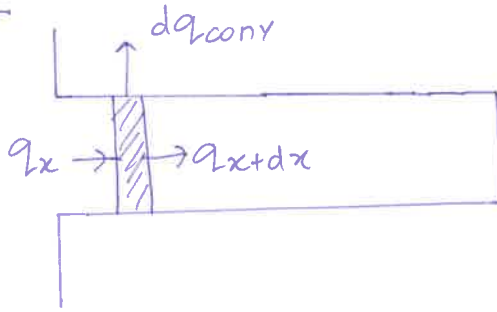


Question 1.

(a)



Balance the flux over the control area

$$q_x = q_{x+dx} + dq_{conv}$$

$$q_x = -\lambda A \frac{dT}{dx}$$

$$\begin{aligned} q_{x+dx} &= -\lambda A \left. \frac{dT}{dx} \right|_{x+dx} \\ &= -\lambda A \left(\frac{dT}{dx} + \frac{d^2T}{dx^2} dx \right) \end{aligned}$$

$$dq_{conv} = hP(T - T_\infty) dx$$

$$\therefore -\lambda A \frac{dT}{dx} = -\lambda A \left(\frac{dT}{dx} + \frac{d^2T}{dx^2} dx \right) + hP(T - T_\infty) dx$$

This simplifies to :

$$0 = -\frac{d^2T}{dx^2} + \frac{hP}{\lambda A} (T - T_\infty)$$

$$\frac{d^2T}{dx^2} - \frac{hP}{\lambda A} (T - T_\infty) = 0 //$$

Boundary conditions

$$x=0 \quad T=T_0$$

$$x=L \quad T=T_L$$

(b) Introduce $\theta = T - T_\infty$

$$\frac{d^2 T}{dx^2} - \frac{hP}{\lambda A} (T - T_\infty) = 0$$

Subst $T = \theta + T_\infty$ to equation above

$$\frac{d^2 \theta}{dx^2} + \cancel{\frac{d^2 T_\infty}{dx^2}} - \frac{hP}{\lambda A} \theta = 0$$

constant

$$\frac{d^2 \theta}{dx^2} - \frac{hP}{\lambda A} \theta = 0$$

$$\theta = C_1 \cosh(mx) + C_2 \sinh(mx)$$

$$\frac{d\theta}{dx} = C_1 m \sinh(mx) + C_2 m \cosh(mx)$$

$$\frac{d^2 \theta}{dx^2} = C_1 m^2 \cosh(mx) + C_2 m^2 \sinh(mx)$$

$$\frac{hP}{\lambda A} \theta = \frac{hP}{\lambda A} (C_1 \cosh(mx) + C_2 \sinh(mx))$$

$$\therefore \frac{d^2 \theta}{dx^2} - \frac{hP}{\lambda A} \theta = C_1 m^2 \cosh(mx) + C_2 m^2 \sinh(mx) - \frac{hP}{\lambda A} C_1 \cosh(mx) - \frac{hP}{\lambda A} C_2 \sinh(mx)$$

This equates to zero if $m^2 = \frac{hP}{\lambda A}$

$$\therefore m = \sqrt{\frac{hP}{\lambda A}}$$

$\therefore \theta = C_1 \cosh(mx) + C_2 \sinh(mx)$ is a solution

BC 1 $x=0 \quad T=T_0$
 $\theta_0 = T_0 - T_\infty$

$$\theta_0 = C_1 \cosh(0) + C_2 \sinh(0)$$

$$C_1 = \theta_0$$

using BC2 $x=L$

$$T_L - T_\infty = \theta_L$$

$$\theta_L = C_1 \cosh(mL) + C_2 \sinh(mL)$$

$$\theta_L = \theta_0 \cosh(mL) + C_2 \sinh(mL)$$

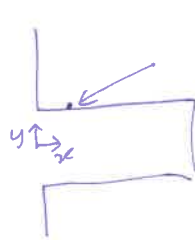
$$C_2 = \frac{\theta_L - \theta_0 \cosh(mL)}{\sinh(mL)}$$

$$\therefore \theta = \theta_0 \cosh(mx) + \frac{\theta_L - \theta_0 \cosh(mL)}{\sinh(mL)} \sinh(mx)$$

