IIA

2020-21

3A6 CRIB #1

Q1 & 3 were prepared for last year but not used; they've been QA'd.

Q2 & 4 are new for this year.

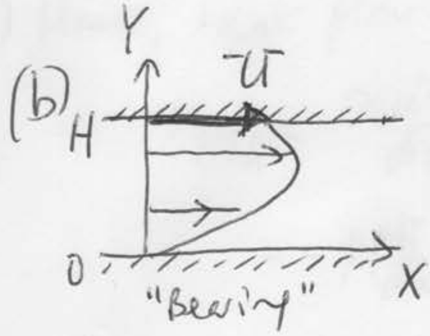
WND.

Feb. 2021.

1 (a) $$\underbrace{u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}}_{\text{convection of u-velocity}} = \underbrace{-\frac{1}{\rho}\frac{dp}{dx}}_{\text{x-wise pressure gradient}} + \underbrace{\nu\nabla^2 u}_{\text{x-wise viscous diffusion (laminar)}} \;;\; \underbrace{u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}}_{\text{convection of enthalpy (from } c_pT)} = \underbrace{\alpha\nabla^2 T}_{\text{thermal diffusion}}$$

$(\nu=\mu/\rho)$

(b)



fully developed, steady, laminar flow:

$$\frac{\partial u}{\partial x}=\frac{\partial^2 u}{\partial x^2}=0 \;;\; v=0 \Rightarrow \frac{dp}{dx}=\mu\frac{d^2u}{dy^2}$$

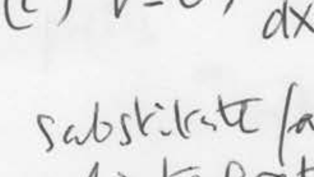
Integrate twice: $\mu u = \left(\frac{dp}{dx}\right)\frac{y^2}{2}+Ay+B$ $\begin{cases} u=0 @ y=0 \therefore \underline{B=0} \\ u=U @ y=H \\ \therefore \mu U = \frac{dp}{dx}\frac{H^2}{2}+\underline{A}H \end{cases}$

$\hookrightarrow A=\mu\frac{U}{H}-\frac{dp}{dx}\frac{H}{2}$

$$\therefore u = \underbrace{U\frac{y}{H}}_{\text{linear term driven by moving bearing pad.}} + \underbrace{\frac{1}{\mu}\left(\frac{dp}{dx}\right)(y-H)\frac{y}{2}}_{\text{quadratic term driven by pressure gradient. increases the bulk flow.}}$$

(c) $v=0$; $\frac{dT}{dx}=\text{constant} \Rightarrow u\left(\frac{dT}{dx}\right)=\alpha\frac{d^2T}{dy^2}$

substitute for u (not coupled as incompressible flow) and integrate twice:



$$\alpha\frac{dT}{dy}=\left(\frac{dT}{dx}\right)\left\{\frac{U}{H}\frac{y^2}{2}+\frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{1}{2}\left(\frac{y^3}{3}-\frac{Hy^2}{2}\right)\right\}+A$$

$\frac{dT}{dy}=0 @ y=0$ (adiabatic) $\therefore \underline{A=0}$

adiabatic wall so $\left.\frac{dT}{dy}\right|_{y=0}=0$

$$\alpha T=\left(\frac{dT}{dx}\right)\left\{\frac{U}{H}\frac{y^3}{6}+\frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{1}{2}\left(\frac{y^4}{12}-\frac{Hy^3}{6}\right)\right\}+B$$

At moving pad $T = T_s$ @ $Y = H$

$$\therefore \; \alpha T_s = \underline{\underline{B}} + \left(\frac{dT}{dx}\right)\left\{\frac{1}{6}U H^2 - \frac{1}{24\mu}\left(\frac{dp}{dx}\right)H^4\right\}$$

$$\therefore \; T - T_s = \frac{1}{\alpha}\left(\frac{dT}{dx}\right)\left\{\frac{1}{6}\frac{U}{H}\left(Y^3 - H^3\right) - \frac{1}{24\mu}\left(\frac{dp}{dx}\right)\left(H^4 - Y^4 + 2HY^3\right)\right\}$$

