3B1 CRIB 2022

1(a) Radn into full sphere

Directivity = 1.5 , $\rho_r = 0.90$, Radn efficiency

Range, R = 86×10^3 m , $P_{trans.} = 25$W

$$P_{density @ gnd} = \frac{25 . 0.9 . 1.5}{4\pi (86\times10^3)^2} = \underline{3.63\times10^{-10}\ \mathrm{W/m^2}}$$

$$= \tfrac{1}{2} E H^* = \tfrac{1}{2}\eta H^2 \quad \therefore \quad H = \underline{1.39\times10^{-6}\ \mathrm{A/m}}$$

Assessor note; Many candidates calculated E rather than H!

(b) Antenna eqn: $G = \frac{4\pi A_e}{\lambda^2}$

Assessor note: Remember to include the antenna efficiency

$$\lambda = \frac{c_0}{f} = \frac{3\times10^8}{920\times10^6} = 0.326\ \mathrm{m}$$

$$\therefore A_e = \frac{G\lambda^2}{4\pi} = \frac{0.9 . 1.5 . 0.326^2}{4\pi}$$

$$= 0.0114\ \mathrm{m^2}$$

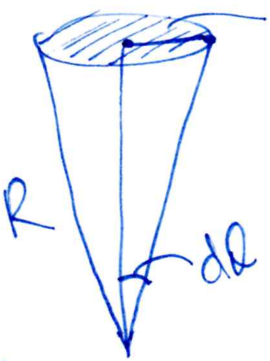
$$P_{density @ space} = \frac{200 . 10^{30/10}}{4\pi\ (86\times10^3)^2} = 2.15\times10^{-6}\ \mathrm{W/m^2}$$

$$\therefore P_{recd.} = 2.15\times10^{-6} . 0.0114 = 2.45\times10^{-8}\ \mathrm{W}$$

$$= \frac{V_r^2}{75} \quad \therefore \quad V_r = 1.36\ \mathrm{mV_{rms}}$$

$$\equiv \underline{3.83\ \mathrm{mV_{pp}}}$$

(c) Beam angle, $d\theta$



$$A = \pi (R d\theta)^2$$

$$G = 1000 = \frac{4\pi R^2}{A} = \frac{4\cancel{\pi}\cancel{R^2}}{\cancel{\pi}\cancel{R^2}\, d\theta^2}$$

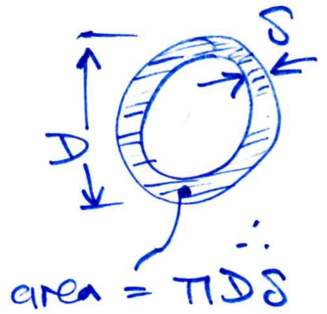
$$\therefore d\theta = \frac{1}{\sqrt{250}}\ \mathrm{rad}$$

$$= 3.6^\circ \quad \underline{\text{full beam } 7.2^\circ}$$

off axis dist. = $\underline{R d\theta = 5.4\ \mathrm{km}}$

1(d) For λ/2 antenna, assuming cosine current dist$\underline{n}$

$$R_r = 30\pi^2\left(\frac{\lambda/2}{\lambda}\right)^2 = 74\,\Omega$$



Skin depth, $\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}} = \sqrt{\frac{\rho}{\pi f\mu_0}}$

$$\therefore\ \delta = \sqrt{\frac{6.92\times10^{-7}}{\pi . 920\times10^{6} . 4\pi\times10^{-7}}} = 1.38\times10^{-5}\,m$$

$$R_{ohmic} \simeq \frac{\frac{\overset{0.326}{\lambda}}{2}\cdot\rho \leftarrow 6.92\times10^{-7}}{\pi D\delta \leftarrow 1.38\times10^{-5}} = \frac{2.60\times10^{-3}}{D}\,\Omega$$

$$\text{Rad}\underline{\text{n}}\text{ efficiency} = \frac{R_r}{R_r+R_{ohmic}} = \frac{74}{74+R_{ohmic}} = 0.90$$

$$\therefore\ R_{ohmic} = 8.2\,\Omega \Rightarrow \underline{D = 0.32\,mm}$$

(e) (i)

$$P_t = \frac{V_t^2}{50} = 200 \quad \therefore\ V_t = 100\,V_{rms} \qquad V_r = 1.36\,mV_{rms}$$

