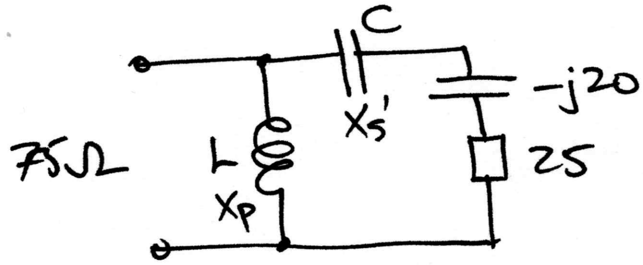
3B1 CRIB 2024

1 (a) $Q = \sqrt{\frac{R_{hi}}{R_{lo}} - 1} = \frac{X_s}{R_{lo}} = \frac{R_{hi}}{X_p} = \sqrt{2}$ with

$R_{hi} = 75\,\Omega$

$R_{lo} = 25\,\Omega$



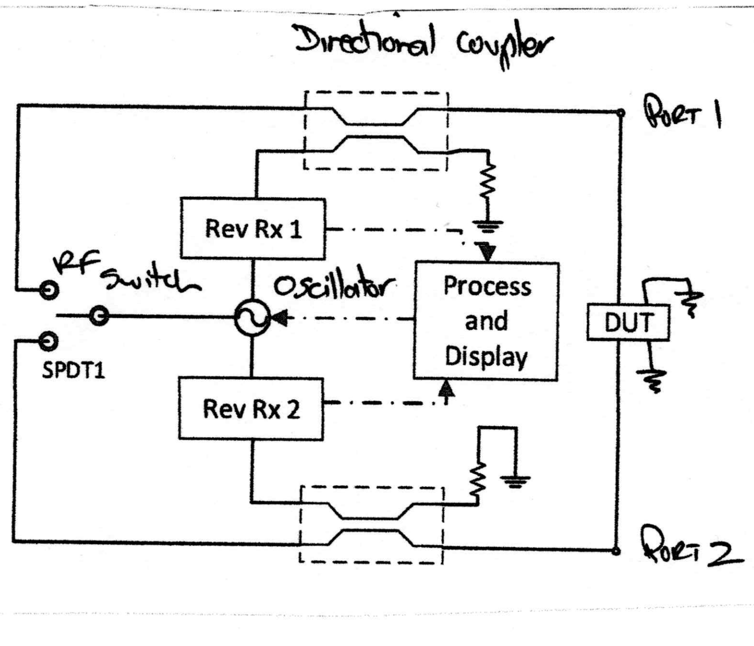
use series C and parallel L to allow both C's to be combined into one for X_s: $X_s' = X_s - 20$ ($\uparrow$ 35.4)

$\therefore X_s' = 15.4 = \frac{1}{2\pi f C}$ $\quad \therefore C = 47\,\text{pF}$

$X_p = 53 = 2\pi f L$ $\quad \therefore L = 38.3\,\text{nH}$

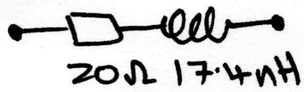
(b) The core of the VNA is an oscillator which can be switched between 2 ports. It is also coupled to two receivers with one connected to each port through a directional coupler so that signals into each port can be measured (it is also common for the termination resistors shown above to be replaced by reference receivers). The receivers normally comprise an arrangement so that both a amplitude of the signal into a port and its phase can be determined. At a basic level S_{11} can be found by considering the signal at Rev Rx 1 when the oscillator is applied to Port 1 and S_{21} by considering the signal at Rev Rx 2 etc. The results are then post-processed for display (often as a smith chart). As the same oscillator is used for transmission and receivers, very narrowband filtering is possible allowing the instrument to have a very wide dynamic range (often 90dB).

The directional couplers are often not ideal, and many measurements require a phase reference at the end of the measurement cables rather than the ports of the device, so calibration should be carried out before any measurements are taken. This is commonly done by attaching, open, short loads to port 1 and 2 (followed by a thru).



