Version SH/5

EGT2
ENGINEERING TRIPOS PART IIA

Monday 6 May 2024 2 to 3.40

Module 3A6

HEAT AND MASS TRANSFER

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Clouds exchange radiative heat with the sun, space and earth. Consider a cloud in steady state, modelled as a thin flat disc for which the lower and upper surfaces have view factors $F_c = 1$ to earth and space, respectively. The earth is at temperature $T_e = 285$ K and radiates as a grey body with a spectrally-independent emissivity $\epsilon_e = 0.95$. Assume that for the area under the cloud the earth has no direct view of space. The sun emits as a black body at temperature $T_s = 5800$ K with an irradiation flux G_S=1000 (in W m^{-2}) incident on the clouds and earth. Space can be considered a black body at $T_\infty = 0$.

(a) Draw a diagram of the radiative interaction between the cloud and sun, neglecting any interaction with the earth. Label the solar irradiation G_S from the sun, transmitted radiation G_τ, reflected radiation G_ρ and the cloud emissive power E. Your labelling should include dependence on the transmissivity τ_c, reflectivity ρ_c, absorptivity α_c and emissivity ϵ_c of the cloud. Identify also the radiosity for the upper and lower cloud surfaces. [25%]

(b) Consider the radiative flux to earth across a cloud for which the reflectivity is zero for radiation emitted by the earth. Determine expressions for the incident radiative flux and the net radiative flux on the earth beneath the cloud. [20%]

(c) Consider a thin cloud at temperature $T_c = 240$ K with emissivity $\epsilon_c = 0.95$. Its transmissivity is $\tau_c = 0.95$ for solar irradiation, but it is opaque to infrared radiation and has zero reflectivity at all wavelengths. All emitted radiation from earth and cloud may be considered infrared. Calculate the net radiative heat flux to the earth, both with and without the cloud. [25%]

(d) Thick clouds have $\epsilon_c = 0.95$ and are opaque at all wavelengths. Consider the surface of the earth at night, with no incident radiation, and assume that thick clouds cover half of that surface. The clouds and uncovered earth both exchange radiative heat with space. Draw a resistor network to represent the heat transfer process and determine the equivalent resistances. Calculate the radiative heat flux from the earth at night. [30%]

2 A hot-wire anemometer is composed of a thin cylindrical wire, of length $2L$ and diameter D, fixed at its ends. An electric current is supplied to the wire, providing a constant and uniform heat production rate $\dot{g}$ per unit volume of wire. Air at ambient temperature T_o flows perpendicular to the wire, the ends of which are also at T_o. The thermal conductivities of the wire and the air are 100 and 0.03 $\mathrm{W\,m^{-1}K^{-1}}$ respectively. The Prandtl number for air is Pr = 0.7 and the Nusselt number for a cylinder in cross flow is given by

$$\mathrm{Nu} = (0.35 + 0.67\,\mathrm{Re}^{0.52})\,\mathrm{Pr}^{0.3}$$

where Re is the Reynolds number based on the wire diameter. Radiation may be neglected.

(a) A typical hot wire has a diameter of a few μm and is used to measure velocities of a few $\mathrm{m\,s^{-1}}$. Show that the wire temperature, T, may be treated as radially uniform. [15%]

(b) Show that the temperature variation along the wire is governed by

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} - m^2\theta = B$$

where x is the distance from the wire centre and $\theta = T - T_o$. Determine B and m in terms of the given parameters. [25%]

(c) Show that the general solution to the equation of part (b) is

$$\theta = C_1 e^{mx} + C_2 e^{-mx} + C_3$$

Determine values for C_1, C_2 and C_3 in terms of m, B and L. Hence obtain the complete solution for θ. Sketch curves of this solution, indicating how it changes with $\dot{g}$ and mL as a function of x/L. [35%]

(d) Using the governing equation given in part (b), or otherwise, show that the total heat generation rate balances the total rate of heat loss from the wire. [25%]

3 Figure 1 shows the plan view of a building of width W and height H subjected to a cross wind with incoming free-stream velocity U. A geometrically similar model is built with length-scale ratio $r_L = H/H_m$ (where subscript m refers to the model) and is tested in a wind tunnel to determine wind forces and heat transfer rates. The measured force on the model, F_m, increases by a factor of 8 when the test air speed, U_m, is increased by a factor of 4.