(from (b))

$$\therefore\ \text{Ratio, dB} = 20\log_{10}\left(\frac{100}{1.36\times10^{-3}}\right)$$

$$= \underline{97.3\ dB}$$

(ii) $P_t = 25 = \frac{V_t^2}{75}$ ∴ $V_t = 43.3\,V_{rms}$

$$A_{e\,grd} = \frac{G\lambda^2}{4\pi} = \frac{1000 . 0.326^2}{4\pi} = 8.46\,m^2$$

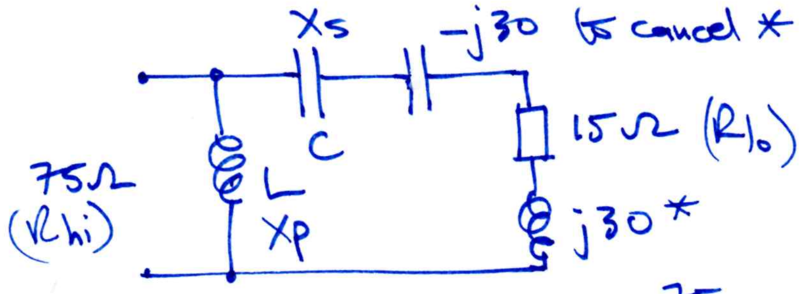
$$\therefore\ P_{recd.} = \underset{\text{from (a)}}{3.63\times10^{-10}} . 8.46 = 3.07\times10^{-9}\,W = \frac{V_r^2}{50}$$

$$\therefore\ V_r = 3.92\times10^{-4}\,V_{rms} \text{ and Ratio, dB} = 20\log_{10}\left(\frac{43.3}{3.92\times10^{-4}}\right)$$

$$\underline{= 100.9\,dB}$$

(not same as (i) due to 50 & 75 Ω difference)

2(a)



$$Q = \sqrt{\frac{R_{hi}}{R_{lo}} - 1} = \frac{\overset{75}{R_{hi}}}{X_p} = \frac{X_s}{R_{lo} \leftarrow 15} = 2 \text{ @ } 10\text{MHz}$$

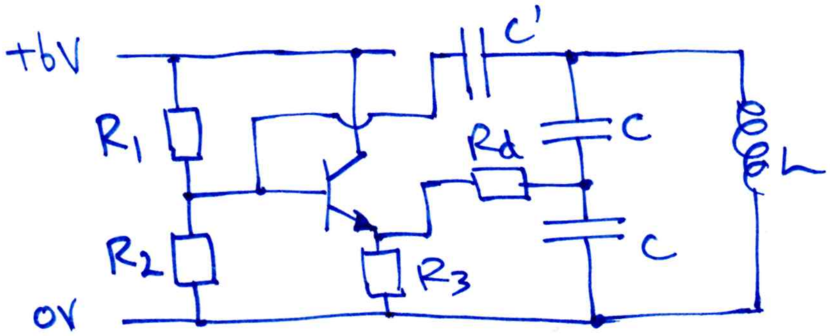
$\therefore X_p = 37.5 = 2\pi f L \quad \therefore \underline{L = 59.7\ \mu H}$

$X_s = 30$ but we can also include the other * capacitor into a single C with Xs

$\therefore X_s' = 30 + 30 = 60 = \frac{1}{2\pi f C}$

$\therefore \underline{C = 265\ pf}$

(b)



$f = 300\text{MHz} = \frac{1}{2\pi\sqrt{L\frac{C}{2}}}$

50nH

$Q = 25$

$\therefore 2\pi f L Q = 2.4\text{k}\Omega$ inductor loss resistance

$\underline{C' = \text{large value} = 10\ nf}$, $\underline{C = 2.81\ pf}$

Apparent load @ C mid-point $= \frac{2.4k}{4} \simeq 600\Omega$

$\therefore$ choose $\underline{R_3 = R_d = 390\ \Omega}$ say $\left(\sim \frac{2}{3} \text{ of } 600\Omega\right)$

