3B1 2018 CRIB

1. (a) Power density $= 1.5\,P / 4\pi R^2 = 150 / 4\pi(345\times10^3)^2$

(assume gain $= 1.5$) $= \underline{1.50\times10^{-10}\ \text{W/m}^2} = \frac{1}{2}E^2/\eta$

η = impedance of free space $= 120\pi$

$\therefore\ E = \sqrt{120\pi \cdot 1.5\times10^{-10}\times 2} = \underline{3.37\times10^{-4}\ \text{V/m}}$ [15%]

(b) Power radiated by an antenna, $P_r = \frac{1}{2}I^2 R_r$ where R_r is the radiation resistance

Gain, $G = \dfrac{\text{max. power density radiated}}{\text{power density from an isotropic antenna}}$

Power delivered into a matched load by an antenna is given by the effective aperture (area) A_e x power density of the incident radio wave.

$G = 4\pi A_e / \lambda^2$ Antenna equation [20%]

(c)(i) Area of 2.5 m diameter circle $= 4.91\ \text{m}^2$

$\therefore$ received power $= 1.5\times4.91\times10^{-10}\ \text{W} = \dfrac{V_r^2}{75}$

$\therefore$ signal voltage, $\underline{V_r = 235\ \mu\text{V}_{rms}}$ [10%]

(ii) When closer, the power density increases by $\times\left(\dfrac{345}{5}\right)^2 = \times 4761$ and the signal amplitude by $\sqrt{4761} = \dfrac{345}{5} = 69$ $\therefore$ dB increase

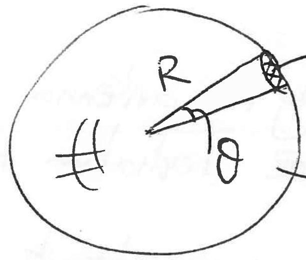
$= 10\log_{10}4761 = 20\log_{10}69 = \underline{36.8\ \text{dB}}$ [10%]

1(c)(iii) Gain of dish antenna = $\frac{4\pi \cdot 4.91}{\lambda^2} = G_d$

where $\lambda = \frac{3 \times 10^8}{913 \times 10^6}$ $\therefore G_d = 570$

$= 0.329$ m

So, if we assume the beam angle is given by $\frac{1}{570} \times 4\pi$ steradians: consider the surface area of a sphere with beam patch on surface:



patch area $= \pi \left(\frac{R\theta}{2}\right)^2$

sphere area $= 4\pi R^2$

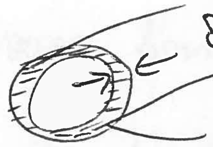
$\therefore 570 = \frac{4\pi \cancel{R^2}}{\frac{\pi}{4} \cancel{R^2} \theta^2}$ $\therefore$ beam angle, $\theta = 0.168$ rad. $(9.6°)$

Assuming the beam centred on Paris, the max. off-axis distance $= \frac{0.168}{2} \times 345 \simeq 29$ km each side off-track.

[20%]

(d) R_r (for cosine dist.) $\simeq 30\pi^2 \left(\frac{\Delta z}{\lambda}\right)^2 \simeq 74\,\Omega$ where Δz is 0.5 effective length, incl. ground plane reflection. (49.3 Ω for lin. dist.)

$R_{ohmic} = \rho \frac{L}{A}$



δ, skin depth

$A = \pi D \delta$

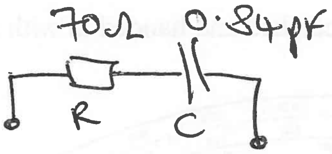
$\delta = \sqrt{\frac{\rho}{\pi f \mu_0}} = 13.9\ \mu m$

$\therefore R_{ohmic} = \frac{6.92 \times 10^{-7} \cdot \frac{0.329}{4}}{13.9 \times 10^{-6} \cdot \pi \cdot 3 \times 10^{-3}} = 1.29\ \Omega$

$\therefore$ Radn. efficiency $= \frac{R_r}{R_r + R_{ohmic}} \simeq 98.3\%$ (or 97.4% linear dist.)

