Version TMO/3

EGT2
ENGINEERING TRIPOS PART IIA

Friday 2 May 2025 9.30 to 11.10

Module 3A6

HEAT AND MASS TRANSFER

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 A fin with a cross-sectional area of A and perimeter of P extends from a base along the x-axis with length L as shown in Fig. 1. The fin has a thickness of D along the y-axis. At the base, the temperature is kept constant at $T = T_0$. Heat along the fin is exchanged by convection with the surroundings at T_∞ with a constant convective heat transfer coefficient h. The fin has a constant thermal conductivity of λ. At steady state, the temperature at the tip of the fin is found to be T_L.

(a) Derive the governing equation for the temperature variation along the fin. State the assumptions, if you make any, and write down the boundary conditions for this system. [25%]

(b) Introduce $\Theta = T - T_\infty$. Show that the general solution for part (a) is

$$\Theta = C_1 \cosh(mx) + C_2 \sinh(mx)$$

Determine C_1, C_2, and m. [25%]

(c) Explain why the variation of T along the y-coordinate can be neglected. [10%]

(d) Obtain a steady state expression for the total rate of heat transferred from the fin. [20%]

(e) Obtain an expression for the fin efficiency. [20%]

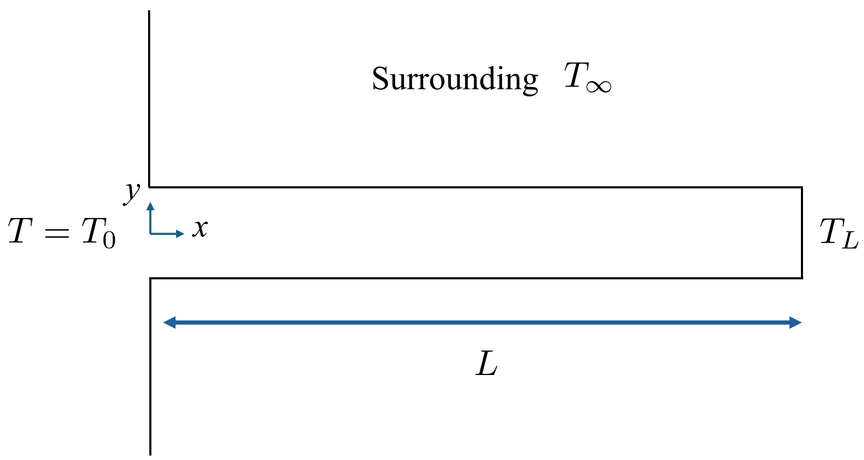

Fig. 1

2 A solid cylindrical Nichrome wire generating heat is placed near a half-cylindrical reflector as shown in Fig. 2. The wire has an emissivity of $\epsilon = 0.8$ and is maintained at 800 °C. The diameter of the wire is 1.0 cm, and the diameter of the reflector is 20 cm. The reflector is insulated and the entire system is placed in a large room with $T = 25$ °C.

(a) Sketch the radiation circuit and write expressions for each components in the radiation circuit. [20%]

(b) Calculate all the view factors for each surface with other surfaces. [20%]

(c) Calculate the effective resistance for the total radiation circuit from part (a). [25%]

(d) Calculate the radiative heat loss per unit length of the wire. [20%]

(e) Calculate the heat loss from the wire without the reflector. Compare this value with that obtained for part (d) and comment on it. [15%]

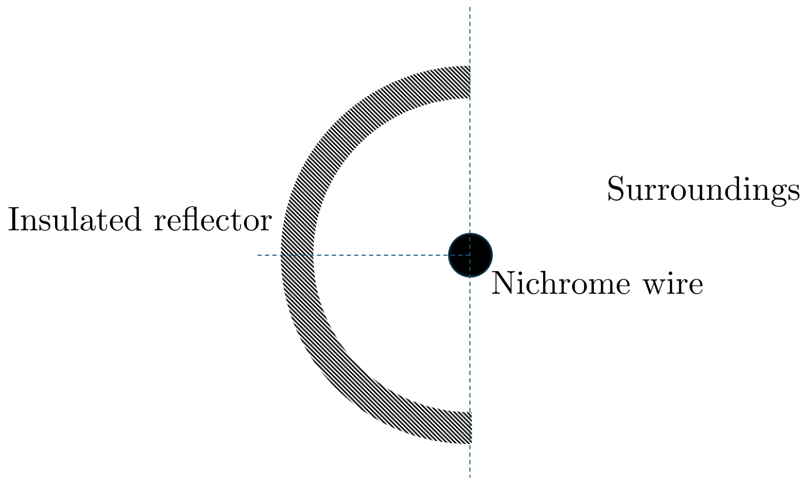

Fig. 2

3 Consider a fluid along the z-coordinate in a cylindrical tube of radius R. The viscous stress τ_{rz} is the only non-zero component of stress and is a function of r. The momentum equation for the z-component in cylindrical polar coordinates is as follows:

$$\rho\left[\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right]=-\frac{\partial P}{\partial z}+\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rz}\right)+\frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta}+\frac{\partial \tau_{zz}}{\partial z}\right]$$

(a) The axisymmetric flow is steady and fully developed with a constant pressure gradient of dP/dz. The fluid is non-Newtonian and follows the power law

$$\tau_{rz}(r)=-m\left|\frac{\partial v_z}{\partial r}\right|^n$$

where m and n are positive constants.

(i) Write down the boundary conditions for this system. Show that

$$v_z=\left(-\frac{1}{2m}\frac{dP}{dz}\right)^{\frac{1}{n}}\frac{n}{n+1}\left(R^{(n+1)/n}-r^{(n+1)/n}\right)$$

is the radial variation of axial velocity. [25%]

(ii) Determine the bulk-mean velocity of the fluid. [20%]

(iii) Describe this flow when $n=1$ and $m=\mu$. [10%]

(b) The non-Newtonian fluid from (a) experiences heat transfer along the tube. The temperature equation is given by

$$\frac{1}{v_z r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right)=\frac{1}{\alpha}\frac{\partial T}{\partial z}$$

where dT/dz is constant. The temperature at the centre of the tube is T_c.

(i) Identify the boundary conditions for T and obtain an expression for its radial variation when $n=1$ and $m=1$. Sketch this variation. [35%]

(ii) Determine the heat flux at the tube wall, which has a constant thermal conductivity λ. [10%]

4 Consider a stagnant liquid film of thickness H on an impermeable and non-reactive solid plate with the coordinate system defined as shown in Fig. 3. There is an irreversible conversion of species M to species N in the liquid film. The concentration of M at the air-liquid interface is C_0 and that of N is zero. The reaction rate in the liquid obeys $R_M = -kC_M$ and $R_N = kC_M$. k is the rate constant. The mass conservation equation for species i in the Cartesian coordinate is

$$\frac{\partial C_i}{\partial t} = D_i \left[\frac{\partial^2 C_i}{\partial x^2} + \frac{\partial^2 C_i}{\partial y^2} + \frac{\partial^2 C_i}{\partial z^2} \right] + R_i$$

where D_i is the diffusivity of species i.

(a) Simplify the above equation for species M, stating all your assumptions and show that its general solution is

$$C_M = A \cosh\left(\frac{y}{\lambda}\right) + B \sinh\left(\frac{y}{\lambda}\right)$$

Also, show that λ is given by $\sqrt{D_M/k}$. [20%]

(b) Write down the boundary conditions for the system in Fig. 3 and obtain an expression for the variation of C_M in the liquid film. Sketch this variation. [35%]

(c) Write down the boundary conditions for C_N and obtain an expression for the variation of C_N in the liquid film. [25%]

(d) What is the rate of species M entering at the liquid film top? [10%]

(e) Explain the implication for the reaction if λ is significantly smaller than H. [10%]

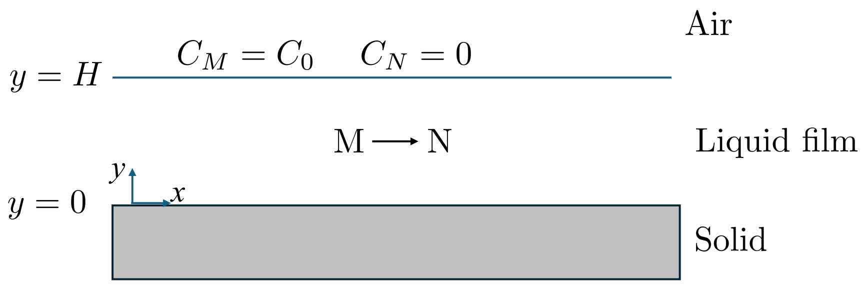

Fig. 3

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