1(c)(i) Point A: $20 + j100\,\Omega$ @ 915 MHz

$\therefore 100 = 2\pi f L \quad \therefore L = 17.4\text{ nH}$



(ii) $A \to B$ on chart $= 0.208\lambda - 0.178\lambda = 0.03\lambda$

Point B: $1 + j3.5 \quad \therefore -j3.5$ series capacitor reqd to match to O. $3.5 \times 50 = \dfrac{1}{2\pi f C}$ with $f = 915$ MHz

$\therefore \underline{C = 1.0\text{ pf}}$ For coax. cable with $Z_0 = 50\,\Omega$ and $C = 85$ pf/m : $v = f\lambda = \dfrac{1}{\sqrt{LC}}$

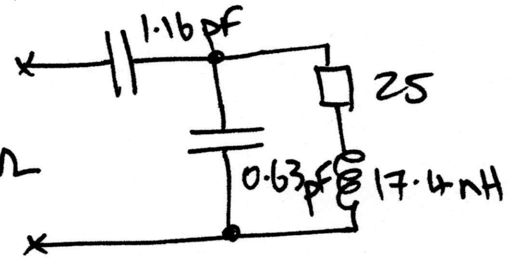
and $Z_0 = \sqrt{L/C} = \dfrac{1}{vC} \quad \therefore 50 = \dfrac{1}{f\lambda C}$ where

$f = 915 \times 10^{6}$, $C = 85 \times 10^{-12} \quad \therefore \lambda = 0.257$ m hence, length of coax. cable $= 0.03 \times 257$ mm $= \underline{7.7\text{ mm}}$

(iii) work with admittance for parallel matching component: point $A \to C$ (through 180°), then parallel capacitor $C \to D$ with susceptance of $(0.48 - 0.30) = 0.18 \quad \therefore 2\pi f C = 0.18/50$

$\therefore C_p = 0.63$ pf. Then change back to impedance $D \to E$ where $E = 1 + j3.0$ and cancel $j3.0$ with a series capacitor of $-j3.0$ where $50 \times 3 = \dfrac{1}{2\pi f C} = 150$ and $C_s = 1.16$ pF.

@ 915 MHz : 50Ω



(or can use $C \to D'$ with larger capacitor in parallel and E' to O with series inductor).

1(c)