(c) variation of T along y -coordinate can be ignored for $Bi \ll 1$

or Temp variation over y -coordinate is smaller than the difference between top of the fin and surrounding.

one can do an analysis on the boundary condition at the top of the fin.



$$-\lambda \frac{dT}{dy}(x, y=w) = h(T(x, y=w) - T_\infty)$$

$$\frac{T(x, y=w) - T(x, y=0)}{w} \sim -\frac{h}{\lambda} (T(x, y=w) - T_\infty)$$

$$-\left(\frac{T(x, y=w) - T(x, y=0)}{T(x, y=w) - T_\infty} \right) \sim \frac{hw}{\lambda}$$

$$\frac{hw}{\lambda} \ll 1$$

$$(d) \quad Q = -\lambda A \left. \frac{d\theta}{dx} \right|_{x=0}$$

$$\frac{d\theta}{dx} = \theta_0 m \sinh(mx) + m \frac{\theta_L - \theta_0 \cosh(mL)}{\sinh(mL)} \cosh(mx)$$

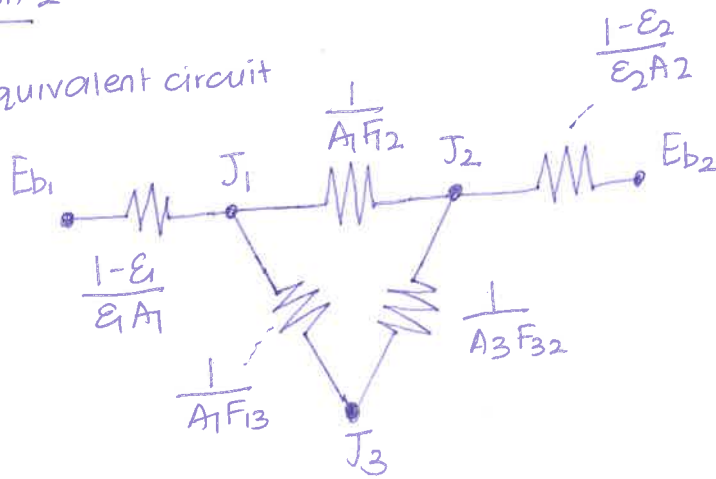
$$Q = -\lambda A \left. \frac{d\theta}{dx} \right|_{x=0} = -\lambda A m \frac{\theta_L - \theta_0 \cosh(mL)}{\sinh(mL)} //$$

(e) Efficiency

$$\epsilon_f = \frac{Q_f}{hPL\theta_0} = \frac{-\lambda A m}{hPL\theta_0} \frac{\theta_L - \theta_0 \cosh(mL)}{\sinh(mL)} //$$

Question 2

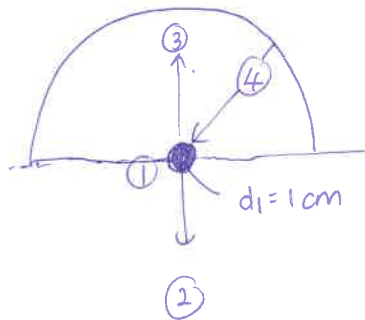
(a) Equivalent circuit



$$\epsilon_1 = 0.80$$

$$\epsilon_2 = 1$$

(b) View Factors



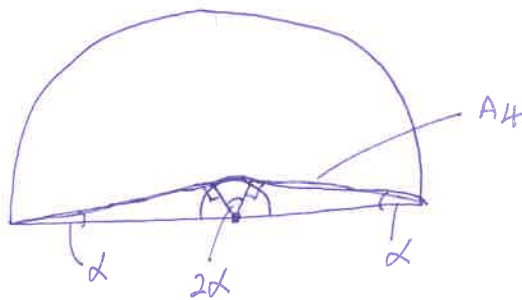
$$F_{12} = 0.50$$

$$F_{13} = 0.50$$

$$F_{43} = 1.0$$

$$A_1 = \pi (0.01) = 0.03$$

$$A_3 = \pi (0.10) = 0.31$$



$$\sin \alpha = \frac{1}{10}$$

$$\alpha = 5.74^\circ$$

$$2\alpha = 11.48^\circ$$

$$A_4 = 2 \times (0.10 \cos 5.74^\circ) + \pi (0.01) \left(\frac{11.48}{360} \right)$$

