Version SH/9

EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 9 May 2023 2 to 3.40

Module 3A6

HEAT AND MASS TRANSFER

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A fluid with thermal diffusivity α flows through a two-dimensional channel of height 2δ, as shown in Fig. 1. The x-direction velocity is fully developed and has the profile

$$u = u_c\left\{1 - \left(\frac{y}{\delta}\right)^2\right\}$$

where y is the distance from the centreline and u_c is the velocity at $y = 0$. The heat fluxes at the top and bottom of the channel (q_+ and q_-, as shown) are uniform along x. It may be assumed that the streamwise temperature gradient $\partial T/\partial x$ is constant and that streamwise conduction is negligible. The fluid density ρ, specific heat capacity c and thermal conductivity λ are all constant.

(a) Starting from the steady flow relation $\boldsymbol{u}\cdot\nabla(\rho c T) = \nabla\cdot(\lambda\nabla T)$, show that the governing equation for temperature is

$$u_c\left\{1 - \left(\frac{y}{\delta}\right)^2\right\}\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial y^2}\ .$$ [20%]

(b) Obtain an expression for the variation of the transverse temperature gradient $\partial T/\partial y$ in terms of the heat fluxes, q_- and q_+, and other quantities defined above. [20%]

(c) Sketch the temperature and velocity profiles and find an expression for the difference between the top and bottom temperatures, $\Delta T = T(\delta) - T(-\delta)$. Explain your result. [30%]

(d) The bulk temperature is defined by

$$\bar{T} = \frac{1}{2\delta\bar{u}}\int_{-\delta}^{+\delta} uT\,\mathrm{d}y$$

where $\bar{u}$ is the average velocity across the channel. Show that $\bar{T}$ increases at a constant rate in the streamwise direction. Explain why this is the case. [30%]

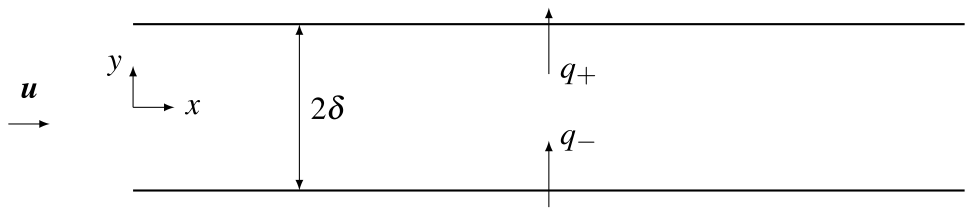

Fig. 1

(TURN OVER

2 Figure 2 shows a tubular condenser. It operates at a steady state, with its internal surface at constant temperature T_s throughout the tube. The tube wall has constant thermal conductivity λ and its inner and outer radii are r_1 and r_2 respectively. Heat transfer is enhanced by fins of thickness t and outer radius r_3, made from the same material as the tube. Heat is convected from the outer surfaces to the surroundings at temperature T_∞, with heat transfer coefficient h. Axial conduction may be neglected.

(a) Consider first the section of tube without fins.

(i) Show that, within the tube wall, the radial variation of temperature may be expressed as $\theta(r) = a \ln r + b$, where $\theta(r) = (T(r) - T_\infty)/\Delta T$ and $\Delta T = T_s - T_\infty$. [15%]

(ii) Noting the convective boundary at r_2, determine the constants a and b in terms of the above-defined quantities. Hence, or otherwise, determine an expression for the rate of heat transfer to the environment. [20%]

(b) Consider now heat transfer within and from the fins.

(i) Starting from first principles, and stating your assumptions, show that the governing equation for the radial temperature variation along a fin may be written in the form

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\theta}{\mathrm{d}r}\right) = \frac{\theta}{\delta^2} \tag{1}$$

and determine δ in terms of the given parameters. [25%]

(ii) The exact solution to equation (1) involves Bessel functions, but for large r/δ it may be approximated by $\theta(r) \approx c\mathrm{e}^{r/\delta} + d\mathrm{e}^{-r/\delta}$. Assuming this approximation is valid and that $T(r_3) \approx T_\infty$, determine the constants c and d in terms of the defined parameters and $\theta(r_2)$. [10%]

(iii) For a particular design, $r_3/\delta = 10$ and $r_2/\delta = 5$. Estimate the fin efficiency, given by

$$\eta_{\text{fin}} = \frac{\text{total heat transfer from the fin}}{\text{heat transfer from the same fin at uniform temperature } T(r_2)} \,. \quad [30\%]$$