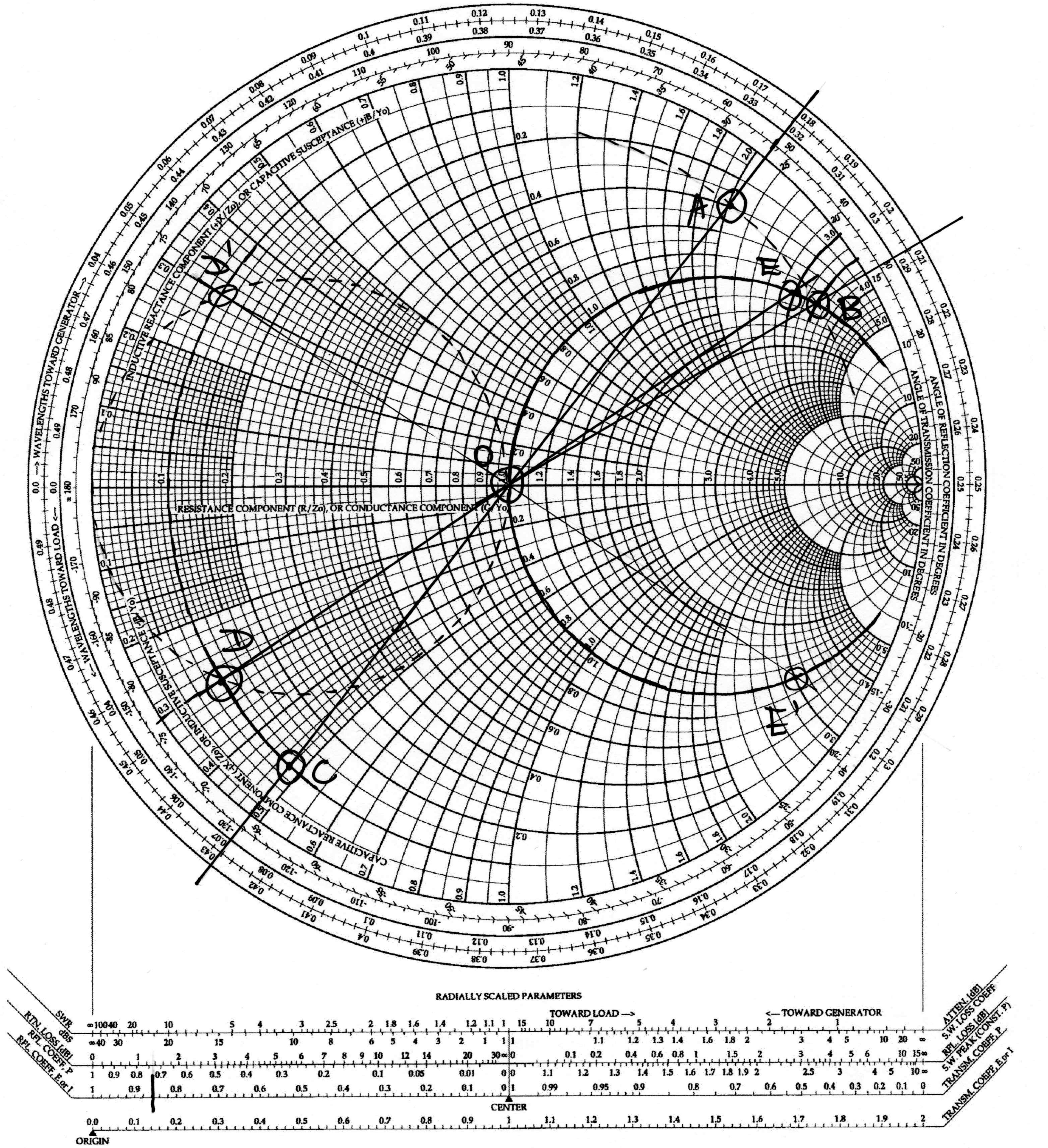
EGT2

ENGINEERING TRIPOS PART IIA

XXday YY April 2024, Module 3B1, Question 1

Candidate No. CRIB

Smith Chart for Question 2 -- to be detached and handed in with script.



Point A : $0{\cdot}86\angle 52^\circ = 0{\cdot}4 + j2{\cdot}0 \rightarrow \underline{20 + j100\ \Omega}$

2. a)

i) XOR gate performs phase detection. Output is 5V or 0V effectively pulse width modulated proportional to phase difference.

RC filter low passes to convert the XOR output to be an analog voltage proportional to phase difference and provides damping

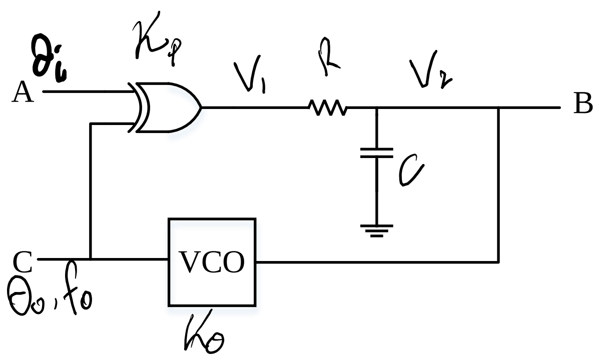
VCO is the voltage controlled oscillator to be phase locked to the input.

X is the input of the reference signal (digital bit stream in this case)

Y is the voltage portional to the phase difference

Z is the frequency output (the clock)

ii)