(a) The drag coefficient is known to vary according to

$$C_D \propto \mathrm{Re}^n$$

where Re is the Reynolds number based on W. Determine the index n. [20%]

(b) Assuming a relation similar to the Reynolds analogy applies – i.e.,

$$\mathrm{St} = \frac{C_D}{2}$$

where St is the Stanton number – determine the form of the dependence of Nusselt number on Reynolds number. [10%]

(c) In a particular model test, $W_m = 0.5$ m, $H_m = 0.8$ m and $U_m = 8$ m s^{-1}, and $r_L = 8$. The measured force on the model is then 7.7 N. Estimate the force on the full-size building if the wind speed is 10 m s^{-1}. Estimate also the heat loss from the building when the temperature difference between its surface and the oncoming air is 15°C. Evaluate air properties at 0°C and 1 bar for both the model and the real flow. [40%]

(d) For the case shown in Fig. 1, sketch the expected rate of heat transfer at the surface as a function of distance along the surface ABC. Justify the shape of your curve, identifying any relevant features. How would the curve change as U increases. [20%]

(e) Discuss briefly the possibility of undertaking the tests in water, highlighting any additional factors that would need to be considered. [10%]

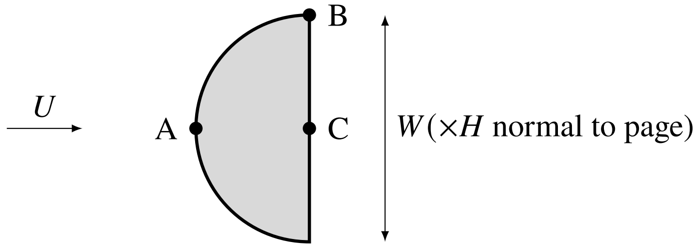

Fig. 1

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4 A spherical water droplet of initial radius R_0 and surface temperature T_s is suspended in stagnant humid air of density ρ. Mass transfer between the droplet and surrounding air may be considered a quasi-steady process such that conservation of water vapour within the humid air is governed by

$$\boldsymbol{u} \cdot \nabla(\rho Y) = \nabla \cdot (D\nabla\{\rho Y\})$$

where $\boldsymbol{u}$ is the convection velocity, Y is the mass fraction of water vapour and D is the diffusion coefficient.

(a) Assuming spherical symmetry, show that the evaporation rate for a droplet of radius R is given by

$$\dot{m} = 4\pi\rho D R \ln\left(\frac{1 - Y_\infty}{1 - Y_s}\right)$$

where Y_s and Y_∞ are the mass fractions of water vapour at the droplet surface and far from the droplet respectively. [35%]

(b) Determine an expression for the rate of change of R^2 in terms of the given parameters and the liquid density, ρ_l. Sketch the resulting variation of R with time, and the corresponding mass transfer rate for the case where T_s is constant. [30%]

(c) Assume now that T_s and the corresponding water saturation pressure, p_{sat}, vary according to

$$\frac{T_s - T_{s,0}}{T_\infty - T_{s,0}} = 1 - e^{-t/\tau} \qquad \text{and} \qquad \frac{p_{\text{sat}}(T_s) - p_{\text{sat}}(T_\infty)}{p_a} = K\frac{T_s - T_\infty}{T_\infty}$$

where τ, is a time constant, p_a is ambient pressure and the subscripts 0 and ∞ denote initial and far-field values respectively. Assume also that $Y \ll 1$ everywhere, such that the molecular weight of the air is approximately that of dry air.

Obtain an expression for $\mathrm{d}R^2/\mathrm{d}t$ as a function of the given quantities and the molecular weights of dry air and water vapour, W_a and W_v respectively. Comment on the nature of the mass transfer in this case. [35%]

END OF PAPER

Answers

1 (a) - , (b) - , (c) 717 $\mathrm{W\,m^{-2}}$ (with clouds) and 645 $\mathrm{W\,m^{-2}}$ (without clouds).

2 (a) - , (b) - , (c) -, (d) - .

3 (a) $n = -0.5$, (b) $\mathrm{Nu}_L \propto \mathrm{Re}_L^0.5$, (c) $F = 243$ N, $Q = 364$ kW, (d) - , (e) -

4 (a) - , (b) - , (c) -, (d) - .