

1. (a) Power density = $1.5 \cdot P / 4\pi r^2 = 150 / 4\pi (345 \times 10^3)^2$

(assume gain = 1.5) = $\frac{1.50 \times 10^{-10} \text{ W/m}^2}{1} = \frac{1}{2} E^2 / \eta$

$\eta = \text{impedance of free space} = 120\pi$

$\therefore E = \sqrt{120\pi \cdot 1.5 \times 10^{-10} \times 2} = \underline{3.37 \times 10^{-4} \text{ V/m}}$

(b) Power radiated by an antenna, $P_r = \frac{1}{2} I^2 R_r$ [15%]
where R_r is the radiation resistance

Gain, $G = \frac{\text{max. power density radiated}}{\text{power density from an isotropic antenna}}$

Power delivered into a matched load by an antenna is given by the effective aperture (area) $A_e \times$ power density of the incident radio wave.

$G = 4\pi A_e / \lambda^2$ Antenna equation [20%]

(c) (i) Area of 2.5 m diameter circle = 4.91 m^2

$\therefore \text{received power} = 1.5 \times 4.91 \times 10^{-10} \text{ W} = \frac{\sqrt{r}^2}{75}$

$\therefore \text{signal voltage, } \underline{\sqrt{r} = 235 \mu\text{V}_{\text{rms}}}$ [10%]

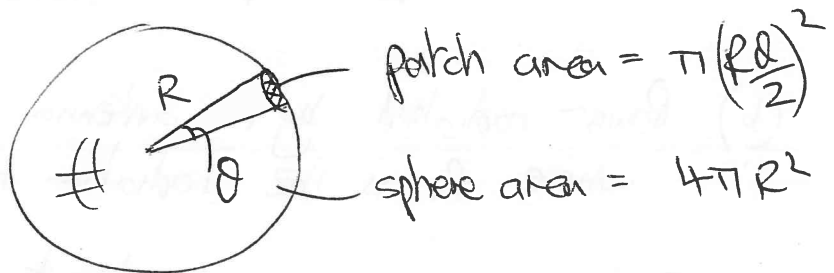
(ii) When closer, the power density increases by
 $\times \left(\frac{345}{5}\right)^2 = \times 4761$ and the signal amplitude
by $\sqrt{4761} = \frac{345}{5} = 69$ \therefore dB increase

$= 10 \log_{10} 4761 = 20 \log_{10} 69 = \underline{36.8 \text{ dB}}$ [10%]

(c)(iii) Gain of dish antenna = $\frac{4\pi \cdot 4.91}{\lambda^2} = G_d$

where $\lambda = \frac{3 \times 10^8}{913 \times 10^6} = 0.329 \text{ m}$ $\therefore G_d = 570$

So, if we assume the beam angle is given by $\frac{1}{570} \times 4\pi$ steradians: consider the surface area of a sphere with beam patch on surface:



$\therefore 570 = \frac{4\pi R^2}{\pi/4 R^2 \theta^2}$

\therefore beam angle, $\theta = 0.168 \text{ rad.}$
(9.6°)

Assuming the beam centred on Paris, the max. off-axis distance = $\frac{0.168}{2} \times 345 \approx 29 \text{ km}$ each side off-track. [20%]

(d) R_r (for cosine dist.) $\approx 30\pi^2 \left(\frac{\Delta z}{\lambda}\right)^2 \approx 74 \Omega$ where Δz is 0.5 effective length, λ incl. ground plane reflection. *Mono pole achieves same current with half the voltage needed for dipole. $\therefore R_{r, \text{mono}} = \frac{74}{2} = 37 \Omega$*

$R_{ohmic} = \frac{\rho L}{A}$ $A = \pi D \delta$, skin depth

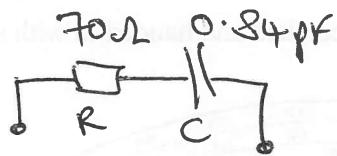
$\delta = \sqrt{\frac{\rho}{\pi f \mu_0}} = 13.9 \mu\text{m}$

$\therefore R_{ohmic} = \frac{6.92 \times 10^{-7} \cdot 0.329}{4} = 1.29 \Omega$

$\frac{13.9 \times 10^{-6} \cdot \pi \cdot 3 \times 10^{-3}}{4}$ (or 95% ~~97.4%~~ linear dist.)