$V_1 = K_p(\theta_0 - \theta_{\text{ref}})$

$$2\pi f_o = \frac{d\theta_o}{dt} = j\omega\theta_o = K_oV_2$$

$$\frac{V_2}{V_1} = \frac{1}{1 + j\omega CR}$$

$$j\omega\theta_o = \frac{V_1K_o}{1 + j\omega CR} = \frac{K_p(\theta_0 - \theta_{\text{ref}})K_o}{1 + j\omega CR}$$

$$j\omega\theta_o - \omega^2CR\theta_o - K_oK_P\theta_o = -K_oK_p\theta_i$$

Since $\theta_o \equiv e^{j(\omega t + \theta)}$, $\omega^2\theta_o = \ddot{\theta}_o, j\omega\theta_o = \dot{\theta}_o$

$$\dot{\theta}_o - \ddot{\theta}_oCR - K_oK_P\theta_o = -K_oK_p\theta_i$$

$$\frac{\ddot{\theta}_o\, CR}{K_oK_p} - \frac{\dot{\theta}_o}{K_oK_p} + \theta_o = \theta_i$$

Standard form from mech data book (p 8 and 9)

$$\ddot{y}/\omega_n^2 + 2\zeta\,\dot{y}/\omega_n + y = x$$

So $\omega_n^2 = \frac{K_oK_p}{CR}, \zeta = -\frac{\omega_n}{2K_oK_p}$

$$\frac{\ddot{\theta_o}}{\omega_n^2} + \frac{2\zeta}{\omega_n}\dot{\theta_o} + \theta_o = \theta_i$$

For 10% overshoot $\zeta = 0.6$

$K_o = 2MHz/V$= $4\pi \times 10^6$rad/s/V

$K_p = 5V$ for ±180 deg. = ±1.59 V/rad

$$0.6 = \frac{\omega_n}{4\pi \times 10^6 * 1.59 * 2}$$

$$\omega_n = 2.4 \times 10^7 \ rad/s$$

$$\omega_n^2 = \frac{K_o K_p}{CR}$$

So $CR = 34$ns

(b)(i)

Assume that fringing fields extend either side of track by d

$$C = \frac{(w + 2d\,)\epsilon_0 \epsilon_r}{d}$$

$Z_0 = \sqrt{\frac{L}{C}}, v = \frac{1}{\sqrt{LC}}, c_o = \frac{1}{\sqrt{\mu_o \epsilon_0}}$

In dielectric

$$v = \frac{1}{\sqrt{\epsilon_o \epsilon_r \mu_0}} = \frac{c_0}{\sqrt{\epsilon_r}}$$

$$Z_0 = \frac{1}{vC} = \frac{\sqrt{\epsilon_r}}{c_0}\frac{d}{(w + 2d\,)\epsilon_0 \epsilon_r}$$

So for 50 ohms we get

$$w = \frac{d}{Z_0 \epsilon_0 \sqrt{\epsilon_r} c_0} - 2d$$

W=1.4mm

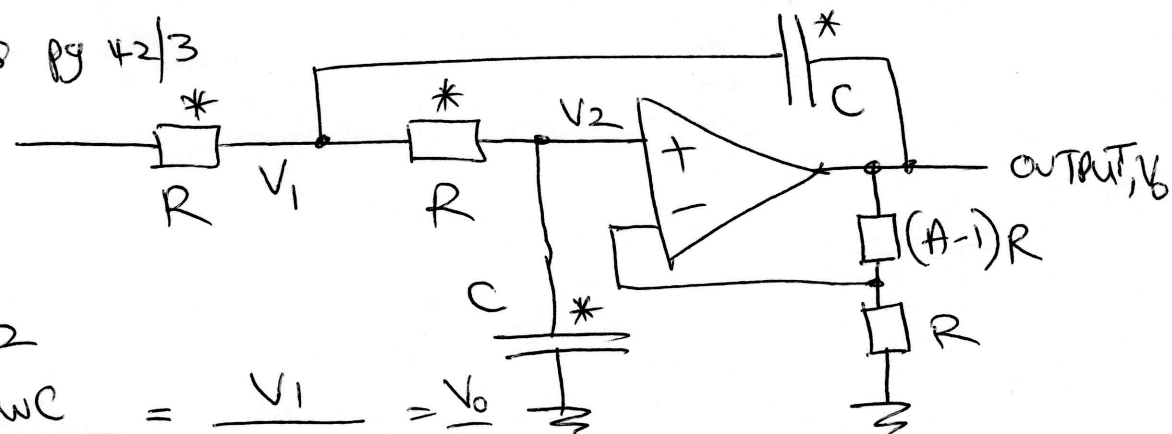
(ii) For circular polarisation need to have a 90° phase shift. A Wilkinson splitter produces a split with zero phase difference.

$$v = \lambda f$$

At 3GHz. $\lambda = 5cm$ in the dielectric.