[25%]

2(a)(i) Point A on Smith Chart → $1.4 - j2.4$ on 50Ω system $= 70 - j120\ \Omega$



$$120 = \frac{1}{2\pi f C}$$

$$f = 1575.42 \text{ MHz}$$

$\therefore C = 0.84 \text{ pF}$ [25%]

(a)(ii) At 1227.60 MHz, the imaginary part will increase by $\times \frac{1.575}{1.228}$ to $-j154\ \Omega$ with the real part staying the same.

$\therefore$ Point B $= \dfrac{70 - j154}{50} = 1.4 - j3.08$

Then from Smith Chart $S_{11} = 0.80\angle{-31^\circ}$ [25%]

(a)(iii) For point B $\lambda = 0.208$ (inner-scale), round to point C at $\lambda = 0.199$ (outer scale) clockwise, is a line length of 0.407λ to $1 + j2.7$ at C.

Then series capacitor of $-j2.7 \times 50 = -j135 = X_c$ for matching to O.

At 1228 MHz (f), $C = \dfrac{1}{2\pi f X_c} = 0.96 \text{ pF}$

[25%]

EGT2

ENGINEERING TRIPOS PART IIA

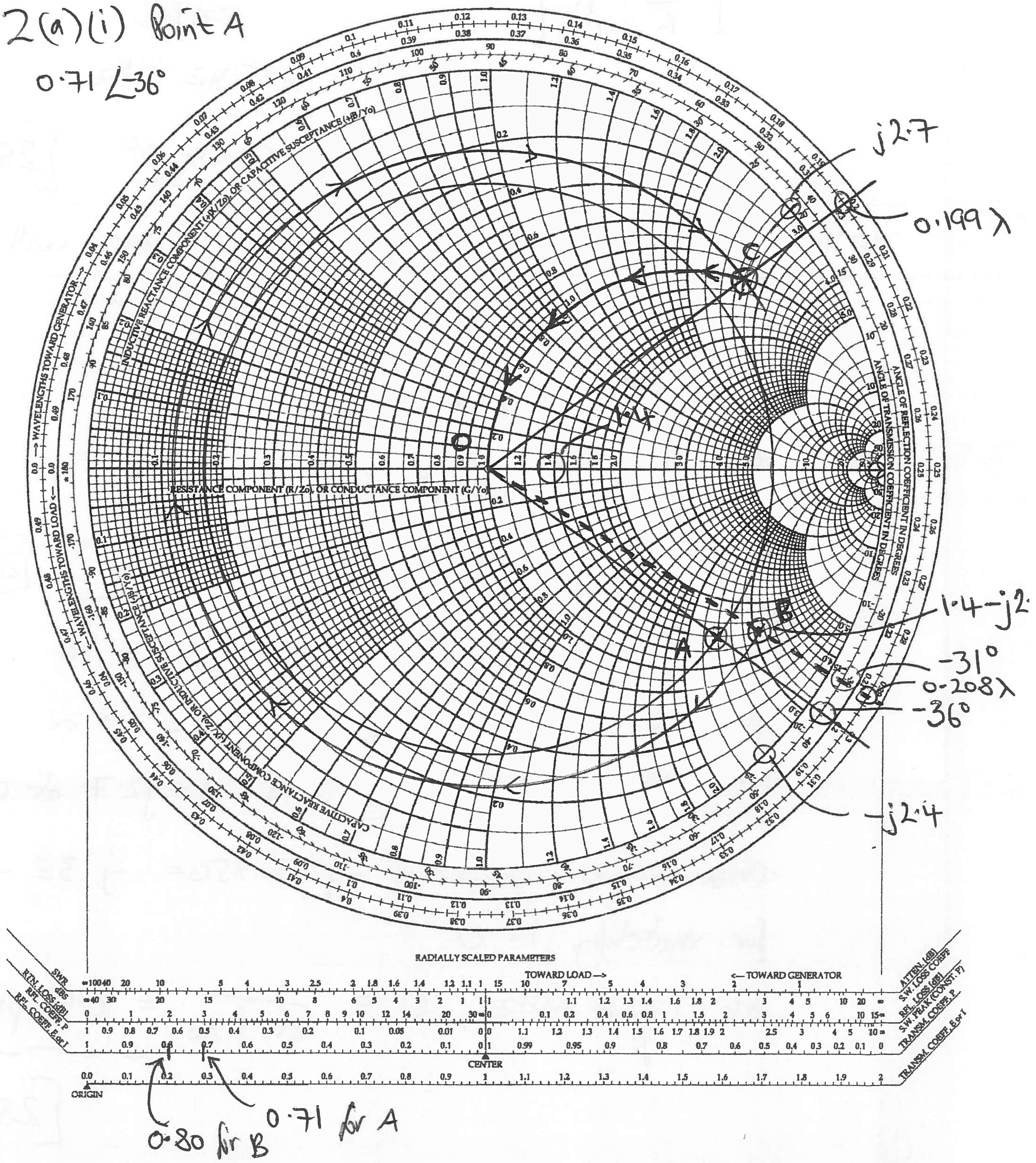
XXday YY April 2018, Module 3B1, Question 2