$$= 0.2$$

$$F_{34} A_3 = A_4 F_{43}$$

$$F_{34} = \frac{0.20 (1.0)}{0.31} = 0.6366$$

$$F_{31} A_3 = F_{13} A_1$$

$$F_{31} = \frac{0.50 (0.03)}{0.31} = 0.05$$

$$F_{34} = F_{31} + F_{32}$$

$$\therefore F_{32} = 0.6366 - 0.05 = 0.5866$$

(c) R_{eff}

Find the R contribution from the parallel part of the circuit

$$\frac{1}{R_{\text{par}}} = \frac{1}{\frac{1}{A_1 F_{12}}} + \frac{1}{\frac{1}{A_1 F_{13}} + \frac{1}{A_3 F_{32}}}$$

$$\frac{1}{R_{\text{par}}} = A_1 F_{12} + \frac{A_1 F_{13} A_3 F_{32}}{A_3 F_{32} + A_1 F_{13}}$$

$$\frac{1}{R_{\text{par}}} = \frac{A_1 F_{12} (A_3 F_{32} + A_1 F_{13}) + A_1 F_{13} A_3 F_{32}}{A_3 F_{32} + A_1 F_{13}}$$

$$R_{\text{par}} = \frac{A_3 F_{32} + A_1 F_{13}}{A_1 F_{12} (A_3 F_{32} + A_1 F_{13}) + A_1 F_{13} A_3 F_{32}}$$

$$R_{\text{tot}} = \frac{1 - \epsilon_1}{\epsilon_1 A_1} + \frac{1 - \epsilon_2}{\epsilon_2 A_2} + \frac{A_3 F_{32} + A_1 F_{13}}{A_1 F_{12} (A_3 F_{32} + A_1 F_{13}) + A_1 F_{13} A_3 F_{32}}$$

Plug in values

$$R_{\text{tot}} = \frac{1 - 0.8}{0.8 (0.03)} + 0 + 33.13 = 41.09$$

(d) Radiative Heat loss

$$E_{b1} = \sigma T_{b1}^4$$

$$= 5.669 \times 10^{-8} (1073)^4$$

$$= 75146 \text{ W/m}^2$$

$$E_{b2} = \sigma T_{b2}^4$$

$$= 5.669 \times 10^{-8} (298)^4$$

$$= 447 \text{ W/m}^2$$

$$q = \frac{75146 - 447}{41.09} = 1818 \text{ W/m}$$

(e) Reflector removed

$$q = \frac{E_{b1} - E_{b2}}{\frac{1}{\epsilon_1 A_1}} = \frac{75146 - 447}{1/0.80(0.03)}$$
$$= 1877 \text{ W/m}$$

More heat is lost if reflector is removed

Question 3

Non Newtonian fluid - power Law.

$$\tau_{rz} = -m \left(\frac{dv_z}{dr} \right)^n$$

$$(i) \quad \frac{1}{r} \frac{d}{dr} (r \tau_{rz}) = \frac{dP}{dz}$$

$$r \tau_{rz} = \frac{dP}{dz} \frac{r^2}{2} + C_1$$

$$\tau_{rz} = \frac{dP}{dz} \frac{r}{2} + \frac{C_1}{r} \xrightarrow{0} \quad \text{BC\#2, } C_1 = 0$$

$$-m \left(\frac{dv_z}{dr} \right)^n = \frac{dP}{dz} \frac{r}{2}$$

$$\left(\frac{dv_z}{dr} \right)^n = -\frac{r}{2m} \frac{dP}{dz} \quad \text{minus sign needed as } \frac{dv_z}{dr} \leq 0$$

$$\frac{dv_z}{dr} = - \left(-\frac{1}{2m} \frac{dP}{dz} \right)^{1/n} r^{1/n}$$

$$v_z = - \left(-\frac{1}{2m} \frac{dP}{dz} \right)^{1/n} \frac{1}{1/n+1} r^{1/n+1} + C_2$$

$$v_z = - \left(-\frac{1}{2m} \frac{dP}{dz} \right)^{1/n} \frac{n}{n+1} r^{\frac{n+1}{n}} + C_2$$