So for 90 deg need 5/4 = 1.25cm difference

3 (a) Notes pg 42/3



$V_o = A V_2$

$$V_2 = \frac{V_1 / j\omega C}{R + \frac{1}{j\omega C}} = \frac{V_1}{(1 + j\omega CR)} = \frac{V_o}{A}$$

$$\frac{V_i - V_1}{R} = \frac{V_1 - V_o}{1/j\omega C} + \frac{V_1 - V_2}{R} \Rightarrow V_i = V_1(2 + j\omega CR) - V_o j\omega CR - V_2$$

$$V_i = \frac{V_o(1 + j\omega CR)(2 + j\omega CR)}{A} - V_o j\omega CR - \frac{V_o}{A}$$

$$= \frac{V_o}{A}\left[1 - (\omega CR)^2 + j\omega CR(3 - A)\right]$$

$$\therefore \left|\frac{V_o}{V_i}\right| = A\left[\left(1 - (\omega CR)^2\right)^2 + (\omega CR)^2(3-A)^2\right]^{-1/2}$$

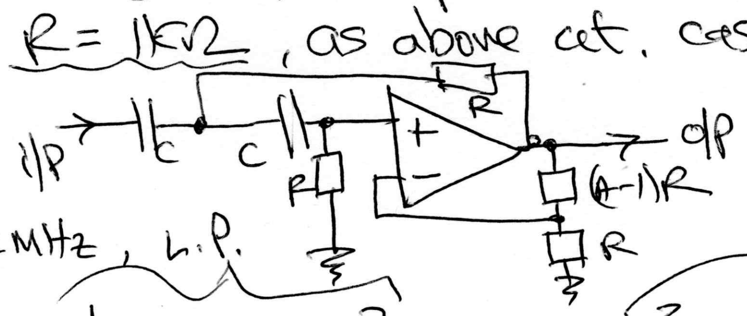
$$= A\left[1 + \underbrace{(\omega CR)^2\left[(3-A)^2 - 2\right]}_{\text{if this } = 0 \text{ then we have Butterworth}} + (\omega CR)^4\right]^{-1/2}$$

ie: $\left|\frac{V_o}{V_i}\right| = \left[1 + \left(\frac{\omega}{\omega_n}\right)^4\right]^{-1/2}$ with $\omega_n = \frac{1}{CR}$ ie $A = 1.586$

and $f = \omega/2\pi$, $f_c = \omega_n/2\pi$

For high pass, swap R's and C's at input (* above)

(b) Use Chebyshev for steepest edges of f cut-off with R = 1kΩ, as above cct. cascaded with H.P. filter