\therefore Radn. efficiency = $\frac{R_r}{R_r + R_{ohmic}} \approx \frac{37}{37 + 1.29} \approx 96.3\%$ [25%]

2(a)(i) Point A on Smith Chart $\rightarrow 1.4 - j2.4$ on 50Ω system = $70 - j120 \Omega$



$$120 = \frac{1}{2\pi f C}$$

$$f = 1575.42 \text{ MHz}$$

$$\therefore C = 0.84 \text{ pF} \quad [25\%]$$

(a)(ii) At 1227.60 MHz , the imaginary part will increase by $\times \frac{1.575}{1.228}$ to $-j154 \Omega$ with the real part staying the same.

$$\therefore \text{Point B} = \frac{70 - j154}{50} = 1.4 - j3.08$$

Then from Smith Chart $S_{11} = 0.80 \angle -31^\circ$ [25%]

(a)(iii) For point B $\lambda = 0.208$ inner-scale, round to point C at $\lambda = 0.199$ outer-scale clockwise, is a line length of 0.407λ to $1 + j2.7$ at C.

Then series capacitor of $-j2.7 \times 50 = -j135 = X_c$ for matching to 0.

$$\text{At } 1228 \text{ MHz, } C = \frac{1}{2\pi f X_c} = 0.96 \text{ pF}$$

[25%]

EGT2

ENGINEERING TRIPOS PART IIA

XXday YY April 2018, Module 3B1, Question 2

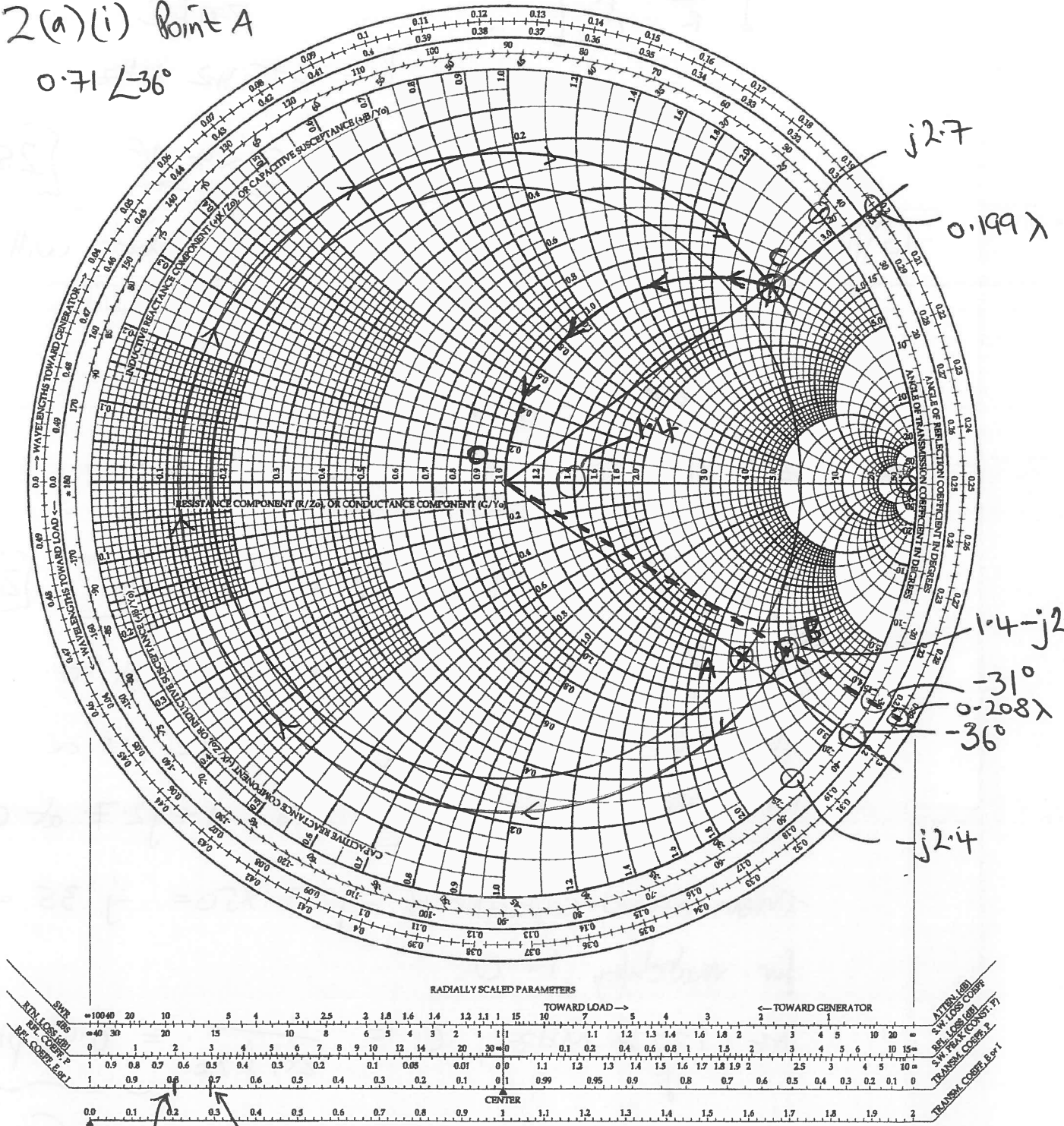
Candidate No.

