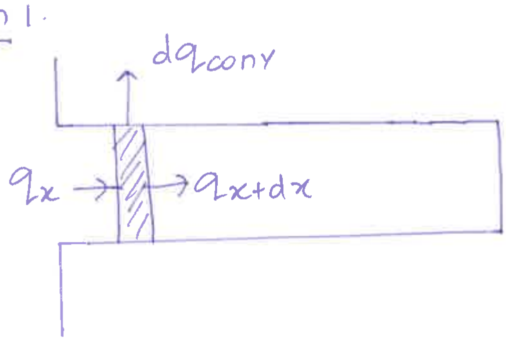
Question 1.

(a)



Balance the flux over the control area

$$q_x = q_{x+dx} + dq_{conv}.$$

$$q_x = -\lambda A \frac{dT}{dx}$$

$$q_{x+dx} = -\lambda A \left.\frac{\delta T}{\delta x}\right|_{x+dx}$$

$$= -\lambda A \left(\frac{\delta T}{\delta x} + \frac{\delta^2 T}{\delta x^2} dx \right)$$

$$dq_{conv} = hP(T - T_\infty)\, dx$$

$$\therefore \quad -\lambda A \frac{dT}{dx} = -\lambda A \left(\frac{\delta T}{\delta x} + \frac{\delta^2 T}{\delta x^2} dx \right) + hP(T - T_\infty)\, dx$$

This simplifies to:

$$0 = -\frac{\delta^2 T}{\delta x^2} + \frac{hP}{\lambda A}(T - T_\infty)$$

$$\frac{\delta^2 T}{\delta x^2} - \frac{hP}{\lambda A}(T - T_\infty) = 0$$

Boundary conditions

$x = 0 \quad T = T_0$

$x = L \quad T = T_L$

(b) Introduce $\theta = T - T_\infty$

$$\frac{\delta^2 T}{\delta x^2} - \frac{hP}{\lambda A}(T - T_\infty) = 0$$

subst $T = \theta + T_\infty$ to equation above

$$\frac{\delta^2 \theta}{\delta x^2} + \cancel{\frac{\delta^2 T_\infty}{\delta x^2}} - \frac{hP}{\lambda A}\theta = 0$$

constant

$$\frac{\delta^2 \theta}{\delta x^2} - \frac{hP}{\lambda A}\theta = 0$$

$$\theta = C_1 \cosh(mx) + C_2 \sinh(mx)$$

$$\frac{\delta \theta}{\delta x} = C_1 m \sinh(mx) + C_2 m \cosh(mx)$$

$$\frac{\delta^2 \theta}{\delta x^2} = C_1 m^2 \cosh(mx) + C_2 m^2 \sinh(mx)$$

$$\frac{hP}{\lambda A}\theta = \frac{hP}{\lambda A}(C_1 \cosh(mx) + C_2 \sinh(mx))$$

$$\therefore \frac{\delta^2 \theta}{\delta x^2} - \frac{hP}{\lambda A}\theta = C_1 m^2 \cosh(mx) + C_2 m^2 \sinh(mx) - \frac{hP}{\lambda A} C_1 \cosh(mx) - \frac{hP}{\lambda A} C_2 \sinh(mx)$$

This equates to zero if $m^2 = \frac{hP}{\lambda A}$

$$\therefore m = \sqrt{\frac{hP}{\lambda A}}$$

$\therefore \theta = C_1 \cosh(mx) + C_2 \sinh(mx)$ is a solution

BC 1 $\quad x = 0 \quad T = T_0$

$$\theta_0 = T_0 - T_\infty$$

$$\theta_0 = C_1 \cancel{\cosh(0)}^{\,1} + C_2 \cancel{\sinh(0)}^{\,0}$$

$$C_1 = \theta_0$$

using <u>BC2</u> $x = L$

$$T_L - T_\infty = \theta_L$$

$$\theta_L = C_1 \cosh(mL) + C_2 \sinh(mL)$$

$$\theta_L = \theta_0 \cosh(mL) + C_2 \sinh(mL)$$

$$C_2 = \frac{\theta_L - \theta_0 \cosh(mL)}{\sinh(mL)}$$

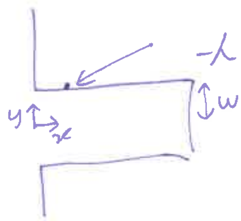
$$\therefore \quad \theta = \theta_0 \cosh(mx) + \frac{\theta_L - \theta_0 \cosh(mL)}{\sinh(mL)} \sinh(mx)$$