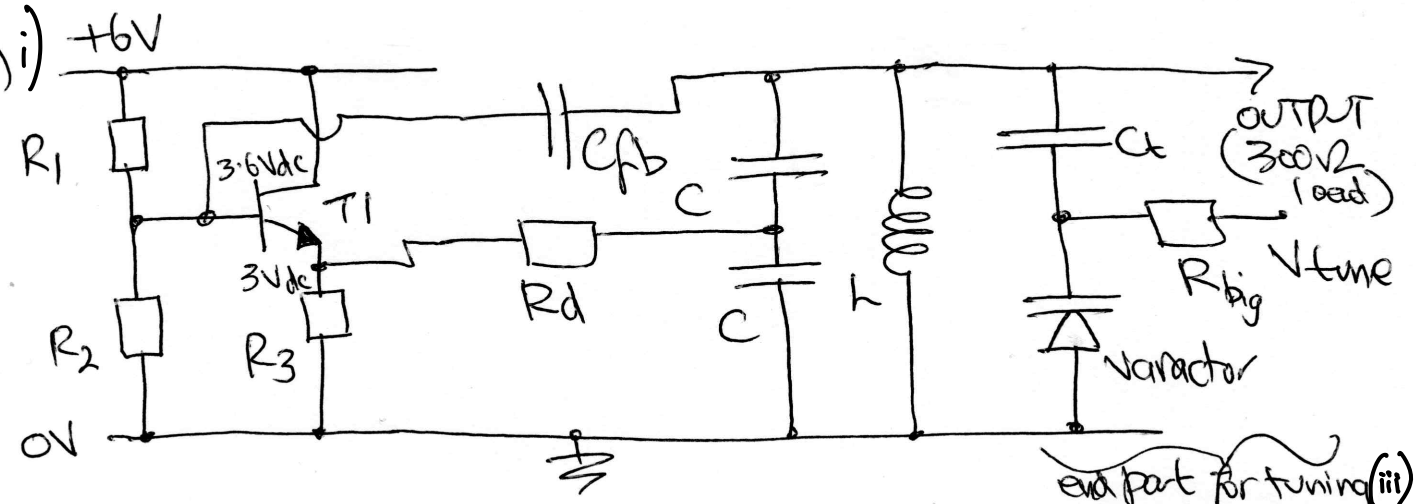


1·2MHz, L.P. H.P., 0·8MHz

STAGE	1	2	3	4
(A-1)R	582	1·66k	582	1·66k
C	222p	129p	119p	205p

$$C_{L.P.} = \frac{1}{2\pi R f_{-3dB} f_n} \qquad C_{H.P.} = \frac{f_n}{2\pi R f_{-3dB}}$$

3 (c) i)



ii)

Check amplitude: $10\,dBm \Rightarrow 0.01W = \frac{V_{rms}^2}{300} \Rightarrow V = 1.7V_{rms} = 4.8V_{pp}$ ✓ o.k.

$f = 1090\,MHz = \frac{1}{2\pi\sqrt{L\frac{C}{2}}}$ with $L = 5\,nH$

$\frac{C}{2} = 4.26pF \Rightarrow C = 8.52pF$

$R_3 = 100\Omega$ say $(\simeq 1.5 \times \frac{300}{4}\Omega$ load).

$R_1 = 1k\Omega$ say, set R_2 for $V_B = 3.6V_{dc} = \frac{6.1}{(1+R_2)}$

$\therefore R_2 = 670\Omega$

(iii)

then: $I_c = \frac{3V}{100\Omega} = 30mA$

$\therefore r_e = \frac{25}{30} = 0.83\Omega$

$C_{fb} = C_t = 10\text{–}100 \times C \simeq 470pF$ or more

Check LC ‖ load for Rd calc., $\omega LQ = 685\Omega$

load $h_{fe}(R_3 + r_e) = 20k\Omega$

$\therefore R_{total} = \underset{R_1}{1000} \| \underset{R_2}{670} \| \underset{\omega LQ}{685} \| \underset{h_{fe}(R_3+r_e)}{20k} \| \underset{load}{300}$

$= 137\Omega$

$\therefore Rd = \frac{137}{5} \simeq 27\Omega$, then $C_{ie} = \frac{1}{2\pi r_e f_t} = 24pF$