$V_E \simeq 3V_{DC} \quad \therefore V_B \simeq 3.6V$ If $h_{fe} \sim 200$ with $I_c \approx \frac{3}{390}$ then $r_e = \frac{kT/q}{7.7\times10^{-3}} \approx 3\Omega$

and input impedance @ base $\sim h_{fe} \times \left[\left(390 // 390+600\right) + 3\right] \approx 56\text{k}\Omega$

so choose R_1 & R_2 ~10's kΩ to give $V_B = 3.6V$

$\therefore \underline{R_2 = 33\text{k}\Omega \ , \ R_1 = 22\text{k}\Omega}$

2(c)

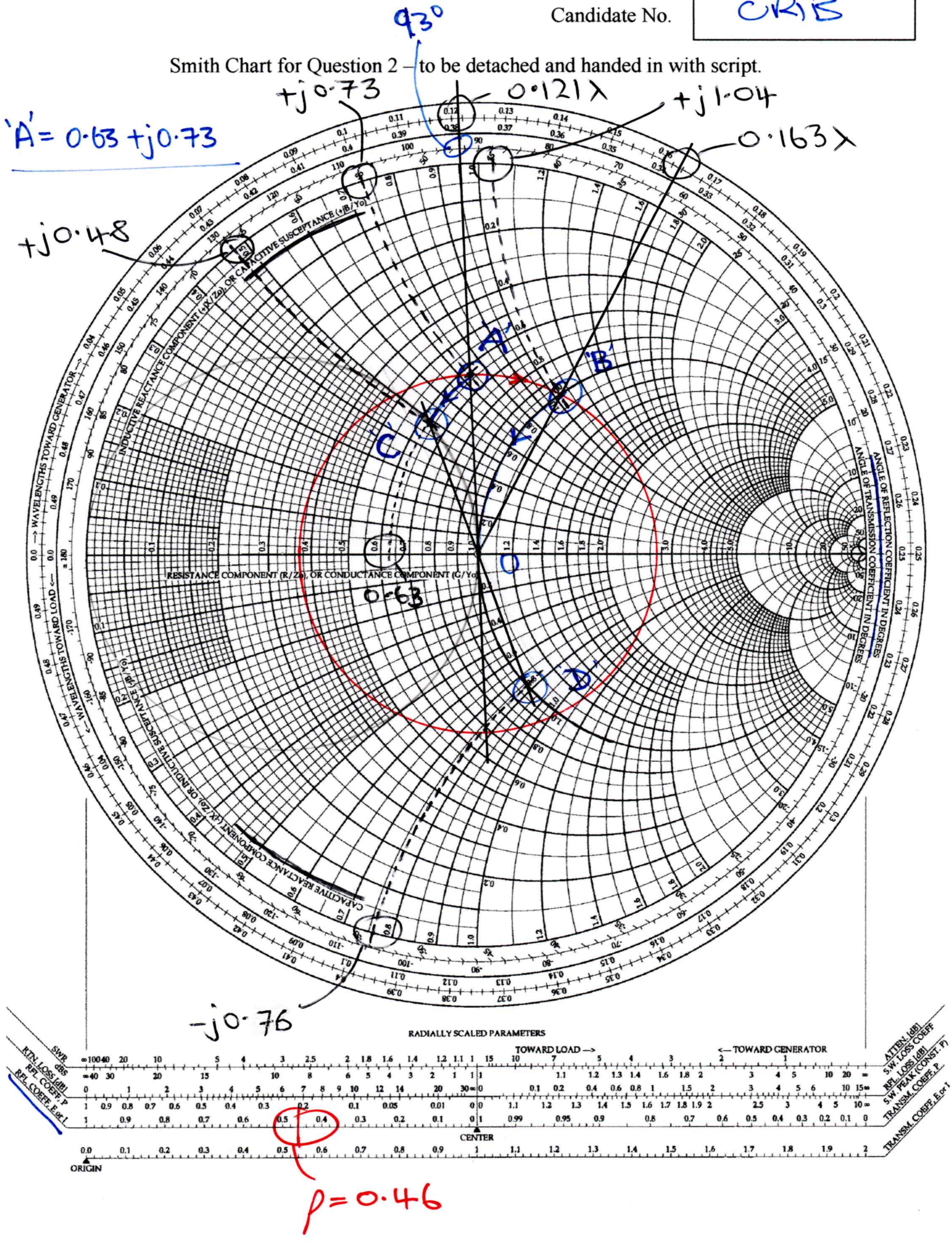
EGT2

ENGINEERING TRIPOS PART IIA

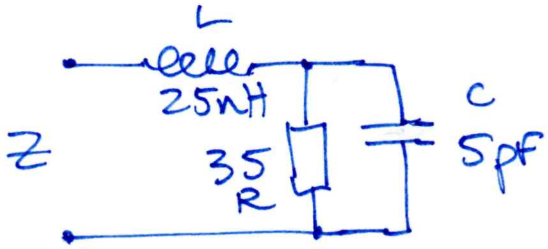
XXday YY April 2022, Module 3B1, Question 2