(d) Hence, heat flow to moving bearing pad:

$$q_w/A = -\lambda \left.\frac{dT}{dY}\right|_{Y=H} \qquad (\alpha = \lambda/\rho)$$

$$= -\rho\left(\frac{dT}{dx}\right)\left\{\frac{1}{6}\frac{U}{H}\left(3H^2 - 0\right) - \frac{1}{24\mu}\left(\frac{dp}{dx}\right)\left(0 - 4H^3 + 6H^3\right)\right\}$$

$$\therefore \; q_w/A = -\rho \frac{H}{2}\left(\frac{dT}{dx}\right)\left\{U - \frac{H^2}{6\mu}\left(\frac{dp}{dx}\right)\right\}$$

Effect of pressure gradient is to $\underline{\underline{\text{reduce}}}$ heat flow.

(e) For particular case with $(dp/dx) = 0$ the bulk velocity, $\underline{U_b = U/2}$ and the bulk temperature

$$T_b = \frac{1}{\underbrace{\dot{m} c_p}_{\rho H U_b}}\int_0^H \rho u c_p T \, dY = \frac{2}{HU}\int \frac{UY}{H}\left(T_s + \frac{1}{\alpha}\left(\frac{dT}{dx}\right)\frac{1}{6}\frac{U}{H}\left(Y^3 - H^3\right)\right)dY$$

$$= \frac{\;}{H^2}\left[T_s\frac{H^2}{2} + \underbrace{\frac{1}{\alpha}\left(\frac{dT}{dx}\right)\frac{U}{6H}\cdot\frac{H^5}{5}}_{30} - \underbrace{\frac{1}{\alpha}\left(\frac{dT}{dx}\right)\frac{U}{6H}\cdot H^3\frac{H^2}{2}}_{12}\right]$$

$$\therefore \; T_b = T_s + \frac{1}{10}\cdot\frac{H^2 U}{\alpha}\cdot\left(\frac{dT}{dx}\right)$$

So, $\frac{q_w}{A} = -\rho\frac{H}{2}\cancel{\left(\frac{dT}{dx}\right)}U = HTC\left(T_s - T_b\right) = HTC\left(-\frac{1}{10}\frac{H^2 U}{\alpha}\cancel{\left(\frac{dT}{dx}\right)}\right)$

$$\therefore \; \underline{HTC = 5\lambda/H} \;!$$

This does $\underline{\text{NOT}}$ depend on U for this special case as $\underline{\text{no}}$ convection of T, just conduction across gap!!!

2. (a) Momentum equation from Q1 with added gravity body force:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + g_x$$

Consider a vertical, heated plate with natural convection: outside the b'layer, $\frac{dp}{dx} = -\rho_\infty g$

$$\therefore -\frac{1}{\rho}\frac{dp}{dx} + g = g - \frac{\rho_\infty}{\rho}g = g\left(1 - \frac{T}{T_\infty}\right)$$

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p = \frac{1}{T} \qquad = g\underbrace{\frac{1}{T_\infty}}_{\beta}(T_\infty - T)$$

$$\therefore u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T_\infty - T) + \nu\frac{\partial^2 u}{\partial y^2}$$

(b) Classic order of magnitude b'layer analysis:

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

$O\left(\frac{U}{H}\right)$ $O\left(\frac{v}{\delta_T}\right)$

$\therefore v \sim U\delta_T/H$

$$\boxed{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T_\infty - T) + \nu\frac{\partial^2 u}{\partial y^2}}$$

$O\left(\frac{U \cdot U}{H}\right)$ $O\left(\frac{U\delta_T U}{H\delta_T}\right)$ $O(g\beta\Delta T)$ $O\left(\frac{\nu U}{\delta_T^2}\right)$

$\div \frac{U^2}{H}$ $O(1)$ $O(1)$ $O\left(\frac{gH\beta\Delta T}{U^2}\right)$ $O\left(\frac{\nu H}{U\delta_T^2}\right)$

$$\therefore \frac{g\beta\Delta T}{U} \sim \frac{\nu}{\delta_T^2} \qquad (1)$$

"induced velocity".