//

(c) variation of T along y-coordinate can be ignored for $Bi \ll 1$

or Temp variation over y-coordinate is smaller than the difference between top of the fin and surrounding.

one can do an analysis on the boundary condition at the top of the fin.



$$-\lambda \frac{dT}{dy}(x, y=w) = h(T(x,y=w) - T_\infty)$$

$$\frac{T(x,y=w) - T(x,y=0)}{w} \sim -\frac{h}{\lambda}(T(x,y=w) - T_\infty)$$

$$-\left(\frac{T(x,y=w) - T(x,y=0)}{T(x,y=w) - T_\infty}\right) \sim \frac{hw}{\lambda}$$

$$\frac{hw}{\lambda} \ll 1$$

(d) $$Q = -\lambda A \frac{d\theta}{\delta x}\bigg|_{x=0}$$

$$\frac{d\theta}{dx} = \theta_0 m \sinh(mx) + m \frac{\theta_L - \theta_0 \cosh(mL)}{\sinh(mL)} \cosh(mx)$$

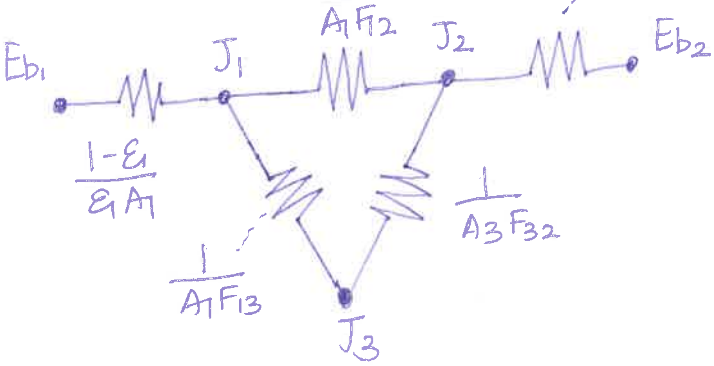
$$Q = -\lambda A \frac{d\theta}{\delta x}\bigg|_{x=0} = -\lambda A m \frac{\theta_L - \theta_0 \cosh(mL)}{\sinh(mL)}$$ //

(e) Efficiency

$$\varepsilon_f = \frac{Q_f}{hPL\theta_0} = \frac{-\lambda A m}{hPL\theta_0} \frac{\theta_L - \theta_0 \cosh(mL)}{\sinh(mL)}$$ //