Candidate No. CRIB

Smith Chart for Question 2 – to be detached and handed in with script.



2(c)(i)



@ 300MHz

$$Z = j2\pi fL + \frac{R}{(1+j2\pi fCR)}$$

$$= j47.1 + \frac{35}{(1+j0.33)}$$

$$\therefore Z = j47.1 + \frac{35(1-j0.33)}{1.11} = \underline{31.5 + j36.7\ \Omega}$$

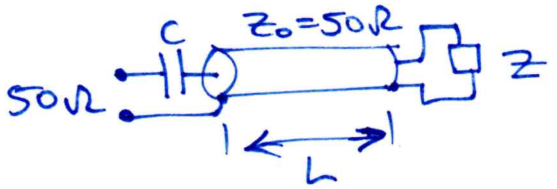
Normalise to 50Ω ⇒ point 'A' @ 0.63 + j0.73 on chart

S_{11} from REF. COEFF. scale = 0.46, angle is 93°

$\therefore \underline{S_{11} = 0.46\angle 93^\circ}$

C(ii) $v = \frac{1}{\sqrt{LC}}$ and $Z_0 = \sqrt{\frac{L}{C}}$ $\therefore v = f\lambda = \frac{1}{Z_0 C}$

$\therefore v = \frac{1}{78\times10^{-12}\times 50} = 2.56\times10^{8}$ m/s and $\lambda = 0.853$ m @ 300 MHz



Point 'A' to 'B' = $(0.163 - 0.121)\lambda = 0.042\lambda \simeq \underline{L = 36\text{mm}}$

'B' → O series capacitor $-j1.04 \Rightarrow -j52\,\Omega = \frac{1}{j2\pi fC}$
×50 denormalise

$\therefore \underline{C = 10.2\ \text{pF}}$

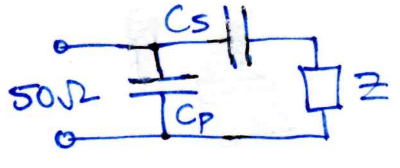
(iii) Matching with 2 capacitors (only possible as load is inductive):
Series cap. 'A' → 'C' such that 1/'C' = 'D' point is on Re=1 circle, so that the admittance = 1 − j? and the −j? will be cancelled with a parallel capacitor (+j?)

$\therefore$ series cap: $\frac{1}{2\pi f C_s} = 50(0.73 - 0.48)$ @ 300MHz

$\therefore \underline{C_s = 42.4\text{pF}}$

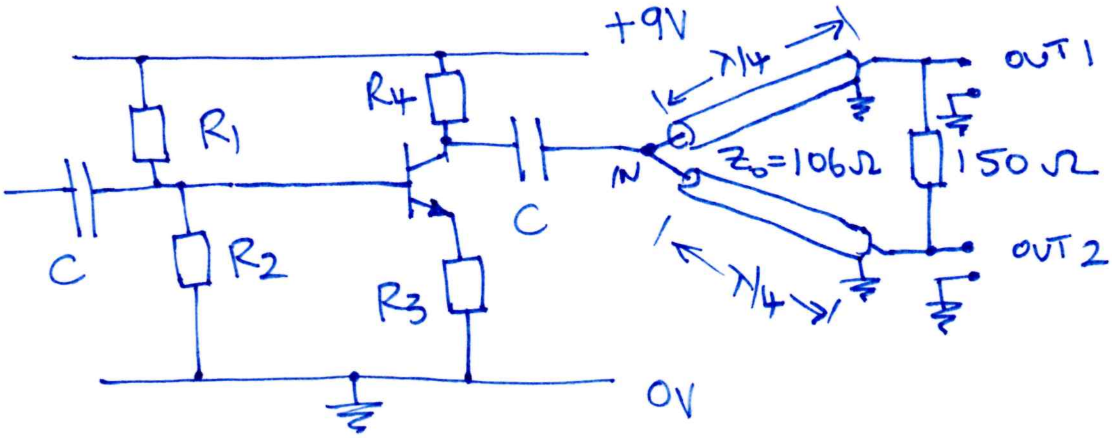
& parallel cap: $\frac{1}{2\pi f C_p} = \frac{50}{0.76}$ $\therefore \underline{C_p = 8.06\ \text{pF}}$