$$\boxed{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}}$$

$O\left(\frac{U\Delta T}{H}\right)$ $O\left(\frac{U\delta_T\Delta T}{H\delta_T}\right)$ $O\left(\frac{\alpha\Delta T}{\delta_T^2}\right)$

$O(1)$ $O(1)$ $O\left(\frac{\alpha H}{U\delta_T^2}\right) \sim O(1)$ $\therefore U \sim \alpha H/\delta_T^2 \qquad (2)$

So, eliminate "U" between ① & ②:

$$g\beta\Delta T\,\delta_T^2 \sim \nu\,\alpha\,H/\delta_T^2 \qquad \text{"Rayleigh \#"}$$

$$\therefore\ \delta_T^4 \sim \frac{\nu\alpha}{g\beta\Delta T}\cdot\frac{H^4}{H^3} \Rightarrow \frac{\delta_T^4}{H^4} \sim \frac{1}{Ra}$$

and, $q/A = h\Delta T = \lambda \left.\frac{dT}{dy}\right|_w \therefore h\Delta T \sim \frac{\lambda\Delta T}{\delta_T} \rightarrow h \sim \frac{\lambda}{\delta_T}$

$$\therefore\ \frac{H^4 h^4}{\lambda^4} = Nu^4 \sim Ra \Rightarrow \underline{Nu \sim Ra^{\frac{1}{4}}}$$

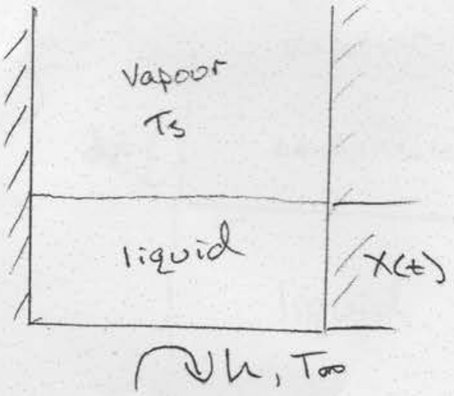
(c) So, induced velocity $U \sim \alpha H \left(\frac{Ra^{1/2}}{H^2}\right)$ ← $\delta_T \sim H/Ra^{1/4}$

$$\therefore\ U \sim \frac{\alpha}{H}\left(\frac{gH^3\beta\Delta T}{\nu\alpha}\right)^{1/2}$$

$$\therefore\ \underline{U \sim (H\Delta T)^{1/2}}$$

(d) So U can get large (potentially damaging) for big H (eg. tall buildings) or big ΔT (eg. cooling tanks associated with nuclear power plant or other heavy process industry.

3 Q1



a) $\dot{q}''_{x+dx} = \dot{q}''_x + \frac{\partial \dot{q}''_x}{\partial x} dx$

dx

$\dot{q}''_x$

Energy per unit area

$$\frac{\partial E''}{\partial t} = \left(\cancel{\dot{q}''_x} - \cancel{\dot{q}''_x} - \frac{\partial \dot{q}_x}{\partial x} dx\right)$$

$$E'' = \frac{m C_p T}{A} = \rho C_p T \, dx$$

where $m = \rho \forall = \rho A dx$

$$\dot{q}''_x = -\lambda \frac{\partial T}{\partial x}$$

$$\rho C_p \cancel{dx} \frac{\partial T}{\partial t} = +\lambda \frac{\partial^2 T}{\partial x^2} \cancel{dx} \Rightarrow \boxed{\frac{\partial T}{\partial t} = \underbrace{\frac{\lambda}{\rho C_p}}_{\alpha} \frac{\partial^2 T}{\partial x^2}}$$

(b) on seperate

c) Boundary Conditions

at $x=0$ $\quad -\dot{q}''_{liq}(x=0) = h\left(T(x=0) - T_\infty\right)$

Energy $\quad \boxed{+\lambda \frac{\partial T}{\partial x}\Big|_{x=0} = h\left(T_{x=0} - T_\infty\right)}$