Candidate No. CRIB

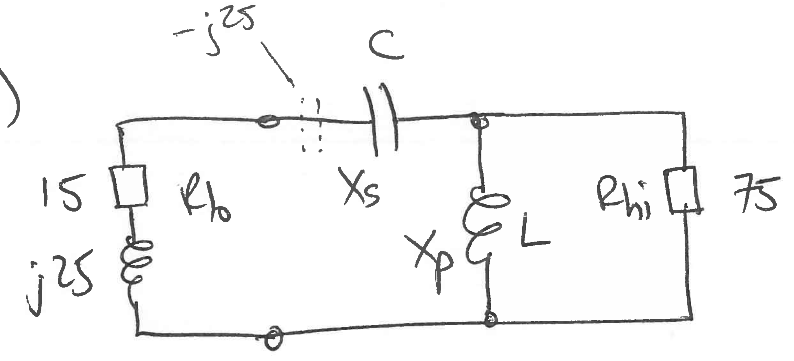
Smith Chart for Question 2 – to be detached and handed in with script.

2(a)(i) Point A

0·71 ∠-36°



2(b)



Matching eqns. $Q = \dfrac{R_{hi}}{X_p} = \dfrac{X_s}{R_{lo}} = \sqrt{\dfrac{R_{hi}}{R_{lo}} - 1}$

$Q = \sqrt{\dfrac{75}{15} - 1} = 2$ (so full ckt. $Q = 1$: very broadband).

$\therefore X_p = \dfrac{75}{2}\,\Omega$, $X_s = 30\,\Omega$ (ignore $j25$ for now)

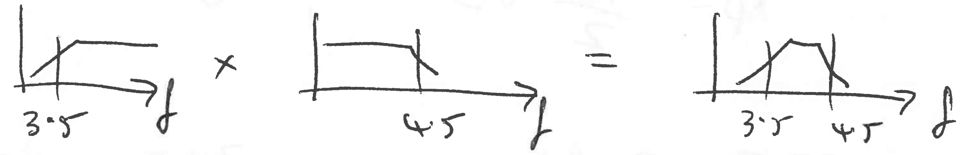
$\therefore 37.5 = 2\pi f L$ $\qquad 30 = \dfrac{1}{2\pi f C}$

