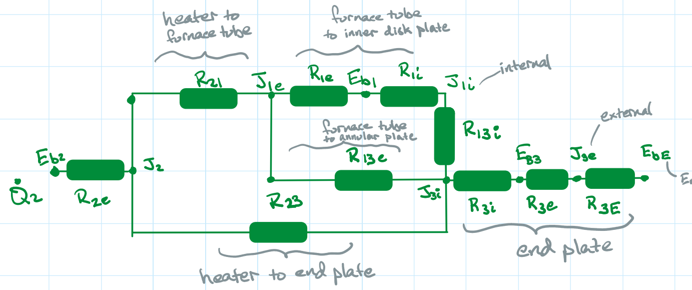
$$\boxed{R'_{N+2} = \frac{1}{2\pi\varepsilon r_2} + \frac{2-\varepsilon}{2\pi\varepsilon r_2}\left(\frac{1}{2}+\frac{1}{4}+\cdots\frac{1}{2^N}\right)}$$

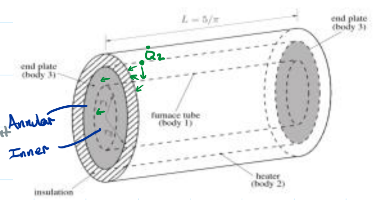
As $N\to\infty$ $\left(\frac{1}{2}+\frac{1}{4}+\cdots\frac{1}{2^N}\right) \Rightarrow 1$

convergant series powers of 2

$$\boxed{R'_{N=\infty} = \frac{3-\varepsilon}{2\pi\varepsilon r_2}}$$

3bi)





3bii)

$$Req = \cancel{R_{2e}}^{\frac{1-\cancel{\varepsilon_2}^{\,0}}{\varepsilon_2 A_2}} + \left[R_{23}^{-1} + \left\{R_{21} + \overbrace{\left(R_{13e}^{-1} + \left(R_{1e}+R_{1i}+R_{13i}\right)^{-1}\right)^{-1}}^{R_{BEQ}}\right\}^{-1}\right]^{-1} + R_{3i} + R_{3e} + R_{3\infty}$$

$$A_2 = 2\pi(1\,m)\left(\tfrac{5}{\pi}m\right) = 10\,m^2 \qquad A_1 = 2\pi\left(\tfrac{1}{2}m\right)\left(\tfrac{5}{\pi}m\right) = 5\,m^2$$

$[m^{-2}]$

$$R_{23} = \frac{1}{A_2 2F_{23}} = \frac{1}{10\cdot 0.3} = \frac{1}{3}$$

2 end plates

$$R_{21} = \frac{1}{A_2F_{21}} = \frac{1}{A_1F_{12}} = \frac{1}{5\cdot 4/5} = \frac{1}{4}$$

$$R_{13e} = \frac{1}{A_{1e}F_{1e3}} = \frac{1}{5\cdot 2/10} = 1$$

$$R_{13i} = \frac{1}{A_{1i}F_{1i3}} = \frac{1}{5\ \ 1/10} = 2$$

$$R_{1i} = R_{1e} = \frac{1-\varepsilon_1}{A_1\varepsilon_1} = \frac{1-1/2}{5\cdot 1/2} = 1/5$$

$$R_{3i} = R_{3e} = \frac{1-\varepsilon_3}{A_3\varepsilon_3} = \frac{1-2/10}{\pi(1/2)^2\, 2/10} = \frac{8/10}{\pi/20} = \frac{16}{\pi}$$

$$R_{3E} = \frac{1}{A_3F_{3E}} = \frac{1}{\pi/4\cdot 1} = \frac{4}{\pi}$$

$$Req = \left[R_{23}^{-1} + \left\{R_{21} + \overbrace{\left(R_{13e}^{-1} + \left(R_{1e}+R_{1i}+R_{B3i}\right)^{-1}\right)^{-1}}^{1\ MEQ}\right\}^{-1}\right]^{-1} + R_{3i}+R_{3e}+R_{3E} = \boxed{11.7\,m^{-2}}$$

$$\left[3 + \left\{\frac{1}{4} + \left(1 + \left(\frac{1}{5}+\frac{1}{5}+2\right)^{-1}\right)^{-1}\right\}^{-1}\right]^{-1} + \frac{16}{\pi} + \frac{16}{\pi} + \frac{4}{\pi}$$

$\frac{12}{17} \sim 0.7$

$3 + \frac{68}{65} \sim 1.05$

$65/263 \sim 0.247$

$+\ 36/\pi \sim 11.46$

$5.7\cdot 10^4$ $\sim 10^{12}$ $\sim 10^{10}$ $10^{12} - 8\cdot 10^9$

order $\sim \frac{5.7\cdot 10^4}{5.8\cdot 10^1}\frac{W}{m^{-2}} \sim 10^5\,W$

$$\dot{Q}_2 = \frac{\sigma\left(T_2^4 - T_E^4\right)}{R_{EQ}} = \frac{5.67\cdot 10^{-8}\frac{W}{m^2K^4}\left((10^3)^4 - (3\cdot 10^2)^4\right)K^4}{11.7\,m^{-2}} = \boxed{\begin{array}{l}4{,}804\ W\\ \sim 4.8\,kW\end{array} = \dot{Q}_2}$$