I.C. Given
$T(x,0) = T_s$

at $x = X(t)$ $\quad \boxed{T(X(t), t) = T_s}$

$\dot{\xi}$

Energy

condensate T_s
liquid
$\dot{q}''_{cond}$

$$\dot{q}''_{latent} = -\dot{q}''_{cond, liquid}$$

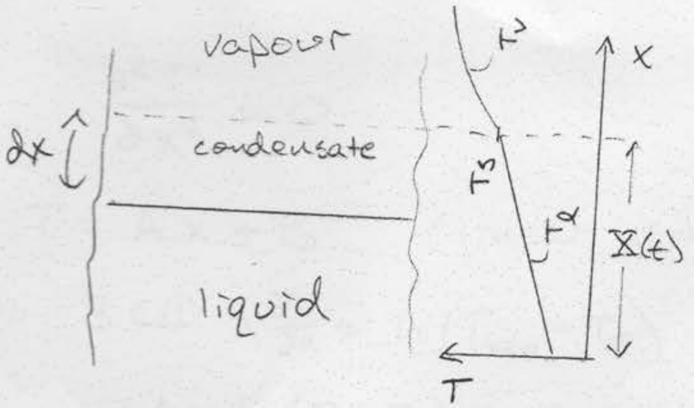
mass flux per unit area

$$\dot{m}'' l_{v \to l} = \lambda \frac{\partial T}{\partial x}\Big|_{x = X(t)}$$

$$\dot{m}'' = \rho \frac{\dot{\forall}}{A} = \rho \frac{dX(t)}{dt}$$

$$\boxed{\rho \, l_{v \to l} \frac{dX}{dt} = \lambda \frac{\partial T}{\partial x}\Big|_{x = X(t)}}$$

b)



$$\dot{q}''_{cond,v} = \dot{q}''_{cond,\ell} + \dot{q}''_{latent}$$

Mass flux per unit area

$$-\lambda \frac{\partial T_v}{\partial x}\bigg|_{x=\bar{X}(t)} dx = -\lambda \frac{\partial T_\ell}{\partial x}\bigg|_{x=\bar{X}(t)} dx + \dot{m}'' l_c$$

$$\dot{m}'' = \frac{\rho \dot{V}}{A} = \frac{\rho A}{A} \frac{d\bar{X}(t)}{dt} l_c$$

total derivative

$$\boxed{-\lambda \frac{\partial T_v}{\partial x}\bigg|_{x=\bar{X}(t)} = -\lambda \frac{\partial T_\ell}{\partial x}\bigg|_{x=\bar{X}(t)} + \rho l_c \frac{d\bar{X}(t)}{dt}}$$

d) $$\frac{\partial^2 T}{\partial x^2} = 0$$

$$T = Ax + B \quad \text{(linear profile)}$$

Into B.C.① $\lambda \frac{\partial T}{\partial x} = h(T_{x=0} - T_\infty)$

$$\lambda A = h(B - T_\infty) \Rightarrow A = \frac{h}{\lambda}(B - T_\infty)$$

B.C.② $T_{x=X(t)} = T_s$

$$T_s = A X(t) + B \Rightarrow T_s = \frac{h}{\lambda}(B - T_\infty) X(t) + B$$

$$B\left(1 + \frac{h}{\lambda} X(t)\right) = T_s + T_\infty \frac{h}{\lambda} X(t)$$

$$B = \frac{T_s + T_\infty\left(\frac{h}{\lambda} X(t)\right)}{1 + \frac{h}{\lambda} X(t)} = \frac{\cancel{h/\lambda}}{\cancel{h/\lambda}}\left(\frac{T_s \lambda/h + T_\infty X(t)}{\lambda/h + X(t)}\right)$$

$$A = \frac{h}{\lambda}\left(\frac{T_s + T_\infty\left(\frac{h}{\lambda} X(t)\right)}{1 + \frac{h}{\lambda} X(t)} - T_\infty\right) = \frac{T_s - T_\infty}{\lambda/h + X[t]}$$