CR1B

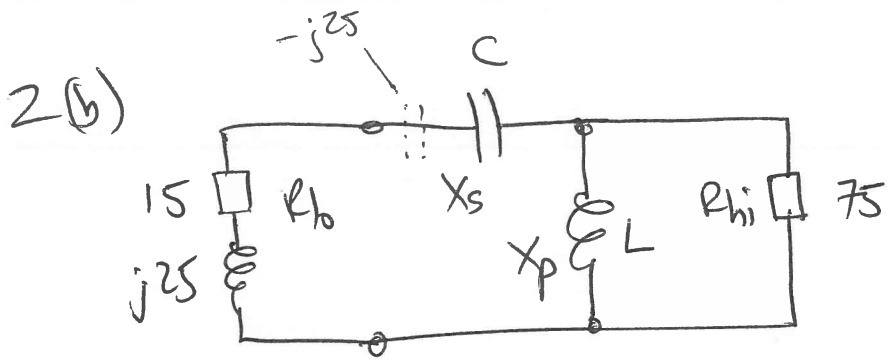
Smith Chart for Question 2 – to be detached and handed in with script.

2(a)(i) Point A

$0.71 \angle 36^\circ$



0.80 for B
 0.71 for A



Matching eqn. $Q = \frac{R_{hi}}{X_p} = \frac{X_s}{R_{b0}} = \sqrt{\frac{R_{hi}}{R_{b0}} - 1}$

$Q = \sqrt{\frac{75}{15} - 1} = 2$ (so all cat. $Q = 1$: very broadband).

$\therefore X_p = \frac{75}{2} \Omega$, $X_s = 30 \Omega$ (ignore $j25$ for now)