3biii)

Film with specular performance

Wein's Law $\quad \lambda^* T = C_3 \quad C = 2{,}897.8 \ \mu m\,K$

λ^*: Max of radiation

$F_{(0-\lambda_{max})} = 0.25$

Determine T_3 to find λ_3^*

$\dot{Q}_2 = \dot{Q}_3 \rightarrow E_{B3} \overset{R_{3e}}{—} J_{3e} \overset{R_{3E}}{—} E_{bE}$

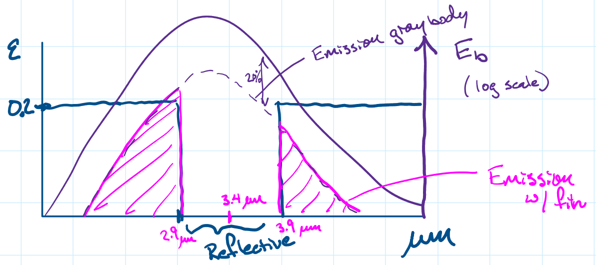
$\therefore \quad \dot{Q}_3 = \dfrac{\sigma\left(T_3^4 - T_E^4\right)}{R_{B3E}}$

$$R_{B3E} = R_{3e} + R_{3E}$$
$$= \frac{16}{\pi} + \frac{4}{\pi} = \frac{20}{\pi}$$

$$T_3^4 = T_E^4 + \frac{\dot{Q}_3 R_{B3E}}{\sigma}$$
$$T_3^4 = \underbrace{81 \cdot 10^8}_{\text{can neglect}} + \underbrace{\frac{4.8 \cdot 10^3 \cdot 20/\pi}{5.67 \cdot 10^{-8}}}_{5 \cdot 10^{11}\,K^4} K^4$$

≈ 6

$$\boxed{T_3 = 860\,K}$$



$$\lambda_{MAX} T_3 = \frac{C_3}{T_3} = \frac{2898\,K\mu m}{860\,K} = \boxed{3.4\,\mu m = \lambda_{mid}}$$

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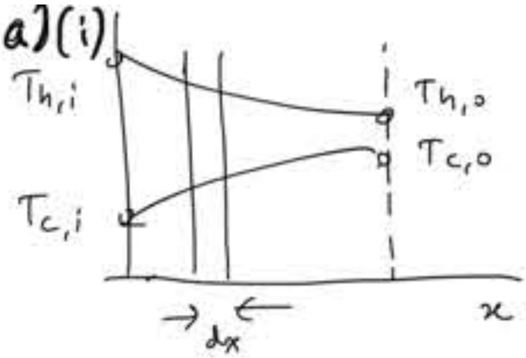
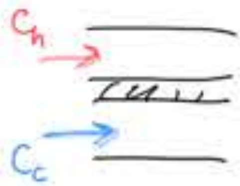
2023 Q4 HX

(a)(i)



co - flow HX



$$\delta\dot{Q} = U\,dA\,(T_h - T_c) = -\overbrace{\dot{m}_h c_h}^{C_h}\,dT_h = \overbrace{\dot{m}_c c_c}^{C_c}\,dT_c$$

$$d(T_h - T_c) = -\frac{\delta Q}{C_h} - \frac{\delta Q}{C_c} = -\delta Q\left(\frac{1}{C_c} + \frac{1}{C_h}\right)$$

$$= -U\,(T_h - T_c)\,dA\left(\frac{1}{C_c} + \frac{1}{C_h}\right)$$

$$d\ln(T_h - T_c) = -UA\left(\frac{1}{C_c} + \frac{1}{C_h}\right)$$

$$\ln\frac{(T_h - T_c)_o}{(T_h - T_c)_i} = -UA\left(\overbrace{\frac{1}{C_h} + \frac{1}{C_c}}^{1/C_{\parallel}}\right)$$

$$\boxed{\frac{\Delta T_o}{\Delta T_i} = \exp\left(-\frac{UA}{C_{\parallel}}\right)}$$ QED