$$T[x] = \frac{(T_s - T_\infty)x + T_s \frac{\lambda}{h} + T_\infty X[t]}{\lambda/h + X[t]}$$

e) $$+\lambda \frac{\partial T}{\partial x} = \rho l_c \frac{dX[t]}{dt}$$

$$\lambda A = \frac{\overbrace{T_s - T_\infty}^{\Delta T}}{1/h + X[t]/\lambda} = \rho l_c X[t]'$$

$$\underbrace{\frac{\Delta T}{\rho l_c}}_{-C} = \underbrace{\frac{1}{h}}_{B} X' + \underbrace{\frac{1}{\lambda}}_{A} XX' = (1/h + X/\lambda)X'$$

Given $(Axx' + Bx + C = 0)$

$$x = \frac{-B \pm \sqrt{B^2 - 2ACt + 2AD}}{A} = \frac{-B \pm \sqrt{B^2 + 2A(D - Ct)}}{A}$$

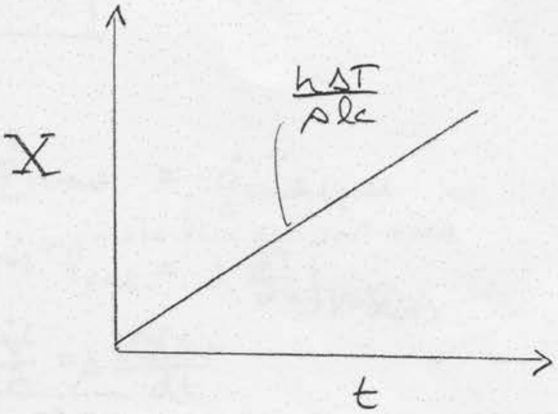
$$X(t) = \frac{-\frac{1}{h} + \sqrt{\frac{1}{h^2} + 2/\lambda\left(D + \frac{\Delta T}{\rho l_c} t\right)}}{1/\lambda}$$

for $X(0) = 0$ $D = 0$

$$X(t) = \frac{\lambda}{h}\left(-1 + \left(1 + \frac{2h^2}{\rho l_c \lambda}(T_s - T_\infty)\, t\right)^{1/2}\right)$$

As $\lambda \to \infty$ $$\frac{\Delta T}{1/h + \cancel{X/\lambda}^{\,\to 0}} = \rho l_c X'[t]$$

$$\boxed{X = \frac{h}{\rho l_c} \Delta T \, t}$$



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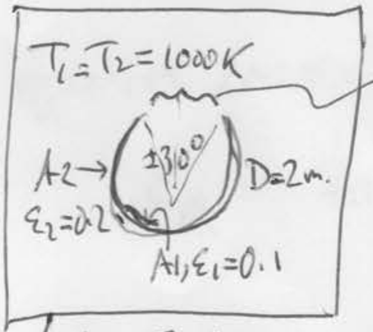