$\therefore 37.5 = 2\pi f L$

$L = 3.79 \text{ nH}$

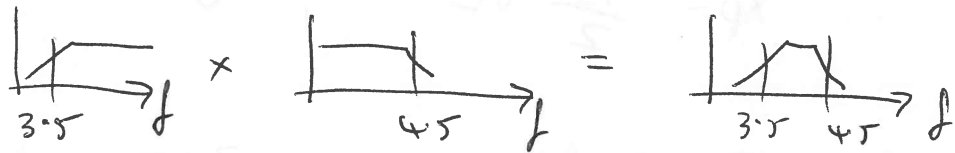
$30 = \frac{1}{2\pi f C}$

$C = 3.37 \text{ pF}$

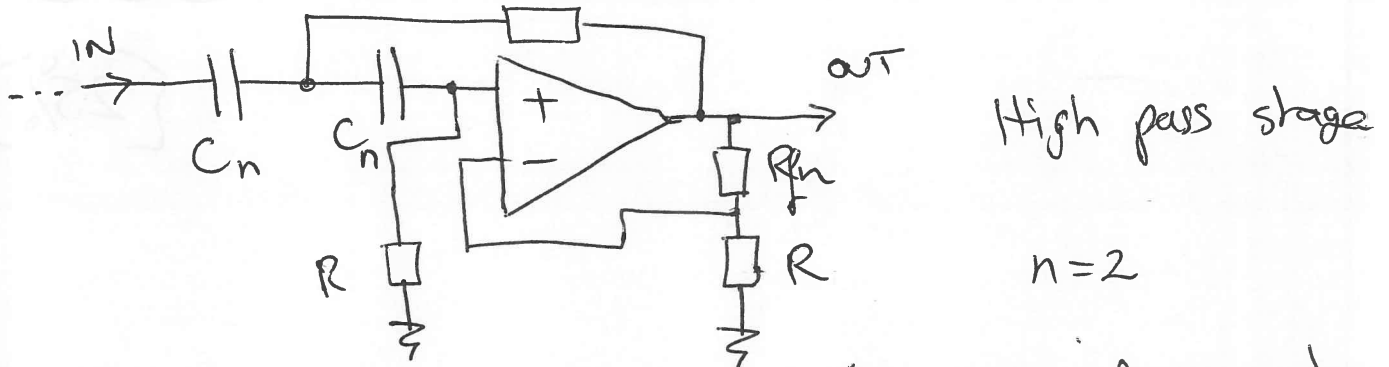
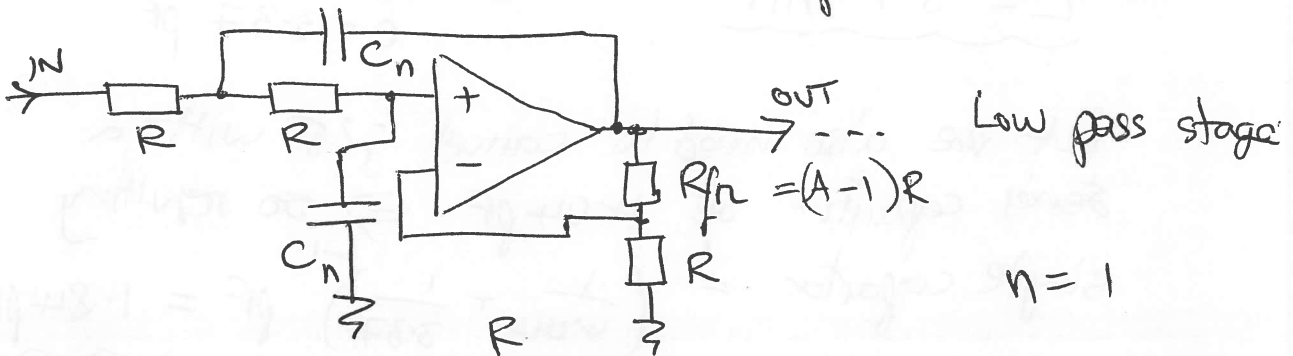
But we also need to cancel $j25$ with a series capacitor of $4.04 \text{ pF} \Rightarrow$ so resulting single capacitor = $\left(\frac{1}{4.04} + \frac{1}{3.37}\right)^{-1} \text{ pF} = \underline{1.84 \text{ pF}}$

$[25\%]$

3(a) For 4MHz band-pass filter with 1MHz bandwidth, select cascaded low pass and high pass filters - with low pass cut-off at 4.5 MHz and high pass cut-off at 3.5 MHz.



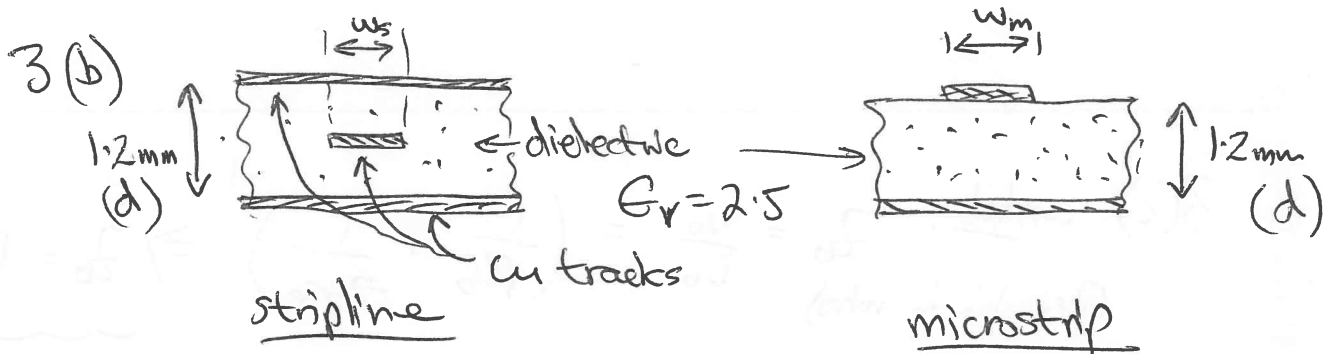
Choose Chebyshev for steepest band edges