⇒



Assessor note: Take care using the Smith Chart in this part.

3(a)

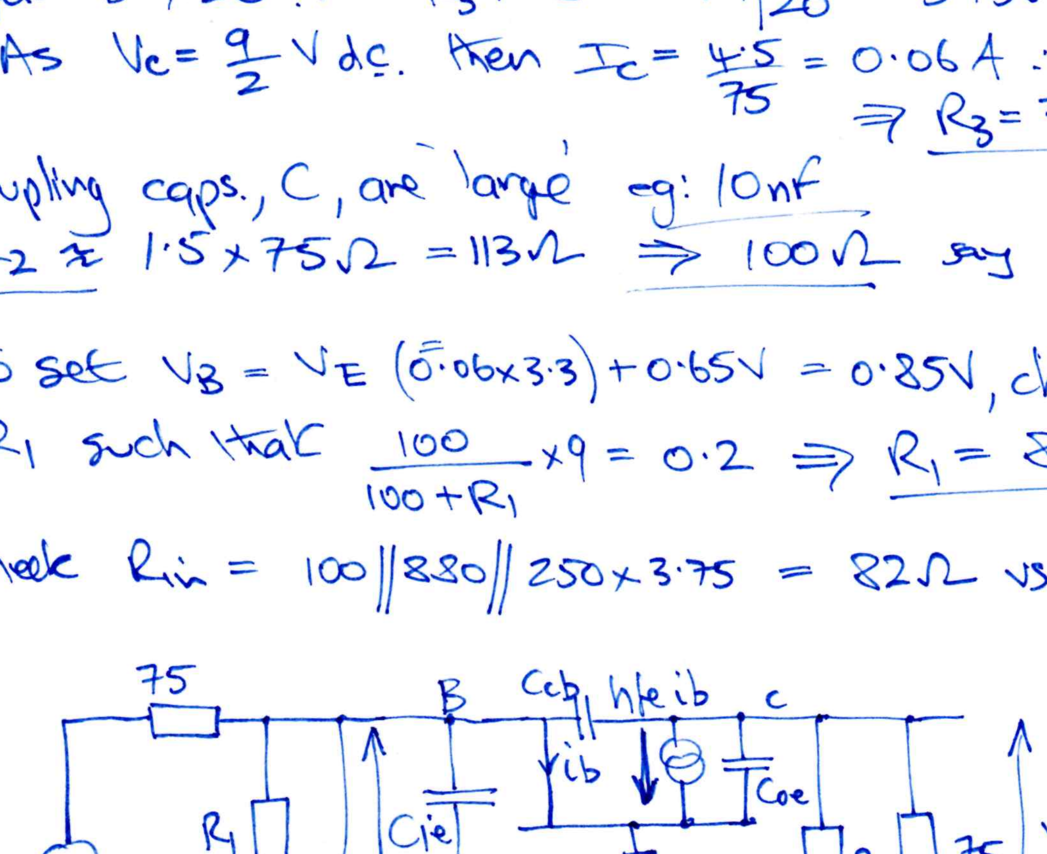


Wilkinson coupler uses a pair of λ/4 lengths of line with $Z_0 = \sqrt{2} \times 75\,\Omega$, with a $2 \times 75\,\Omega$ link resistor between the output ports. This evenly splits power to IN between OUT 1 and OUT 2 with isolation between the 2 outputs.