Question 2

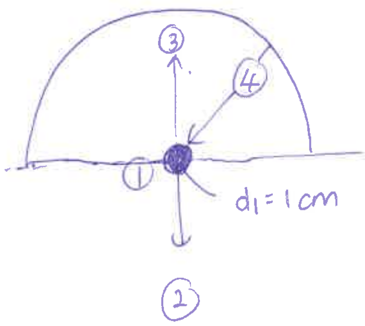
(a) Equivalent circuit



$\varepsilon_1 = 0.80$

$\varepsilon_2 = 1$

(b) View Factors



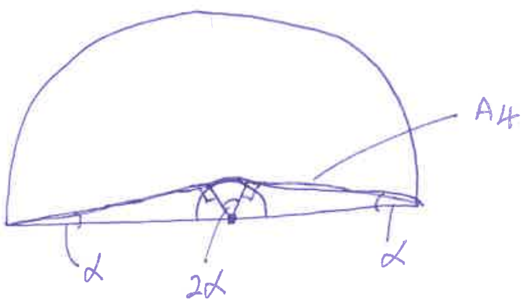
$F_{12} = 0.50$

$F_{13} = 0.50$

$F_{43} = 1.0$

$A_1 = \pi(0.01) = 0.03$

$A_3 = \pi(0.10) = 0.31$



$\sin\alpha = \frac{0.5}{10}$

$\alpha = 2.866°$

$2\alpha = 5.732°$

$A_4 = 2\times(0.10\cos 2.866°) + \pi(0.01)\left(\frac{5.732}{360}\right)$

$= 0.2$

$F_{34}A_3 = A_4F_{43}$

$F_{34} = \frac{0.20(1.0)}{0.31} = 0.6366$

$F_{31}A_3 = F_{13}A_1$

$F_{31} = \frac{0.50(0.03)}{0.31} = 0.05$

$F_{34} = F_{31} + F_{32}$ $\therefore$ $F_{32} = 0.6366 - 0.05 = 0.5866$

(c) Reff

Find the R contribution from the parallel part of the circuit

$$\frac{1}{R_{par}} = \frac{1}{1/A_1F_{12}} + \frac{1}{\frac{1}{A_1F_{13}} + \frac{1}{A_3F_{32}}}$$

$$\frac{1}{R_{par}} = A_1F_{12} + \frac{A_1F_{13}A_3F_{32}}{A_3F_{32} + A_1F_{13}}$$

$$\frac{1}{R_{par}} = \frac{A_1F_{12}(A_3F_{32} + A_1F_{13}) + A_1F_{13}A_3F_{32}}{A_3F_{32} + A_1F_{13}}$$

$$R_{par} = \frac{A_3F_{32} + A_1F_{13}}{A_1F_{12}(A_3F_{32} + A_1F_{13}) + A_1F_{13}A_3F_{32}}$$

$$R_{tot} = \frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2} + \frac{A_3F_{32} + A_1F_{13}}{A_1F_{12}(A_3F_{32}+A_1F_{13}) + A_1F_{13}A_3F_{32}}$$

Plug in values

$$R_{tot} = \frac{1-0.8}{0.8(0.03)} + 0 + 33.13 = 41.09$$

(d) Radiative Heat loss

$$E_{b1} = \sigma T_{b1}^4$$
$$= 5.669 \times 10^{-8}(1073)^4$$
$$= 75146 \text{ W/m}^2$$

$$E_{b2} = \sigma T_{b2}^4$$
$$= 5.669 \times 10^{-8}(298)^4$$
$$= 447 \text{ W/m}^2$$

$$q = \frac{75146 - 447}{41.09} = 1818 \text{ W/m}$$

(e) Reflector removed

$$q = \frac{E_{b1} - E_{b2}}{\frac{1}{\varepsilon_1 A_1}} = \frac{75146 - 447}{1/0.80(0.03)}$$

$$= 1877 \text{ W/m}$$

More heat is lost if reflector is removed

Question 3

Non Newtonian fluid - power Law.

$$\tau_{rz} = -m\left(\frac{dv_z}{\delta r}\right)^n$$

(i) $$\frac{1}{r}\frac{\delta}{\delta r}(r\tau_{rz}) = \frac{dP}{dz}$$

$$r\tau_{rz} = \frac{dP}{dz}\frac{r^2}{2} + C_1$$

$$\tau_{rz} = \frac{dP}{dz}\frac{r}{2} + \cancel{\frac{C_1}{r}}^{\,0} \qquad BC\#2, C_1 = 0$$

$$-m\left(\frac{dv_z}{\delta r}\right)^n = \frac{dP}{dz}\frac{r}{2}$$

$$\left(\frac{\delta v_z}{\delta r}\right)^n = -\frac{r}{2m}\frac{dP}{dz}$$

minus sign needed as $\frac{dv_z}{dr} \leq 0$

$$\frac{dv_z}{\delta r} = -\left(-\frac{1}{2m}\frac{dP}{dz}\right)^{1/n} r^{1/n}$$

$$v_z = -\left(-\frac{1}{2m}\frac{dP}{dz}\right)^{1/n}\frac{1}{1/n+1} r^{1/n+1} + C_2$$

$$v_z = -\left(-\frac{1}{2m}\frac{dP}{dz}\right)^{1/n}\frac{n}{n+1} r^{\frac{n+1}{n}} + C_2$$

using BC#1, $v_z = 0$ at $r = R$

$$0 = -\left(-\frac{1}{2m}\frac{dP}{dz}\right)^{1/n}\frac{n}{n+1} R^{\frac{n+1}{n}} + C_2$$