R = 1kΩ throughout :

Low pass: $f_{3dB} = \frac{1}{2\pi RC f_n}$
 High pass: $f_{3dB} = \frac{f_n}{2\pi RC}$

Stage (n)	C_n	R_{fn}	Type	Attenuation
1	28.7 pF	842 Ω	low pass	35%
2	56.0 nF	842 Ω	high pass	



Capacitance per m

$$C = \frac{2(w_s + d)\epsilon_0\epsilon_r}{d}$$

(allows for some fringing width of E field)

$$C = \frac{(w_m + 2d)\epsilon_0\epsilon_r}{d}$$

[wave velocity]

$$Z_0 = \sqrt{L/C}, \quad v = \frac{1}{\sqrt{LC}}, \quad v = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_r}}$$

$$\therefore L = \mu_0\epsilon_0\epsilon_r / C \quad \text{and} \quad Z_0 = \frac{\sqrt{\mu_0\epsilon_0\epsilon_r}}{C} = \frac{1}{vC}$$

with $v = 1.90 \times 10^8$ m/s

$$\therefore 75 = \frac{1.2 \times 10^{-3}}{1.90 \times 10^8 \cdot 2.5 \cdot 8.854 \times 10^{-12} \cdot (w_s + 1.2 \times 10^{-3}) \cdot 4}$$

$\therefore w_s = -0.25 \text{ mm}$

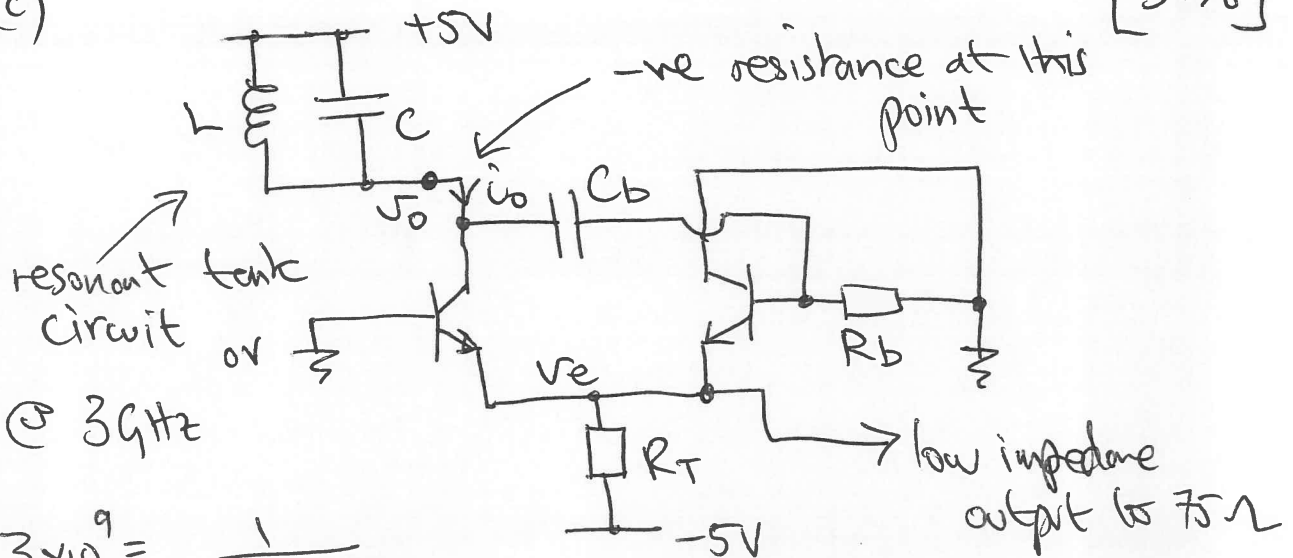
or

$$75 = \frac{1.2 \times 10^{-3}}{1.90 \times 10^8 \cdot 2.5 \cdot 8.854 \times 10^{-12} \cdot (w_m + 2.4 \times 10^{-3})}$$