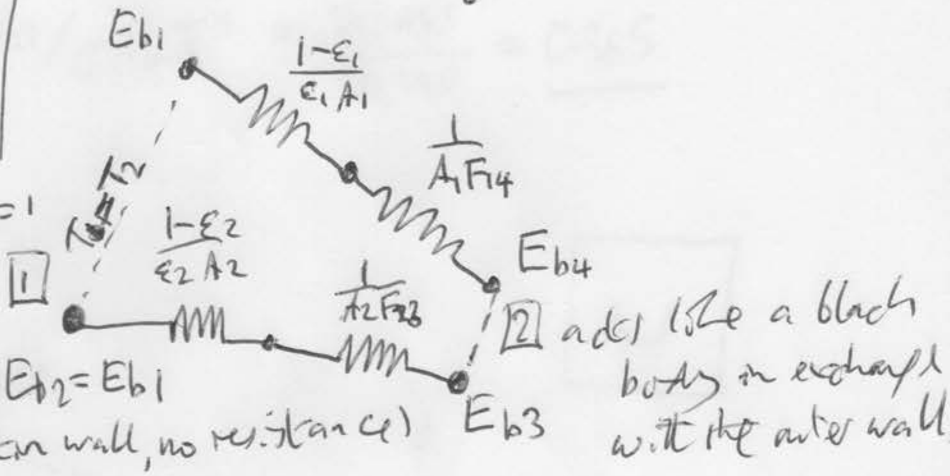
$A_3,\ T_3 = 300K,\ \varepsilon_3 = 1$

$A_4 = DL \sin \pi/6 = \frac{1}{2} DL$

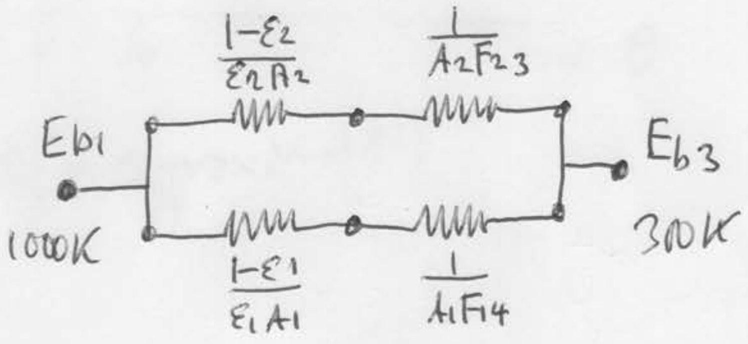
$A_1 = A_2 = DL(\pi - \theta) = \frac{5}{6}\pi DL$

(a)

$\theta = 30° = \pi/6$



Note $\boxed{1}$ & $\boxed{2}$ and re-cast:



(b) $A_1 F_{14} = A_4 F_{41} \longrightarrow \underline{F_{14}} = \frac{A_4}{A_1} \cancel{F_{41}}^{\,1} = \frac{1}{2} \cdot \frac{6}{5\pi} = \underline{0.191}$

$F_{41} = 1 = F_{43}\ ;\ F_{23} = 1$

(c) Total heat to duct (resistors in parallel...)

$\dot{q}_3$ $\quad \dot{Q}_{13_{TOT}} = \frac{E_{b3} - E_{b1}}{R_{TOT}}\ ;\ R_{TOT} = \left[\left\{ \frac{1-\varepsilon_2}{\varepsilon_2 A_2} + \frac{1}{A_2 F_{23}} \right\}^{-1} + \left\{ \frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{14}} \right\}^{-1} \right]^{-1}$

$= --- = 0.707\ 1/m$

"σT^4" ... $= \frac{\overbrace{5.67 \cdot 10^{-8} (1000^4 - 300^4)}^{56,240}}{0.707} \quad \frac{\frac{W}{m^2 K^4} K^4}{1/m} = \underline{79,581\ W/m}$

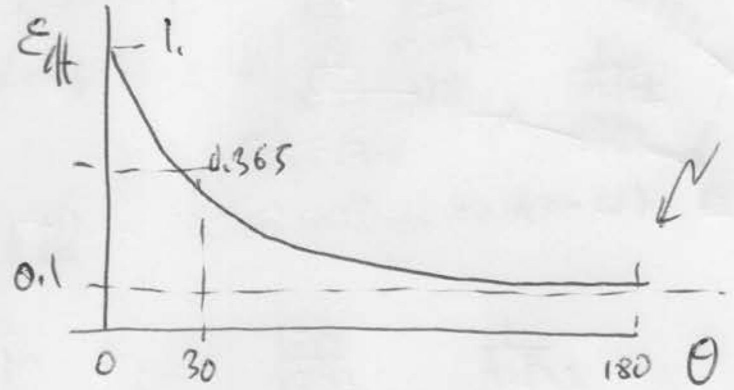
Total heat lost from inner tube

$\dot{q}_1$ $\quad \dot{Q}_{13} = \frac{E_{b1} - E_{b3}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{14}}} = \frac{56,240}{2.72} = \underline{20,685\ W/m}$

(d) effective emissivity

$$\varepsilon_{eff} = \dot{Q}_{13} / \sigma A_4 T_1^4 = \frac{20,685}{56,240} = \underline{0.365}$$

(e)



Sketch (~approximate).

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