(a)(ii)

$$\dot{Q} = \int_i^o U\,(T_h - T_c)\, dA =$$

$$= \int_i^o U\,\Delta T_i \exp\left(-\frac{UA}{C_{||}}\right) dA =$$

$$= -C_{||}\,\Delta T_i \exp\left(-\frac{UA}{C_{||}}\right)\Bigg|_i^o$$

$$= -C_{||}\,\Delta T_i\left[\exp\left(-\frac{UA}{C_{||}}\right) - 1\right] \quad \{A(0)=0\}$$

$$\dot{Q}_{||} = C_{||}\Delta T_i\left[1 - \exp\left(-\frac{UA}{C_{||}}\right)\right]$$

$$\boxed{\frac{\dot{Q}}{UA\Delta T_i} = \frac{C_{||}}{UA}\left[1 - \exp\left(-\frac{UA}{C_{||}}\right)\right]} \quad \text{QED}$$

ALTERNATIVELY:

$$(T_h - T_c)_o = \overbrace{(T_{h,o} - T_{h,i})}^{-\dot{Q}/C_{||}} + \overbrace{(T_{h,i} - T_{c,i})}^{\Delta T_i} + \overbrace{(T_{c,i} - T_{c,o})}^{-\dot{Q}/C_c}$$

$$= \Delta T_i \exp\left(-UA/C_{||}\right)$$

$$\therefore\ -\frac{Q_{||}}{C_{||}} = -\Delta T_i + \Delta T_i \exp\left(-UA/C_{||}\right)$$

$$Q_{||} = C_{||}\,\Delta T_i\left(1 - \exp\left(-UA/C_{||}\right)\right) \quad \ldots \text{etc.}$$

— · —

(b) COUNTERFLOW HX

NOTE THE SIGN ≠

(i) $\delta \dot{Q} = - C_h \, dT_h = - C_c \, dT_c$

$$d(T_h - T_c) = -\delta \dot{Q} \left(\frac{1}{C_h} - \frac{1}{C_c} \right) = -\frac{U}{C_x} (T_h - T_c) \, dA$$

where $\boxed{C_x^{-1} = C_h^{-1} - C_c^{-1}}$

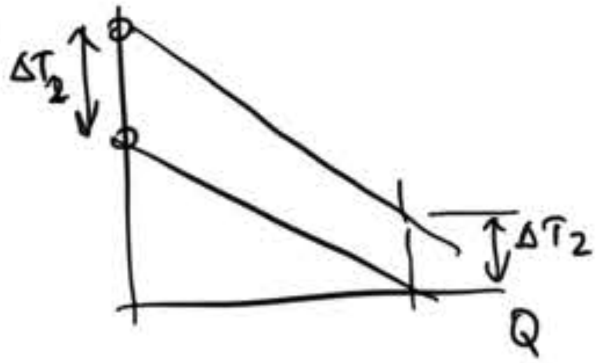
$$d \ln (T_h - T_c) = -UA / C_x$$

$$\ln \frac{(T_h - T_c)_2}{(T_h - T_c)_1} = -UA / C_x$$

$$\boxed{\Delta T_2 = \Delta T_1 \exp\left(-UA / C_x\right)}$$

QED

(ii)



$$d \Delta T = -\frac{\delta Q}{C_x}$$

$$(T_h - T_c)_2 = (T_{h,o} - T_{c,i}) = (T_{h,o} - T_{h,i}) + (T_{h,i} - T_{c,i})$$

$$= -\frac{\dot{Q}_x}{C_h} + \Delta T_i$$

$$(T_h - T_c)_1 = (T_{h,i} - T_{c,o}) = (T_{h,i} - T_{c,i}) + (T_{c,i} - T_{c,o})$$

$$= \Delta T_i - \frac{Q_x}{C_c}$$

$$\Delta T_i - \frac{\dot{Q}_x}{C_h} = \left(\Delta T_i - \frac{\dot{Q}_x}{C_c}\right) \exp\left(-\frac{UA}{C_x}\right)$$

$$\dot{Q}_x\left[\frac{1}{C_h} - \frac{1}{C_c}\exp\left(-\frac{UA}{C_x}\right)\right] = \Delta T_i\left(1-\exp\left(\frac{-UA}{C_x}\right)\right)$$

$$\boxed{\frac{\dot{Q}_x}{UA\,\Delta T_i} = \left(\frac{C_x}{UA}\right)\left[\frac{1-\exp(-UA/C_x)}{\frac{C_x}{C_h} - \frac{C_x}{C_c}\exp\left(-\frac{UA}{C_x}\right)}\right]}$$

QED

(c) For $C_h \ll C_c$: $C_{11} = C_h$; $C_x = C_h$

$$\frac{1}{C_{11}} = \cancel{\frac{1}{C_c}} + \frac{1}{C_h} \qquad \frac{1}{C_x} = \frac{1}{C_h} - \cancel{\frac{1}{C_c}}$$

$$\dot{Q}_{11} = C_h\,\Delta T_i\left(1-\exp\left(-UA/C_h\right)\right)$$

$$\dot{Q}_x = C_h\,\Delta T_i\left(1-\exp\left(-UA/C_h\right)\right)$$

$$\dot{Q}_{11} \approx \dot{Q}_x$$