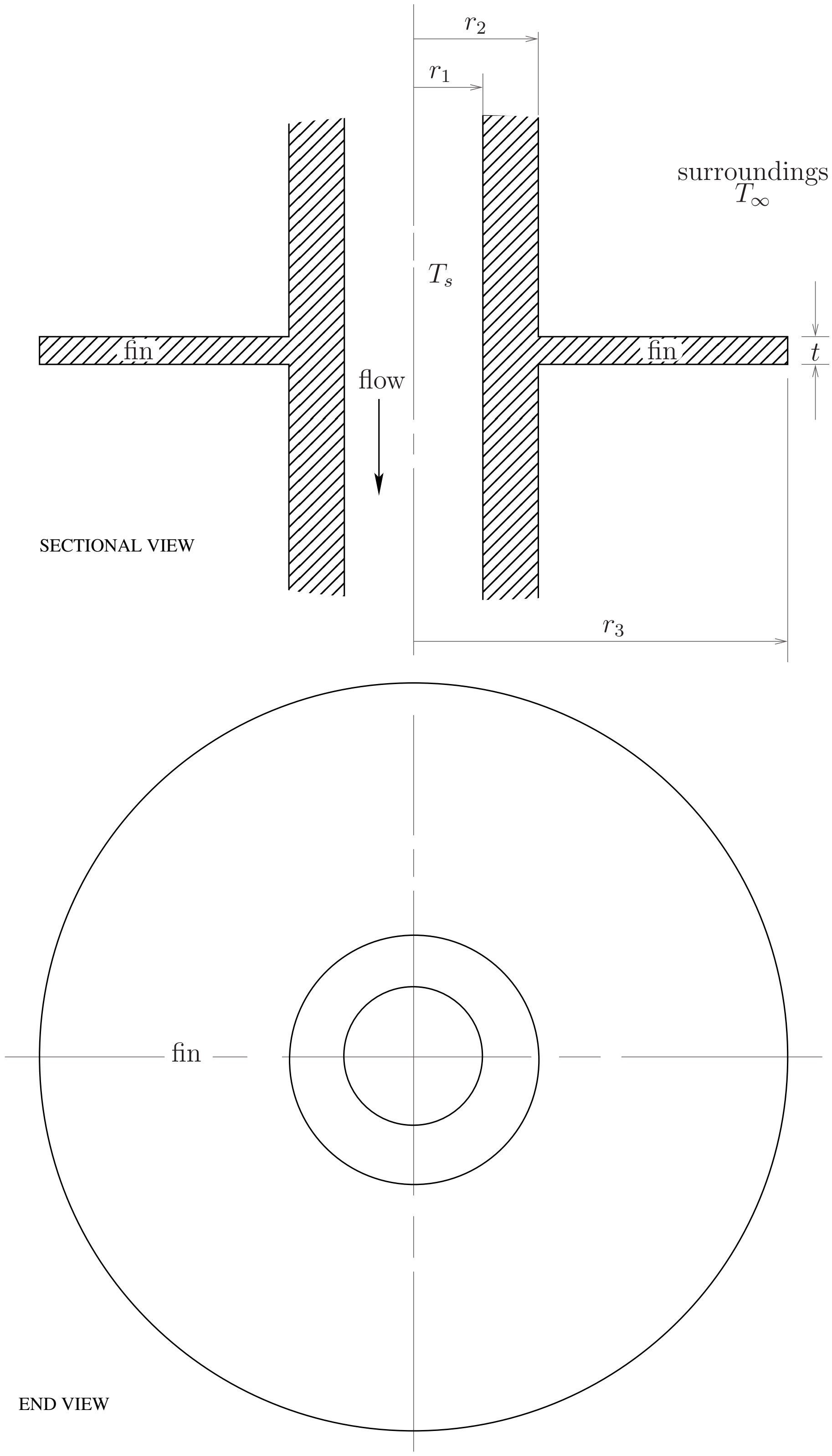
r_2
r_1
surroundings
T_∞
T_s
fin
fin
flow
t
SECTIONAL VIEW
r_3
fin
END VIEW

Fig. 2

3 *It may be assumed throughout this question that heat transfer is by radiation only.*

(a) A furnace comprises a circular tube of radius r_1 and a co-axial heater of radius r_2. In order to reduce heat losses to the environment the heater is surrounded by a series of N thin cylindrical radiation shields at radii r_3, r_4 ... r_{N+2}. The heater and both sides of each shield have emissivity ε. The furnace may be assumed of infinite length.