Using BC\#1, $v_z = 0$ at $r = R$

$$0 = - \left(-\frac{1}{2m} \frac{dP}{dz} \right)^{1/n} \frac{n}{n+1} R^{\frac{n+1}{n}} + C_2$$

Rearranging and inserting back to eqn.

$$\therefore v_z = - \left(-\frac{1}{2m} \frac{dP}{dz} \right)^{1/n} \frac{n}{n+1} \left(r^{\frac{n+1}{n}} - R^{\frac{n+1}{n}} \right)$$

$$v_z = \left(-\frac{1}{2m} \frac{dP}{dz} \right)^{1/n} \frac{n}{n+1} \left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}} \right)$$

$$(ii) \quad U = \frac{\int_0^R \int_0^{2\pi} v_z \, r \, d\theta \, dr}{\int_0^R \int_0^{2\pi} r \, d\theta \, dr}$$

$$U = \frac{2}{R^2} \int_0^R v_z \, r \, dr$$

$$= \frac{2}{R^2} \int_0^R \left(-\frac{1}{2m} \frac{dp}{dz} \right)^{1/n} \frac{n}{n+1} \left(r R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}+1} \right) dr$$

$$= \frac{2}{R^2} \left(-\frac{1}{2m} \frac{dp}{dz} \right)^{1/n} \frac{n}{n+1} \left(\frac{1}{2} r^2 R^{\frac{n+1}{n}} - \frac{1}{\frac{2n+1}{n}+1} r^{\frac{3n+1}{n}} \right) \Big|_0^R$$

$$= \frac{2}{R^2} \left(-\frac{1}{2m} \frac{dp}{dz} \right)^{1/n} \frac{n}{n+1} \left(\frac{1}{2} R^{2+\frac{n+1}{n}} - \frac{n}{3n+1} R^{\frac{3n+1}{n}} \right)$$

$$U = \left(\frac{n}{3n+1} \right) \left(-\frac{1}{2m} \frac{dp}{dz} \right)^{1/n} R^{\frac{n+1}{n}}$$

(iii) if $n=1$, $m=\mu$,
this becomes the Poiseuille flow.

$$(c) \quad \frac{1}{V_z r} \frac{\partial}{\partial r} r \frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{\partial T}{\partial z}$$

$$\text{From (b)} \quad V_z = \left(-\frac{1}{2m} \frac{dP}{dz} \right)^{1/n} \frac{n}{n+1} \left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}} \right)$$

$$\text{For } m=1, n=1$$

$$V_z = \frac{dP}{dz} \frac{1}{4} (r^2 - R^2)$$

$$\text{BC} \\ \#1 \quad r=0 \quad \frac{\partial T}{\partial r} = 0$$

$$\#2 \quad r=R \quad T=T_c$$

$$\therefore \frac{1}{V_z r} \frac{\partial}{\partial r} r \frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{dT}{dz}$$

$$\frac{\partial}{\partial r} r \frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{dT}{dz} \frac{dP}{dz} \frac{1}{4} (r^3 - R^2 r)$$

$$r \frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{dT}{dz} \frac{dP}{dz} \frac{1}{4} \left(\frac{r^4}{4} - \frac{R^2 r^2}{2} \right) + C_1$$

Using BC#1

$$r=0 \quad \frac{\partial T}{\partial r} = 0 \quad C_1 = 0$$

$$\frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{dT}{dz} \frac{dP}{dz} \frac{1}{4} \left(\frac{r^3}{4} - \frac{R^2 r}{2} \right)$$