Rearranging and inserting back to eqtn.

$$\therefore \quad v_z = -\left(-\frac{1}{2m}\frac{dP}{dz}\right)^{1/n}\frac{n}{n+1}\left(r^{\frac{n+1}{n}} - R^{\frac{n+1}{n}}\right)$$

$$v_z = \left(\frac{-1}{2m}\frac{dP}{dz}\right)^{1/n}\frac{n}{n+1}\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right)$$

(ii) $$U = \frac{\int_0^R \int_0^{2\pi} v_z \, r \, d\theta \, dr}{\int_0^R \int_0^{2\pi} r \, d\theta \, dr}$$

$$U = \frac{2}{R^2} \int_0^R v_z \, r \, dr$$

$$= \frac{2}{R^2} \int_0^R \left(-\frac{1}{2m} \frac{dp}{dz}\right)^{1/n} \frac{n}{n+1} \left(r R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}+1} \right) dr$$

$$= \frac{2}{R^2} \left(-\frac{1}{2m} \frac{dp}{dz}\right)^{1/n} \frac{n}{n+1} \left(\frac{1}{2} r^2 R^{\frac{n+1}{n}} - \frac{1}{\frac{2n+1}{n}+1} r^{\frac{3n+1}{n}} \right) \Bigg|_0^R$$

$$= \frac{2}{R^2} \left(-\frac{1}{2m} \frac{dp}{dz}\right)^{1/n} \frac{n}{n+1} \left(\frac{1}{2} R^{2+\frac{n+1}{n}} - \frac{n}{3n+1} R^{\frac{3n+1}{n}} \right)$$

$$U = \left(\frac{n}{3n+1}\right) \left(-\frac{1}{2m} \frac{dp}{dz}\right)^{1/n} R^{\frac{n+1}{n}}$$

(iii) if $n=1$, $m=\mu$,

this becomes the Poiseuille flow.

(c) $\frac{1}{V_z r}\frac{\delta}{\delta r} r \frac{\delta T}{\delta r} = \frac{1}{\alpha}\frac{\delta T}{\delta z}$

From (b) $V_z = \left(-\frac{1}{2m}\frac{dP}{dz}\right)^{1/n}\frac{n}{n+1}\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right)$

For $m=1$, $n=1$

$V_z = \frac{dP}{dz}\frac{1}{4}(r^2 - R^2)$

BC

#1 $r=0$ $\frac{\delta T}{\delta r} = 0$

#2 $r=0$ $T = T_c$

$\therefore$ $\frac{1}{V_z r}\frac{\delta}{\delta r} r \frac{\delta T}{\delta r} = \frac{1}{\alpha}\frac{dT}{dz}$

$$\frac{\delta}{\delta r} r \frac{\delta T}{\delta r} = \frac{1}{\alpha}\frac{dT}{dz}\frac{dP}{dz}\frac{1}{4}(r^3 - R^2 r)$$

$$r\frac{\delta T}{\delta r} = \frac{1}{\alpha}\frac{dT}{dz}\frac{dP}{dz}\frac{1}{4}\left(\frac{r^4}{4} - \frac{R^2 r^2}{2}\right) + C_1$$

using BC#1

$r=0$ $\frac{\delta T}{\delta r} = 0$ $\quad C_1 = 0$

$$\frac{\delta T}{\delta r} = \frac{1}{\alpha}\frac{dT}{dz}\frac{dP}{dz}\frac{1}{4}\left(\frac{r^3}{4} - \frac{R^2 r}{2}\right)$$

$$T = \frac{1}{\alpha}\frac{dT}{dz}\frac{dP}{dz}\frac{1}{4}\left(\frac{r^4}{16} - \frac{R^2 r^2}{4}\right) + C_2$$