(i) Draw an equivalent resistor network to represent radiative heat transfer between the heater and the environment. Derive expressions for the resistance of the i-th shield and for the overall resistance between the heater and the environment. [25%]

(ii) Determine the overall resistance in terms of ε and r_2 for the case where $r_i = 2r_{i-1}$ $(i = 3, 4, \ldots N+2)$ and $N \to \infty$. [10%]

(b) In an alternative design the radiation shields are replaced by a layer of perfect insulation, but in this case the furnace is of finite length, $L = 5/\pi\,\text{m}$, as shown in Fig. 3. Heat is lost to the environment by radiation from the endplates, denoted body 3 and shaded grey in Fig. 3.

(i) Draw an equivalent circuit and identify the appropriate resistances. [20%]

(ii) The furnace tube and heater are of radii $r_1 = 0.5\,\text{m}$ and $r_2 = 1\,\text{m}$ respectively and the corresponding emissivities are $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 1$. The emissivity of the endplates is $\varepsilon_3 = 0.2$ on either side. Calculate the rate of heat supply required to maintain the heater at 1000 K if the environment temperature is 300 K. Take the view factors as $F_{12} = 0.8$, $F_{11} = 0.9$ and $F_{23} = 0.15$, where this last view factor is between the heater and a single endplate. [25%]

(iii) Losses from the endplates are to be reduced by applying a reflective film that has zero emissivity in the wavelength range $\lambda_{\text{mid}} \pm 0.5\,\mu\text{m}$. Outside this range it has no effect on the emissivity. Without further solution of the heat flow equations, estimate the best choice of λ_{mid}. Assume Wien's law, $\lambda^* = C/T$, for the wavelength λ^* at which the black-body radiation curve peaks, where $C = 2898\,\mu\text{m}\,\text{K}$ and T is the body temperature. [20%]

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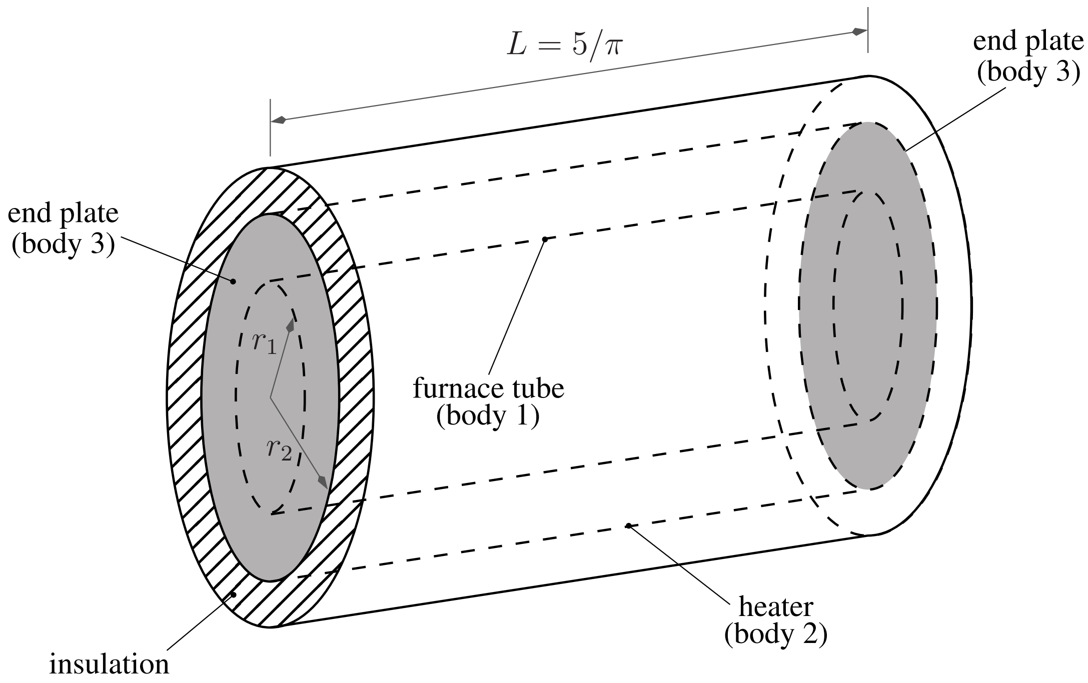

$L = 5/\pi$
end plate
(body 3)
end plate
(body 3)
r_1
r_2
furnace tube
(body 1)
heater
(body 2)
insulation

Fig. 3

4 A heat exchanger is to be designed to operate with fixed inlet temperatures of $T_{h,i}$ and $T_{c,i}$ for the hot and cold streams respectively. The corresponding heat capacities of the two streams are $C_h = \dot{m}_h c_h$ and $C_c = \dot{m}_c c_c$, where $\dot{m}$ is the mass flow rate and c is the isobaric specific heat capacity; subscripts h and c denote hot and cold streams respectively. The total heat-exchange area between the streams is A and the overall heat-transfer coefficient U may be assumed constant.