$\therefore w_m = 1.40 \text{ mm}$

[35%]

3(c)



$$3 \times 10^9 = \frac{1}{2\pi\sqrt{LC}}$$

eg: choose $L = 1 \text{ nH}$, $C = 2.81 \text{ pF}$

$C_b = 10 \text{ nF}$ say (large)

3(c) contd. $Z_o = \frac{V_o}{I_o} = \left(\frac{1}{R_b} + \frac{1}{-2r_e} \right)^{-1} \Rightarrow Z_o = R_b \parallel -2r_e$
 (derivation in notes)

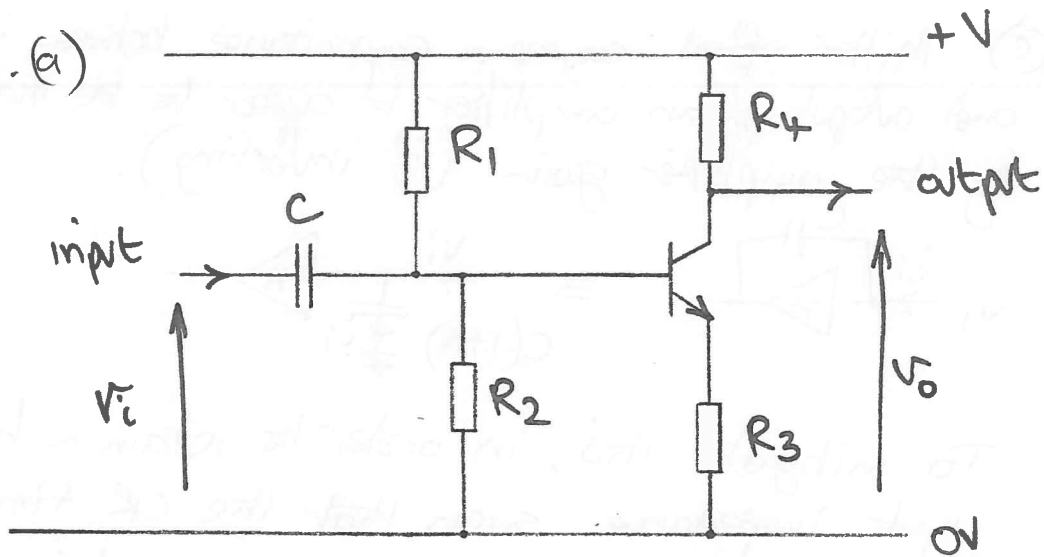
Parasitic loss resistance across $L \approx 2\pi f L Q$ with $Q \sim 20$ say. Hence with $L = 1 \mu\text{H}$, loss resistance $\approx 377 \Omega$, so we want a -ve resistance of say less than -50Ω to be sure of oscillation with $R_b = 470 \Omega$ and $R_T = 470 \Omega$ too. Now $r_e = \frac{0.025}{I_c}$ where I_c is

d.c. collector current = $\frac{(5 - 0.6)}{2 \times 470} = 4.7 \text{ mA}$

$\therefore Z_o = 470 \parallel -10.6 \Omega \therefore$ about -10Ω which will oscillate oscillate [30%]



4. (a)



Function of passive components:-

C is the ac signal coupling capacitor - it isolates the dc bias levels between stages

R1 and **R2** form a voltage source to provide a d.c. base bias current,

R3 provides negative feedback to the base bias current to stabilise the bias point (more on this later),

R4 is the collector output load resistor: changes in collector current create an output voltage swing.

(b) 20dB of power gain = 40 linear gain [20%]
 $R_4 = 75 \Omega$ = output impedance (loaded)

for $V_0 = +6V$ dc. bias, then $I_C = \frac{6V}{75\Omega} = 0.08A$

$r_e = \frac{0.025}{I_C} = 0.31 \Omega$ \therefore for gain of 20, unloaded.