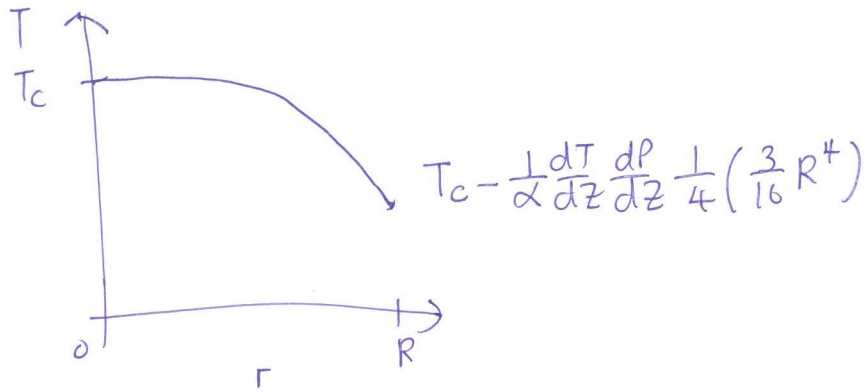
$$T = \frac{1}{\alpha} \frac{dT}{dz} \frac{dP}{dz} \frac{1}{4} \left(\frac{r^4}{16} - \frac{R^2 r^2}{4} \right) + C_2$$

Using BC#2

$$r=R, T=T_c \quad \therefore C_2 = T_c$$

$$T - T_c = \frac{1}{\alpha} \frac{dT}{dz} \frac{dP}{dz} \frac{1}{4} \left(\frac{r^4}{16} - \frac{R^2 r^2}{4} \right)$$

(b) (ii)



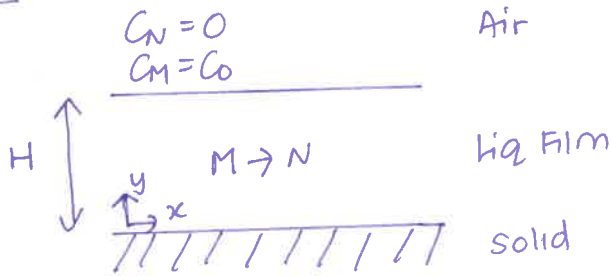
(iii) $q = -\kappa \left. \frac{\partial T}{\partial r} \right|_{r=R}$

$$\frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{dT}{dz} \frac{dp}{dz} \frac{1}{4} \left(\frac{r^3}{4} - \frac{R^2 r}{2} \right)$$

$$q = -\kappa \left(\frac{1}{\alpha} \frac{dT}{dz} \frac{dp}{dz} \frac{1}{4} \left(\frac{R^3}{4} - \frac{R^3}{2} \right) \right)$$

$$q = \frac{\kappa}{\alpha} \frac{dT}{dz} \frac{dp}{dz} \frac{1}{4} \left(\frac{R^3}{4} \right)$$

Question 4



Rectangular coordinate mass cons.:

$$\frac{\cancel{\delta C_M}}{\cancel{\delta t}} = D_M \left(\cancel{\frac{\delta^2 C_M}{\delta x^2}} + \frac{\delta^2 C_M}{\delta y^2} + \cancel{\frac{\delta^2 C_M}{\delta z^2}} \right) + R_M$$

steady state NO dep on x NO dep on z

$$0 = D_M \frac{\delta^2 C_M}{\delta y^2} - k C_M$$

BC
(1) $y = H$ $C_M = C_0$
(2) $y = 0$ $\frac{\delta C_M}{\delta y} = 0$

$$0 = \frac{\delta^2 C_M}{\delta y^2} - \frac{k}{D_M} C_M$$

$$C_M(y) = A \cosh(y/\lambda) + B \sinh(y/\lambda)$$

$$\frac{\delta C_M}{\delta y} = \frac{A}{\lambda} \sinh(y/\lambda) + \frac{B}{\lambda} \cosh(y/\lambda)$$

$$\frac{\delta^2 C_M}{\delta y^2} = \frac{A}{\lambda^2} \cosh(y/\lambda) + \frac{B}{\lambda^2} \sinh(y/\lambda)$$

$$\therefore \frac{A}{\lambda^2} \cosh(y/\lambda) + \frac{B}{\lambda^2} \sinh(y/\lambda) - \left[\frac{k}{D_M} \right] A \cosh(y/\lambda) - \frac{k}{D_M} B \sinh(y/\lambda) = 0$$