$\underline{L = 3.79\text{ nH}}$ $\qquad C = 3.37\text{ pF}$

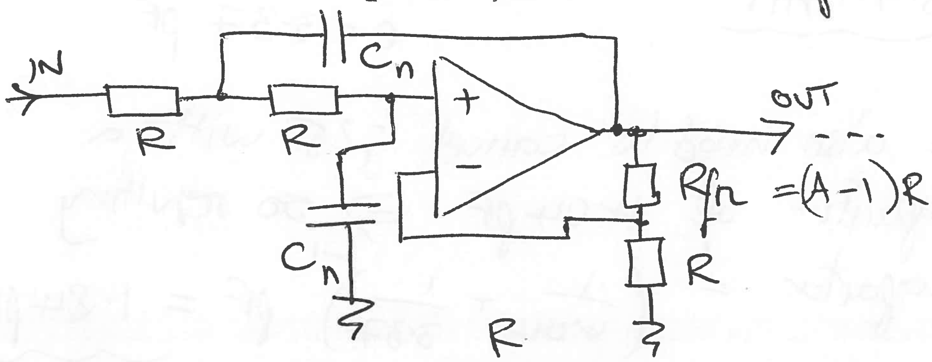
But we also need to cancel $j25$ with a series capacitor of 4.04 pF $\Rightarrow$ so resulting single capacitor $= \left(\dfrac{1}{4.04} + \dfrac{1}{3.37}\right)^{-1}\text{ pF} = \underline{1.84\text{ pF}}$

[25%]

3(a) For 4MHz band-pass filter with 1MHz bandwidth, select cascaded low pass and high pass filters – with low pass cut-off at 4.5 MHz and high-pass cut-off at 3.5 MHz.

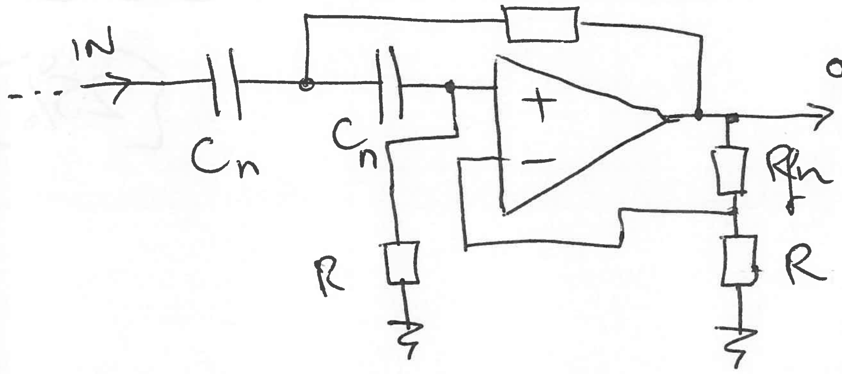


Choose Chebyshev for steepest band edges



Low pass stage

$n = 1$



High pass stage

$n = 2$

Low pass: $f_{3dB} = \dfrac{1}{2\pi RC f_n}$

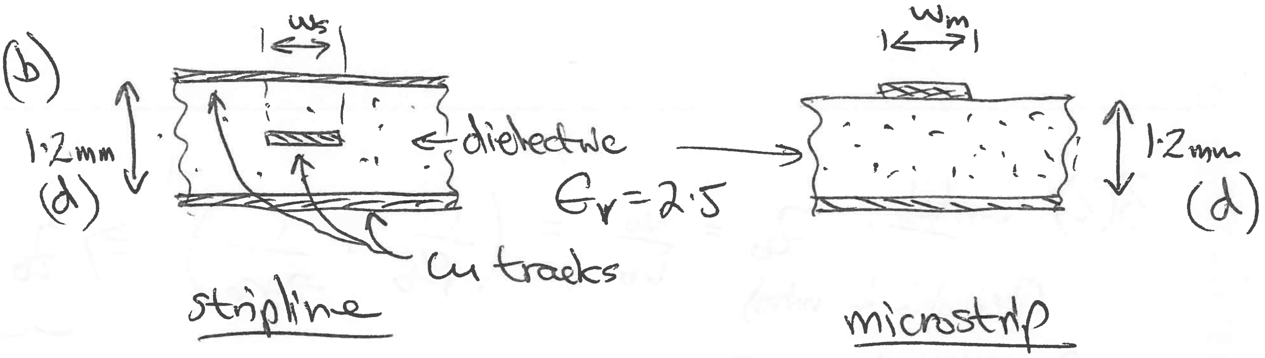
R = 1kΩ througout :