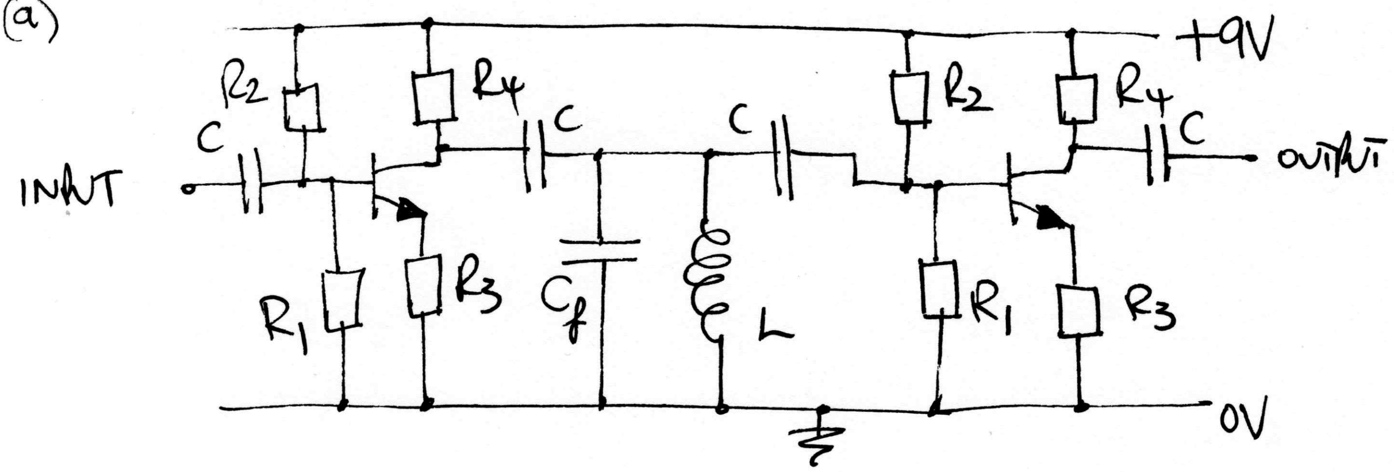
Effect of C_{ie} is reduced by emitter feedback: $G_e = \frac{R_3}{(R_3 + r_e)} = 0.9918$

$\therefore C_{ie}(1 - G_e) = 0.20\,pF$

$C_{cb} = 0.25\,pF$ $\therefore 0.45\,pF$ on top of $\frac{C}{2}$ (4.26pF)

This is about 10%, so freq. will drop by ~5%. To compensate, reduce C by about 1pF and add back with varactor tuning range. Can compensate for temperature changes too.

4 (a)



C, coupling caps. block d.c. C_f, L resonant // filter
R4 output resistor : sets Rout + gain
R3 emitter fback : sets gain
R1, R2 base bias resistors

(b) For LC resonant cct. $f = 1030 \times 10^6 = \frac{1}{2\pi\sqrt{1.2\times10^{-9} \cdot C}}$
$\Rightarrow$ $C_f = 19.9\ pF$ with $L = 1.2\,nH$

$R_4 = 75\,\Omega$ for output impedance, calculate net gain required : $\frac{10^{35/20} \leftarrow \text{gain}}{0.5^2 \leftarrow \text{coupling losses}} = 225$ ∴ x15 each stage

∴ $R_3 = \frac{75}{15} = 5\,\Omega$, $I_c = \frac{4.5V}{75\,\Omega} = 0.064 \Rightarrow r_e = 0.42\,\Omega$

$V_E = 0.3V$ ∴ $V_B = 0.3 + 0.65 = 0.95V$ (say 1V to allow for base current).

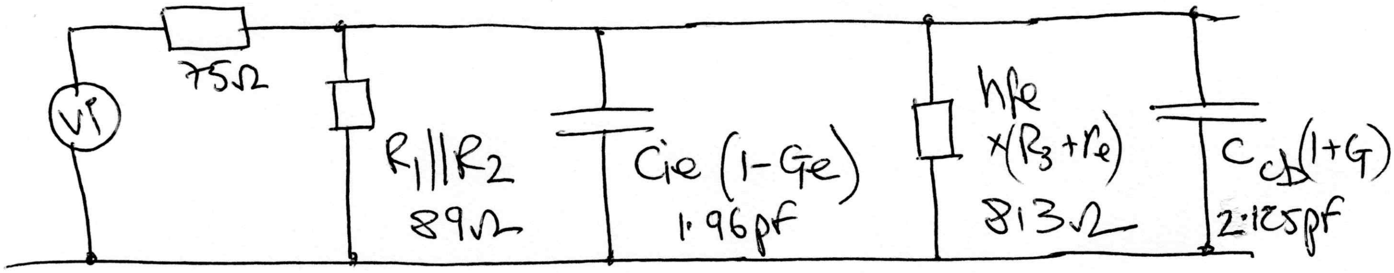
Choose $R_1 = 100\,\Omega$ R_2 from $1 = 9 \cdot \frac{R_1}{(R_1+R_2)}$ ∴ $R_2 = 800\,\Omega$

check $R_{in} \simeq 100 \| 800 \| h_{fe} \times 5.42$ = 82 Ω ✓ ok
(h_{fe} =200 assume; 5.42 = $R_3 + r_e$)

$C = 1nF$ coupling caps.