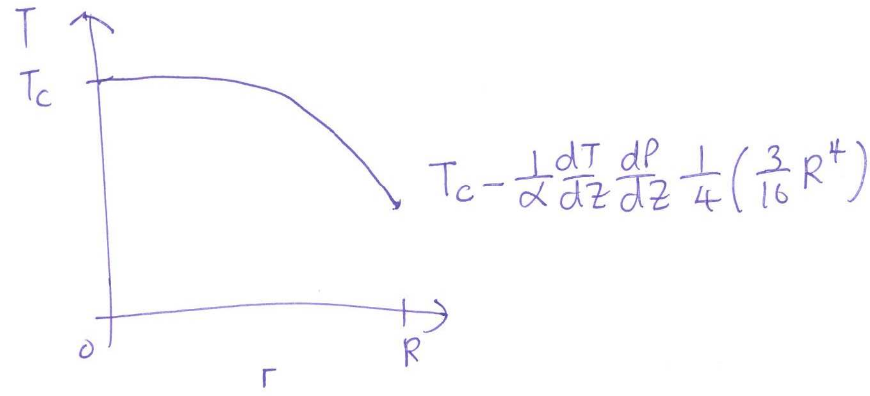
using BC#2

$r=0$, $T = T_c$ $\quad \therefore C_2 = T_c$

$$T - T_c = \frac{1}{\alpha}\frac{dT}{dz}\frac{dP}{dz}\frac{1}{4}\left(\frac{r^4}{16} - \frac{R^2 r^2}{4}\right)$$

//

(b) (ii)



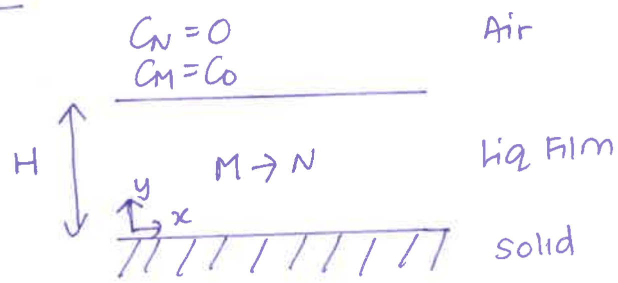
(iii) $q = -\lambda \left.\frac{\delta T}{\delta r}\right|_{r=R}$

$$\frac{\delta T}{\delta r} = \frac{1}{\alpha}\frac{dT}{dz}\frac{dp}{dz}\frac{1}{4}\left(\frac{r^3}{4} - \frac{R^2 r}{2}\right)$$

$$q = -\lambda\left(\frac{1}{\alpha}\frac{dT}{dz}\frac{dp}{dz}\frac{1}{4}\left(\frac{R^3}{4} - \frac{R^3}{2}\right)\right.$$

$$q = \frac{\lambda}{\alpha}\frac{dT}{dz}\frac{dp}{dz}\frac{1}{4}\left(\frac{R^3}{4}\right)$$

Question 4



Rectangular coordinate mass cons :

$$\cancel{\frac{\delta C_M}{\delta t}}^{\,0} = D_M\left(\cancel{\frac{\delta^2 C_M}{\delta x^2}}^{\,0} + \frac{\delta^2 C_M}{\delta y^2} + \cancel{\frac{\delta^2 C_M}{\delta z^2}}^{\,0}\right) + R_M$$

steady state   NO dep on x No dep on z

$$0 = D_M \frac{\delta^2 C_M}{\delta y^2} - k C_M$$

BC

(1) $y = H \quad C_M = C_0$

(2) $y = 0 \quad \frac{\delta C_M}{\delta y} = 0$

$$0 = \frac{\delta^2 C_M}{\delta y^2} - \frac{k}{D_M} C_M$$

$$C_M(y) = A\cosh(y/\lambda) + B\sinh(y/\lambda)$$

$$\frac{\delta C_M}{\delta y} = \frac{A}{\lambda}\sinh(y/\lambda) + \frac{B}{\lambda}\cosh(y/\lambda)$$

$$\frac{\delta^2 C_M}{\delta y^2} = \frac{A}{\lambda^2}\cosh(y/\lambda) + B/\lambda^2 \sinh(y/\lambda)$$

$$\therefore \frac{A}{\boxed{\lambda^2}}\cosh(y/\lambda) + B/\lambda^2\sinh(y/\lambda) - \boxed{\frac{k}{D_M}} A\cosh(y/\lambda) - \frac{k}{D_M} B\sinh(y/\lambda) = 0$$