High pass: $f_{-3dB} = \dfrac{f_n}{2\pi RC}$

stage (n)	C_n	R_{fn}	type
1	28.7 pF	842 Ω	low pass
2	56.0 pF	842 Ω	high pass

(35%)

3(b)



Capacitance per m (allows for some fringing width of E field) $C = 2(w_s + d)\dfrac{\epsilon_0 \epsilon_r}{d}$ $C = (w_m + 2d)\dfrac{\epsilon_0 \epsilon_r}{d}$

[wave velocity]

$Z_0 = \sqrt{L/C}$, $v = \dfrac{1}{\sqrt{LC}}$, $v = \dfrac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}}$

$\therefore L = \mu_0 \epsilon_0 \epsilon_r / C$ and $Z_0 = \dfrac{\sqrt{\mu_0 \epsilon_0 \epsilon_r}}{C} = \dfrac{1}{vC}$

with $v = 1·90 \times 10^8$ m/s

$\therefore 75 = \dfrac{1·2 \times 10^{-3}}{1·90 \times 10^8 . 2·5 . 8·854 \times 10^{-12} . (w_s + 1·2 \times 10^{-3}) . 4}$ $\therefore w_s = -0·25$mm

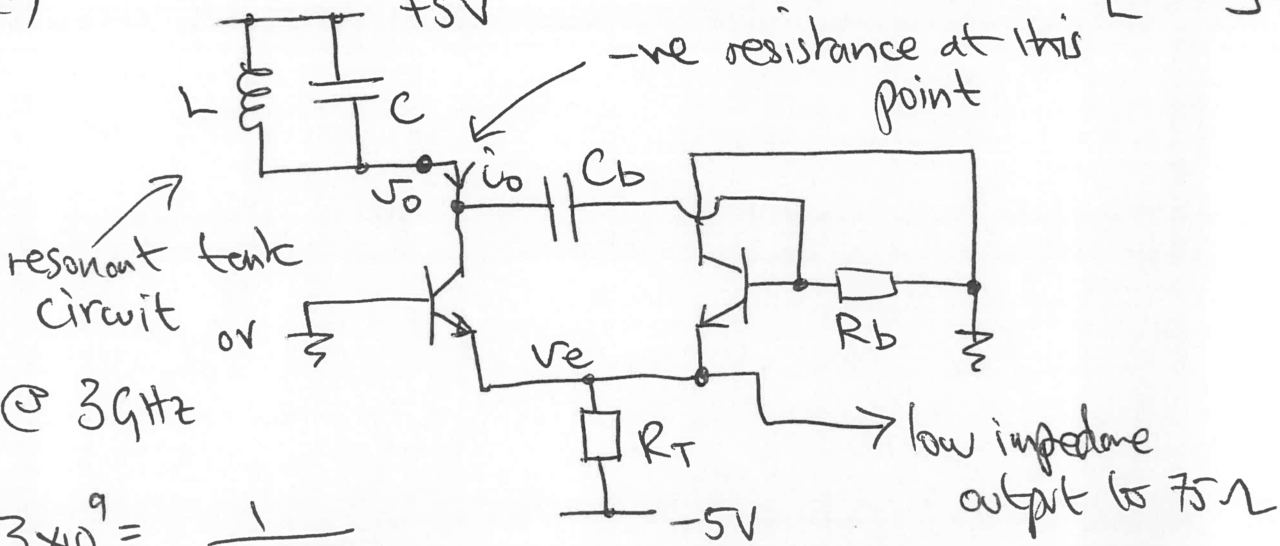
[$\therefore$ not possible in stripline]

or

$75 = \dfrac{1·2 \times 10^{-3}}{1·90 \times 10^8 . 2·5 . 8·854 \times 10^{-12} (w_m + 2·4 \times 10^{-3})}$ $\therefore w_m = 1·40$mm

[35%]

3(c)