(c) Miller effect increases apparent input capacitance due to current reqd. to charge/discharge feedback capacitor by (1 + Gain). See notes pg.20
mitigate with cascode or long-tail pair : input transistor has no gain, output transistor has low source impedance.

4 (d) ssm: for input only (output has no miller effect)



$$Ge = \frac{R_3}{R_3 + re} = \frac{5}{5.42} = 0.923 \quad \therefore (1-Ge) = 0.0775$$

Cie from $f_t = \frac{1}{2\pi\, Cie\, re} \quad \therefore \quad 15 \times 10^9 = \frac{1}{2\pi . Cie . 0.42} \quad \therefore Cie = 25.3\,pf$

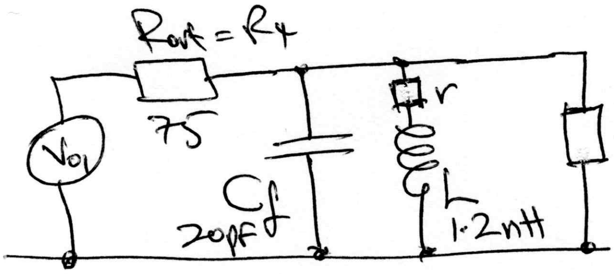
$hfe = 150 \quad \Rightarrow \quad R' = 75 \| 89 \| 813 = 38.8\ \Omega$

$C' = 1.96 + 2.13\,pf = 4.09\,pf$

$\Rightarrow f_{-3dB} = \frac{1}{2\pi R' C'} = \underline{1.00\ GHz}$ ∴ Marginal ($\approx -3dB$ each stage) @ 1030 MHz

(need to push gain up a bit: say reduce R_3 by 1/3 to 3.3Ω may just suffice: we have a bit of spare gain from $R_{in} > 75\Omega$ about $2 \times 4\% = 8\%$ overall).

(e) Consider impedances at ~~the~~ inter-stage coupling:



$R_{in_2} = R_1 \| R_2 \| hfe(R_3 + re)$

Now $R_{out} \| R_{in} = R'$ above $= 38.8\,\Omega$

$r = 0.2\Omega$, so Q for inductor $= \frac{\omega L}{r} = \frac{7.77}{0.2} = 38.8$

∴ equiv. parallel loss $R = 7.77 \times 38.8 = 302\ \Omega$ for inductor only

∴ total $\| R = 38.8 \| 302 = 34.4\Omega$ for LCR cct.

∴ $Q_{LC\ cct} = \frac{34.4}{7.77} = \frac{R}{\omega L} = 4.42$

∴ Bandwidth $= \frac{1030}{4.42} = \underline{233\ MHz}$.

Examiners Comments:

Q1:

VNA part of the question was answered poorly in general with very few able to draw the basic block diagram. Smith charts were generally done well, although some drawing was rather inaccurate. A common mistake was forgetting to denormalise to calculate component values. In (b) and (c) there are several solutions, so circuit diagrams are important to show the component arrangements..

Q2:

Generally well answered. In the PLL the most common errors were in finding the phase gain constant of the XOR gate and also converting the VCO constant to radians. Characteristic impedance was well done, but the wilkison splitter in the final part caused confusion.

Q3:

VCVS – some mistook the number of poles for number of op-amps in both (a) and (b). In the colpitts question, few realised that the transistor capacitance would increase the resonant capacitance and lower the frequency.

Q4:

Almost all could draw the amplifier, although many missed the required decoupling capacitors to prevent the interstage inductor messing up the biasing. Descriptions of the functions of the components lacked awareness that most have 2 function (e.g. R1 and R2 set input impedance as well as base bias). Most common mistakes were working out the coupling losses and compensating the amplifier gains, after that most could pick sensible resistor values. In the final part only a few realised that the Rin and Rout needed to be considered in the bandwidth calculation.