$$\lambda^2 = \frac{D_M}{k} \qquad \lambda = \sqrt{\frac{D_M}{k}}$$

$$C_M(y) = A\cosh(y/\lambda) + B\sinh(y/\lambda)$$

using BC #2

$$\frac{\delta C_M}{\delta y} = 0 \quad \text{for } y = 0$$

$$\frac{\delta C_M}{\delta y} = \frac{A}{\lambda}\sinh(y/\lambda) + B/\lambda \cosh(y/\lambda)$$

$$0 = \frac{A}{\lambda}\sinh(0) + B/\lambda \cosh(0)$$

$$0 = 0 + B/\lambda$$

$$\therefore B = 0$$

$$\therefore C_M = A\cosh(y/\lambda)$$

using BC #1

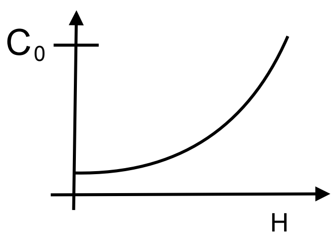
$$C_M = C_0 \quad \text{for } y = H$$

$$C_0 = A\cosh(H/\lambda)$$

$$A = \frac{C_0}{\cosh(H/\lambda)}$$

$$\therefore C_M = C_0 \frac{\cosh(y/\lambda)}{\cosh(H/\lambda)}$$

(b) Plot



(c) Concentration profile of C_N

$$0 = D_N \frac{\delta^2 C_N}{\delta y^2} + k C_M$$

Rate of formation of N

BC

#1 $C_N = 0$ at $y = H$

#2 $\frac{\delta C_N}{\delta y} = 0$ at $y = 0$

$$\frac{\delta^2 C_N}{\delta y^2} = -\frac{k}{D_N} C_o \frac{\cosh(y/\lambda)}{\cosh(H/\lambda)}$$

$$\frac{\delta C_N}{\delta y} = -\frac{k}{D_N} C_o \lambda \frac{\sinh(y/\lambda)}{\cosh(H/\lambda)} + C_1$$

using BC#2

$$\frac{\delta C_N}{\delta y} = 0 \quad \text{for } y = 0 \qquad C_1 = 0$$

$$\therefore \frac{\delta C_N}{\delta y} = -\frac{k}{D_N} C_o \lambda \frac{\sinh(y/\lambda)}{\cosh(H/\lambda)}$$

$$C_N = -\frac{k}{D_N} C_o \lambda^2 \frac{\cosh(y/\lambda)}{\cosh(H/\lambda)} + C_2$$

using BC #1

$$0 = -\frac{k}{D_N} C_o \lambda^2 \cancel{\frac{\cosh(H/\lambda)}{\cosh(H/\lambda)}}^{1} + C_2$$

$$C_2 = \frac{k}{D_N} C_o \lambda^2$$

$$\therefore C_N = \frac{k}{D_N} C_o \lambda^2 \left(1 - \frac{\cosh(y/\lambda)}{\cosh(H/\lambda)}\right)$$

(d) Rate of M entering the liquid. at $y = H$

$$N_M = -D_M \frac{\delta C_M}{\delta y}$$

$$N_M (y=H) = -\frac{D_M}{\lambda} \frac{C_0 \sinh(H/\lambda)}{\cosh(H/\lambda)}$$

(e) $$0 = D_M \frac{\delta^2 C_M}{\delta y^2} - k C_M \quad \text{from (a)}$$

$$k C_M = D_M \frac{\delta^2 C_M}{\delta y^2}$$

Introduce non-dimensional variables

$$\tilde{y} = \frac{y}{y_c} \qquad \tilde{C} = \frac{C}{C_c}$$ characteristic

$$\therefore \quad k \tilde{C}_M \cancel{C_c} = D_M \frac{\cancel{C_c} \, \delta^2 \tilde{C}_M}{y_c^2 \, \delta \tilde{y}^2}$$

$$\tilde{C}_M = \frac{D_M}{k y_c^2} \frac{\delta^2 \tilde{C}_M}{\delta \tilde{y}^2}$$

$$y_c^2 = \frac{D_M}{k}$$

$$y_c = \sqrt{\frac{D_M}{k}}$$

$y_c << H$, reaction occurs at the interface

$y_c > H$, reaction occurs throughout the film