$75 / (R_3 + 0.31) = 20 \therefore R_3 \approx 3.4 \Omega$
 say 3.3Ω std.

$C = 10nF$ say (large)

R_2 should be $\leq 1.5 \times 75 \Omega = 120 \Omega$ std. (113)

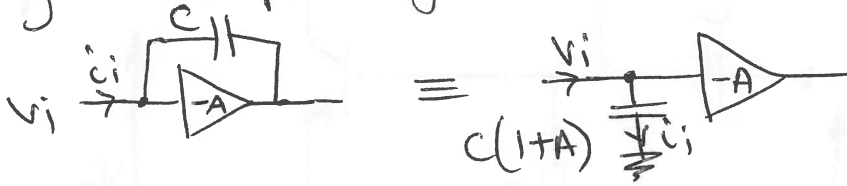
Then for $V_B = 0.65 + 0.08 \times 7.2 = 1.2V$ with +12V

Supply: $1.2 = \frac{R_2}{R_1 + R_2} \times 12 \Rightarrow R_1 = 1k\Omega$

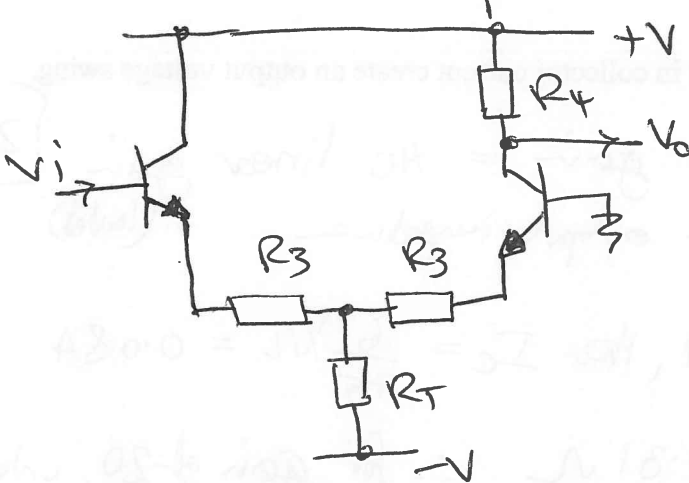
check $R_{in} = R_1 \parallel R_2 \parallel h_{fe} \times (R_3 + r_e) = 100 \Omega$; a bit high so reduce R_1 and R_2 by $\approx 25\%$ so $R_2 = 91 \Omega$,

$R_1 = 750 \Omega$ then $R_{in} \approx 77 \Omega$ (close enough) [25%]

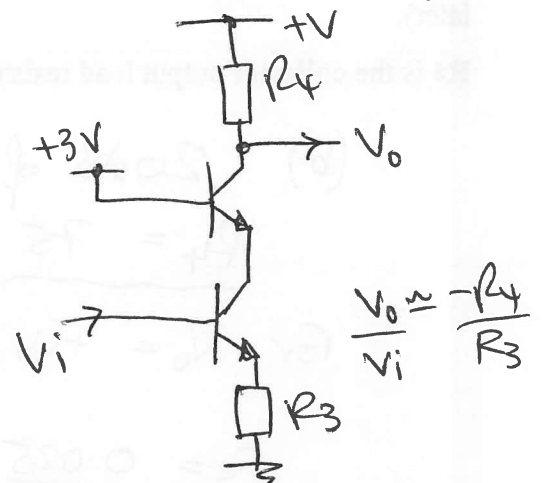
4(c) Miller effect causes a capacitance between input and output of an amplifier to appear to be magnified by the amplifier gain (if inverting).