@ 3GHz

$3 \times 10^9 = \dfrac{1}{2\pi\sqrt{LC}}$

eg: choose L = 1nH, C = 2·81 pF C_b = 10nF say (large)

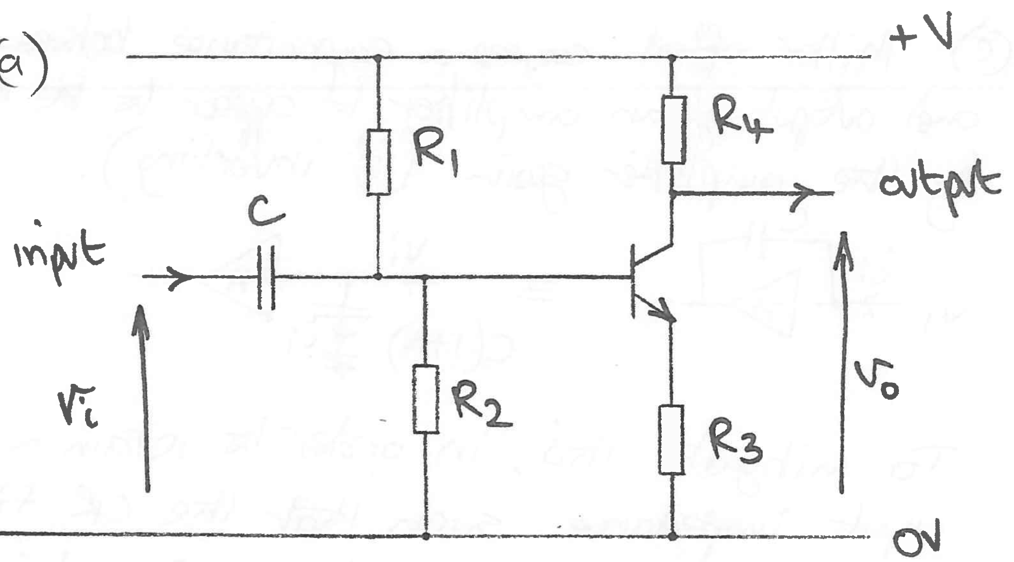
3(c) contd. $Z_o = \frac{V_o}{i_o} = \left(\frac{1}{R_b} + \frac{1}{-2r_e}\right)^{-1} \Rightarrow Z_o = R_b \| -2r_e$

(derivation in notes)

Parasitic loss resistance across L $\approx 2\pi f L Q$ with $Q \sim 20$ say. Hence with $L = 1nH$, loss resistance $\simeq 377\Omega$, so we want a -ve resistance of say less than -50Ω to be sure of oscillation with $R_b = 470\Omega$ and $R_T = 470\Omega$ too. Now $r_e = \frac{0.025}{I_c}$ where I_c is d.c. collector current $= \frac{(5-0.6)}{2 \times 470} = 4.7mA$

$\therefore Z_o = 470 \| -10.6\Omega \therefore$ about -10Ω which will oscillate [30%]

4. (a)



Function of passive components:-

C is the ac signal coupling capacitor - it isolates the dc bias levels between stages

R1 and **R2** form a voltage source to provide a d.c. base bias current,

R3 provides negative feedback to the base bias current to stabilise the bias point (more on this later),

R4 is the collector output load resistor: changes in collector current create an output voltage swing.

[20%]

(b) 20dB of power gain = ×10 linear gain (loaded)

$R_4 = 75\,\Omega$ = output impedance

for $V_o = +6$V dc. bias, then $I_C = \frac{6}{75}\,V/\Omega = 0.08A$

$r_e = \frac{0.025}{I_c} = 0.31\,\Omega$ ∴ for gain of 20, unloaded

$75/(R_3+0.31) = 20$ ∴ $R_3 \simeq 3.4\,\Omega$ say $3.3\,\Omega$ std.