(a) Consider first a co-flow heat exchanger.

(i) Starting from a differential energy balance, show that the temperature differences at outlet (subscript o) and inlet (subscript i) are related by

$$\Delta T_o = \Delta T_i \exp(-UA/C_{\|})$$

where $\Delta T = T_h - T_c$ and $C_{\|}^{-1} = C_h^{-1} + C_c^{-1}$. [20%]

(ii) Show that the total rate of heat transfer $\dot{Q}_{\|}$ may be written in the dimensionless form

$$\frac{\dot{Q}_{\|}}{UA\Delta T_i} = \frac{C_{\|}}{UA}\left\{1 - \exp\left(\frac{-UA}{C_{\|}}\right)\right\}.$$ [15%]

(b) Now consider a counter-flow heat exchanger.

(i) Show that the temperature differences at the hot and cold ends, ΔT_2 and ΔT_1 respectively, are related by

$$\Delta T_2 = \Delta T_1 \exp(-UA/C_{\times})$$

and determine $C_{\times}$ in terms of C_h and C_c. [15%]

(ii) Sketch a *T-Q* (temperature vs. heat transfer) diagram and mark on it ΔT_1, ΔT_2, ΔT_i, and the quantities $\dot{Q}_{\times}/C_h$ and $\dot{Q}_{\times}/C_c$, where $\dot{Q}_{\times}$ is the total heat transfer rate. Hence show that

$$\frac{\dot{Q}_{\times}}{UA\Delta T_i} = \frac{C_{\times}}{UA}\left\{\frac{1 - \exp(-UA/C_{\times})}{(C_{\times}/C_c) - (C_{\times}/C_h)\exp(-UA/C_{\times})}\right\}.$$ [30%]

(c) Determine the ratio $\dot{Q}_{\|}/\dot{Q}_{\times}$ for the limiting case where $C_h \ll C_c$. Provide an interpretation of your result with reference to a *T-Q* diagram. [20%]

END OF PAPER

Answers

1

(a) -

(b) $\frac{\partial T}{\partial y} = \frac{u_c}{\alpha}\frac{\partial T}{\partial x} y \left(1 - \left(\frac{y}{\delta}\right)^2\right) - \frac{q_+ + q_-}{2\lambda}$

(c) $\Delta T = (q_+ + q_-)\frac{\delta}{\lambda}$. The temperature rise is constant in x, so the temperature difference across the channel must remain constant.

(d) Integration of the original governing equation yields:

$$\rho \bar{u} c_p \frac{\partial T}{\partial x} = -(q_+ - q_-)$$

2

(a)

(i) -

(ii) $a = \left(\frac{\lambda}{hr_2} + \ln r_2 r_1\right)^{-1}$, $b = 1 - \frac{\ln r_1}{\frac{\lambda}{hr_2} + \ln r_2 r_1}$

(b) -

(c)

(i) -

(ii) $c = \left(e^{r_1/\delta} - e^{-r_1/\delta} e^{2r_3/\delta}\right)^{-1}$, $d = -c\, e^{2r_3/\delta}$

(iii) $\eta = \frac{\lambda}{h\delta} \frac{e^{r_2/\delta} + e^{2r_3/\delta} e^{-r_2/\delta}}{e^{2r_3/\delta} e^{-r_2/\delta} - e^{r_2/\delta}}$

(d) $\eta \approx 0.26$

3

(a)

(i) -

(ii) $R_{\text{eq}} = \frac{1}{2\pi r_2}\frac{(3-\varepsilon)}{\varepsilon}$

(b)

(i) -

(ii) 4.8 kW

(iii) $\lambda_{\text{mid}} \approx 3.4\mu\text{m}$

4

(a)

(i) -

(ii) -

(b)

(i) $C_{\times} = \left(\frac{1}{C_h} - \frac{1}{C_c}\right)^{-1}$

(ii) -

(c) $\frac{\dot{Q}_{||}}{\dot{Q}_{\times}} \approx 1$. In this case, the heat transfer is determined by the available heat capacity C_h.