To mitigate this, in order to retain a higher input impedance such that the CR time constant due to the source impedance R and input cap. C remains small, a pair of transistors can be used: one provides voltage gain (with a low input impedance) and the other provides an input path with low capacitance (but no voltage gain). Eg:

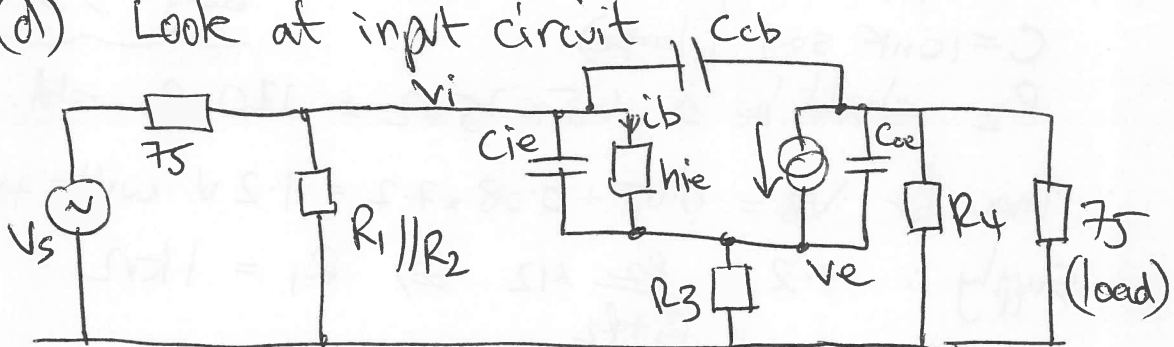


Differential amp.



Cascode ckt. [20%]

(d) Look at input circuit



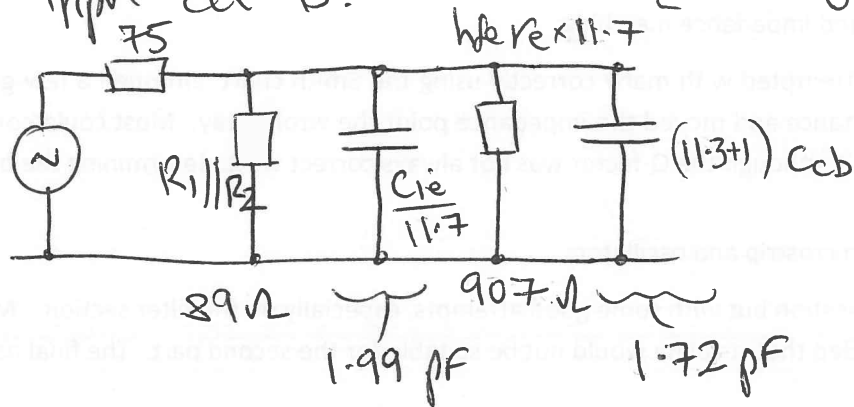
$$f_c = 22 \times 10^9 = \frac{1}{2\pi C_{ie} R_e} \quad \therefore C_{ie} = 23.3 \text{ pF}$$

$$\frac{v_e}{v_i} = \frac{R_3}{R_2 + R_3} = 0.914 \quad \therefore \text{impedances } C_{ie} \text{ \& } h_{ie}$$

4(d) contd.

are multiplied by $\times \frac{1}{(1-0.914)} = \times 11.7$ when referred to ground from base input: $h_{ie} = h_{fe} r_e$

So input ckt is: [loaded gain = $\times 11.3$]



$$\therefore R' = 75 \parallel 89 \parallel 907 = 39.0 \Omega$$

$$C' = 1.99 + 1.72 = 3.71 \text{ pF}$$

$$f_{-3dB} = \frac{1}{2\pi R' C'} = \underline{\underline{1.10 \text{ GHz}}}$$

Note: output ckt. has similar R' but smaller C' \therefore is not the limiting case.

[35%]

Q1 Antennas:

A popular question with many good attempts. Most candidates calculated the field strengths correctly and knew the bookwork terms. Some got a little confused with the dB calculation and the beam angle estimate, although there were some model answers too. A few erroneously neglected the skin depth in the final section.

Q2 Smith chart and impedance matching:

Generally well attempted with many correctly using the Smith chart, although a few got the wrong sign for the reactance and moved the impedance point the wrong way. Most could correctly design a matching circuit although the Q-factor was not always correct w.r.t. determining the bandwidth.

Q3 VCVS filter, microstrip and oscillator:

Least popular question but with some good attempts, especially on the filter section. Many correctly concluded that stripline would not be suitable for the second part. The final oscillator section

Q4 RF amplifier and Miller effect:

Most popular question, and well handled by most. A number of attempts used the wrong gain, either too high or too low but many were correct and calculated the bandwidth correctly.