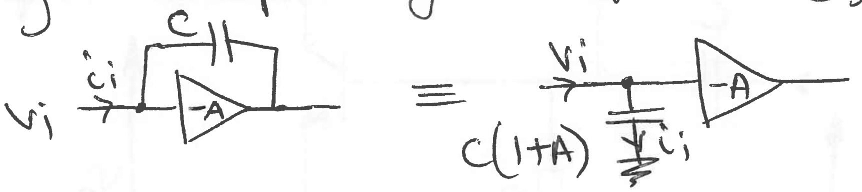
C = 10nF say (large)

R_2 should be $\simeq 1.5 \times 75\,\Omega = 120\,\Omega$ std. (113)

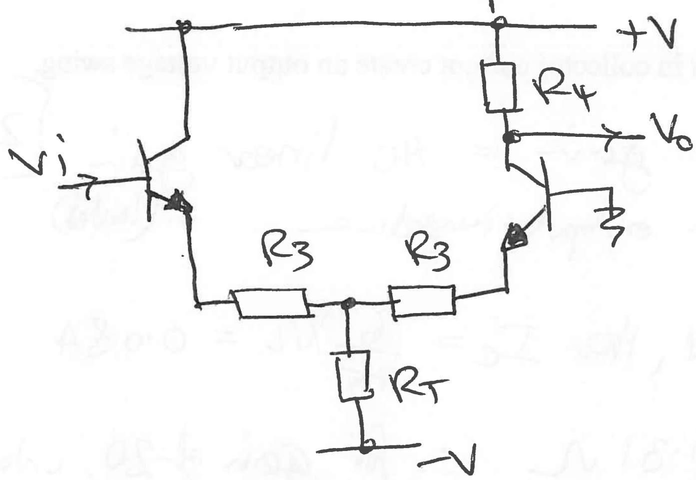
Then for $V_B = 0.65 + 0.08 \times 7.2 = 1.2V$ with +12V supply : $1.2 = \frac{R_2}{R_1+R_2} \times 12 \Rightarrow R_1 = 1k\Omega$

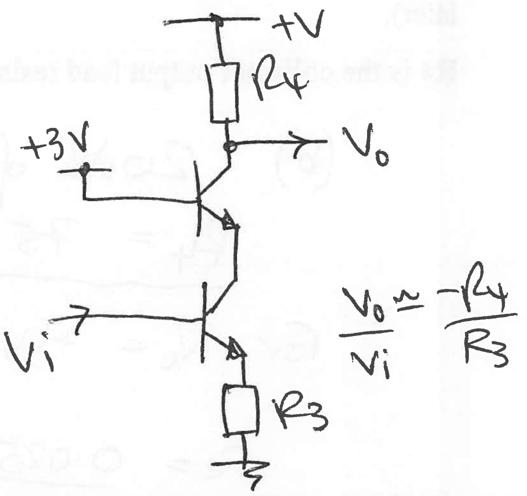
Check $R_{in} = R_1 || R_2 || h_{fe} \times (R_3 + r_e) = 100\,\Omega$; a bit high so reduce R_1 and R_2 by $\simeq 25\%$ to $R_2 = 91\,\Omega$, $R_1 = 750\,\Omega$ then $R_{in} \simeq 77\,\Omega$ (close enough) [25%]

4(c) Miller effect causes a capacitance between input and output of an amplifier to appear to be magnified by the amplifier gain (if inverting).