$$\lambda^2 = \frac{D_M}{k} \quad \lambda = \sqrt{\frac{D_M}{k}}$$

$$C_M(y) = A \cosh(y/\lambda) + B \sinh(y/\lambda)$$

using BC #2

$$\frac{dC_M}{dy} = 0 \text{ for } y=0$$

$$\frac{dC_M}{dy} = \frac{A}{\lambda} \sinh(y/\lambda) + B/\lambda \cosh(y/\lambda)$$

$$0 = \frac{A}{\lambda} \sinh(0) + B/\lambda \cosh(0)$$

$$0 = 0 + B/\lambda$$

$$\therefore B = 0$$

$$\therefore C_M = A \cosh(y/\lambda)$$

using BC #1

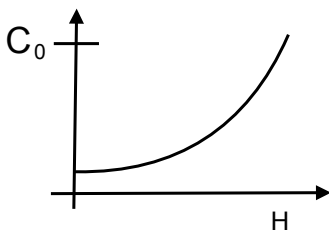
$$C_M = C_0 \text{ for } y=H$$

$$C_0 = A \cosh(H/\lambda)$$

$$A = \frac{C_0}{\cosh(H/\lambda)}$$

$$\therefore C_M = C_0 \frac{\cosh(y/\lambda)}{\cosh(H/\lambda)}$$

(b) Plot



(c) Concentration profile of C_N

$$0 = D_N \frac{d^2 C_N}{dy^2} + k C_N \quad \leftarrow \begin{array}{l} \text{Rate of} \\ \text{formation} \\ \text{of } N \end{array}$$

BC
#1 $C_N = 0$ at $y = H$

#2 $\frac{dC_N}{dy} = 0$ at $y = 0$

$$\frac{d^2 C_N}{dy^2} = -\frac{k}{D_N} C_0 \frac{\cosh(y/\lambda)}{\cosh(H/\lambda)}$$

$$\frac{dC_N}{dy} = -\frac{k}{D_N} C_0 \lambda \frac{\sinh(y/\lambda)}{\cosh(H/\lambda)} + C_1$$

Using BC #2

$$\frac{dC_N}{dy} = 0 \quad \text{for } y = 0 \quad C_1 = 0$$

$$\therefore \frac{dC_N}{dy} = -\frac{k}{D_N} C_0 \lambda \frac{\sinh(y/\lambda)}{\cosh(H/\lambda)}$$

$$C_N = -\frac{k}{D_N} C_0 \lambda^2 \frac{\cosh(y/\lambda)}{\cosh(H/\lambda)} + C_2$$

Using BC #1

$$0 = -\frac{k}{D_N} C_0 \lambda^2 \frac{\cosh(H/\lambda)}{\cosh(H/\lambda)} + C_2$$

$$C_2 = \frac{k}{D_N} C_0 \lambda^2$$

$$\therefore C_N = \frac{k}{D_N} C_0 \lambda^2 \left(1 - \frac{\cosh(y/\lambda)}{\cosh(H/\lambda)} \right)$$

(d) Rate of M entering the liquid. at $y = H$

$$N_M = -D_M \frac{\delta C_M}{\delta y}$$

$$N_M (y=H) = -\frac{D_M}{\lambda} \frac{C_0 \sinh(H/\lambda)}{\cosh(H/\lambda)}$$

(e) $0 = D_M \frac{\delta^2 C_M}{\delta y^2} - k C_M$ from (a)

$$k C_M = D_M \frac{\delta^2 C_M}{\delta y^2}$$

Introduce non-dimensional variables

$$\tilde{y} = \frac{y}{y_c} \quad \tilde{C} = \frac{C}{C_0} \text{ characteristic}$$

$$\therefore k \tilde{C}_M \cancel{y_c} = D_M \frac{\cancel{y_c} \delta^2 \tilde{C}_M}{y_c^2 \delta \tilde{y}^2}$$

$$\tilde{C}_M = \frac{D_M}{k y_c^2} \frac{\delta^2 \tilde{C}_M}{\delta \tilde{y}^2}$$

$$y_c^2 = \frac{D_M}{k}$$

$$y_c = \sqrt{\frac{D_M}{k}}$$

$y_c \ll H$, reaction occurs at the interface

$y_c > H$, reaction occurs throughout the film