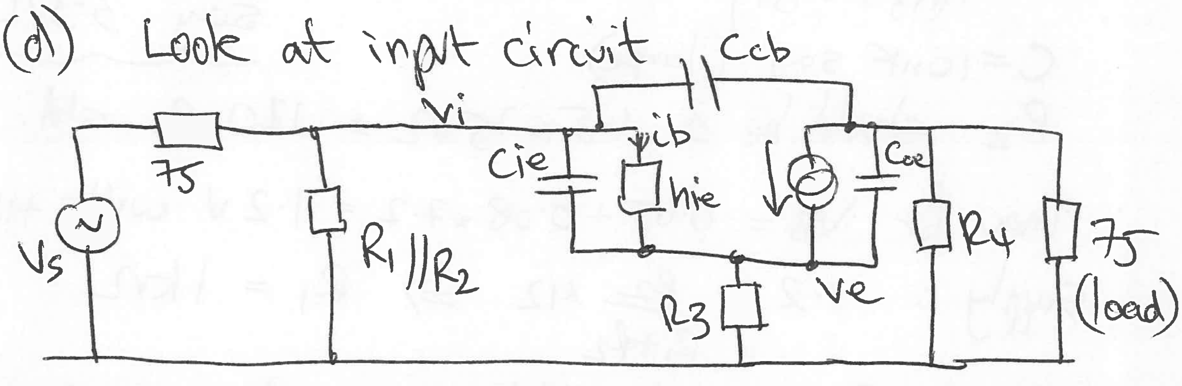
To mitigate this, in order to retain a higher input impedance such that the CR time constant due to the source impedance R and input capac. C remains small, a pair of transistors can be used: one provides voltage gain (with a low input impedance) and the other provides an input path with low capacitance (but no voltage gain). eg:





Differential amp. cascode cct. [20%]

(d) Look at input circuit



$f_t = 22 \times 10^9 = \frac{1}{2\pi C_{ie} r_e}$ ∴ $C_{ie} = 23.3$ pF

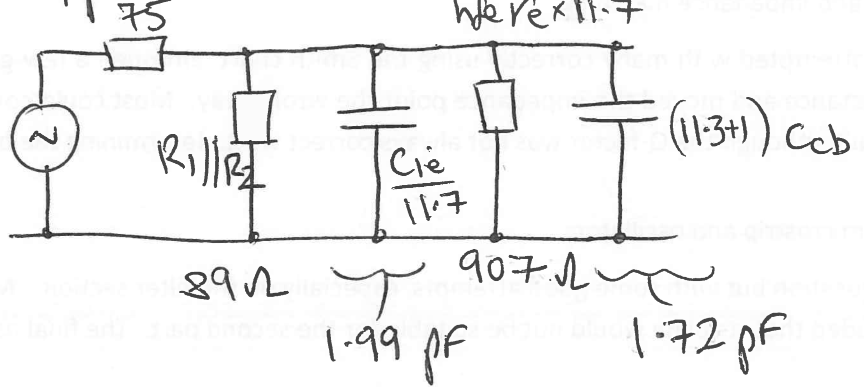
$\frac{V_e}{V_i} = \frac{R_3}{R_3 + r_e} = 0.984$ ∴ impedances C_{ie} & hie

4(d) contd.

are multiplied by $\times \dfrac{1}{(1-0.914)} = \times 11.7$ when referred to ground from base input: $h_{ie} = h_{fe}r_e$

[loaded gain $= \times 11.3$]

So input cct is:



$$\therefore \quad R' = 75 \| 89 \| 907 = 39.0\,\Omega$$

$$C' = 1.99 + 1.72 = 3.71\,\text{pF}$$

$$f_{-3dB} = \frac{1}{2\pi R'C'} = \underline{1.10\ \text{GHz}}$$

Note: output cct. has similar R' but smaller C' $\therefore$ is not the limiting case.

[35%]

Q1 Antennas:

A popular question with many good attempts. Most candidates calculated the field strengths correctly and knew the bookwork terms. Some got a little confused with the dB calculation and the beam angle estimate, although there were some model answers too. A few erroneously neglected the skin depth in the final section.

Q2 Smith chart and impedance matching:

Generally well attempted with many correctly using the Smith chart, although a few got the wrong sign for the reactance and moved the impedance point the wrong way. Most could correctly design a matching circuit although the Q-factor was not always correct w.r.t. determining the bandwidth.

Q3 VCVS filter, microstrip and oscillator:

Least popular question but with some good attempts, especially on the filter section. Many correctly concluded that stripline would not be suitable for the second part. The final oscillator section

Q4 RF amplifier and Miller effect:

Most popular question, and well handled by most. A number of attempts used the wrong gain, either too high or too low but many were correct and calculated the bandwidth correctly.