(b)
- $R_4 = 75\,\Omega$ for correct output impedance
- with 20dB net gain (×10) when loaded, open cct. gain is ×20 ∴ $R_3 + r_e = R_4/20 = 3.75\,\Omega$
- As $V_c = \frac{9}{2}$ V d.c. then $I_c = \frac{4.5}{75} = 0.06\,A$ ∴ $r_e = \frac{0.025}{0.06}\,\Omega$ $\Rightarrow R_3 = 3.3\,\Omega$
- Coupling caps., C, are large eg: 10nf
- $R_2 \approx 1.5 \times 75\,\Omega = 113\,\Omega \Rightarrow 100\,\Omega$ say
- To set $V_B = V_E\,(0.06 \times 3.3) + 0.65V = 0.85V$, choose R_1 such that $\frac{100}{100 + R_1} \times 9 = 0.2 \Rightarrow R_1 = 880\,\Omega$
- check $R_{in} = 100 \| 880 \| 250 \times 3.75 = 82\,\Omega$ vs. $75\,\Omega$ o.k.

(c)

SSM



Assessor note: Don't forget to include the emitter resistance

3(c) contd. $r_e = 0.42\,\Omega$ from (b), $f_t = 18\times10^9 = \frac{1}{2\pi r_e C_{ie}}$

∴ $C_{ie} = 21\,pF$. Also $v_e = \frac{R_3}{R_3+r_e} . v_i = 0.888\,v_i$

So equivalent value of C_{ie} to gnd. $= (1-0.888).21\,pF$

∴ $C_{ie}' = 2.35\,pF$

and from Miller effect with gain of 10 (loaded) $C_{cb}' = (1+10)C_{cb}$

∴ $C_{cb}' = 2.42\,pF$

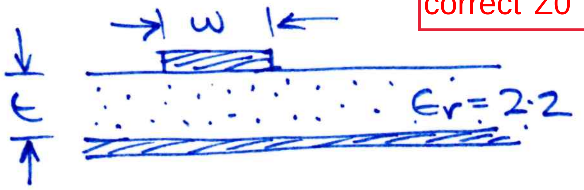
Hence input time constant $\simeq \underbrace{75 \| 82\,\Omega}_{39\,\Omega} \times C_{ie}' + C_{cb}' \; (=4.77\,pF)$

∴ $\underline{f_{-3dB}} = \frac{1}{2\pi . 39 . 4.77\times10^{-12}} = \underline{852\,MHz}$

∴ ok @ 520 MHz ✓

(Note: output side is higher freq. due to lower capacitance ~ $(0.24+0.15)\,pF$ with $37.5\,\Omega$ ⇒ >10 GHz)

(d)



Assessor note: Take care to use the correct Z0

for $Z_0 = 106\,\Omega = \sqrt{\frac{L}{C}}$

$v = \frac{1}{\sqrt{LC}}$ ∴ $Z_0 = \frac{1}{vC}$

With $\epsilon_r = 2.2$, $v = \frac{1}{\sqrt{\epsilon_0 \epsilon_r \mu_0}} = \frac{3\times10^8}{\sqrt{2.2}} = 2.02\times10^8\,m/s$

section lengths:

@ $f = 520\times10^6\,Hz$, $\lambda = \frac{2.02\times10^8}{520\times10^6} = 0.389\,m$ ∴ $\underline{\frac{\lambda}{4} = 97.2\,mm}$

$C = \frac{A\epsilon_0\epsilon_r}{t} \simeq \frac{(w+2t)\epsilon_0\epsilon_r}{t}$, capacitance / unit length

∴ $Z_0 = \frac{t}{(w+2t)\epsilon_0\epsilon_r v} = 106 = \frac{1.6\cancel{\times10^{-3}}}{(w+2.40)\cancel{\times10^{-3}} . 8.854\times10^{-12} . 2.2 . 2.02\times10^{8}}$

∴ $(w+2.40) = 3.84$ ⇒ $\underline{w = 1.44\,mm}$

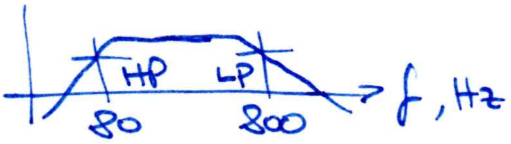
with $\underline{150\,\Omega \text{ resistor}}$ between the 2 output nodes

4(a)(i) $f_{Dop.} = \frac{2fv}{C_o}$, $f = 24\times10^{9}$ Hz

$v = 0.5 - 5$ m/s

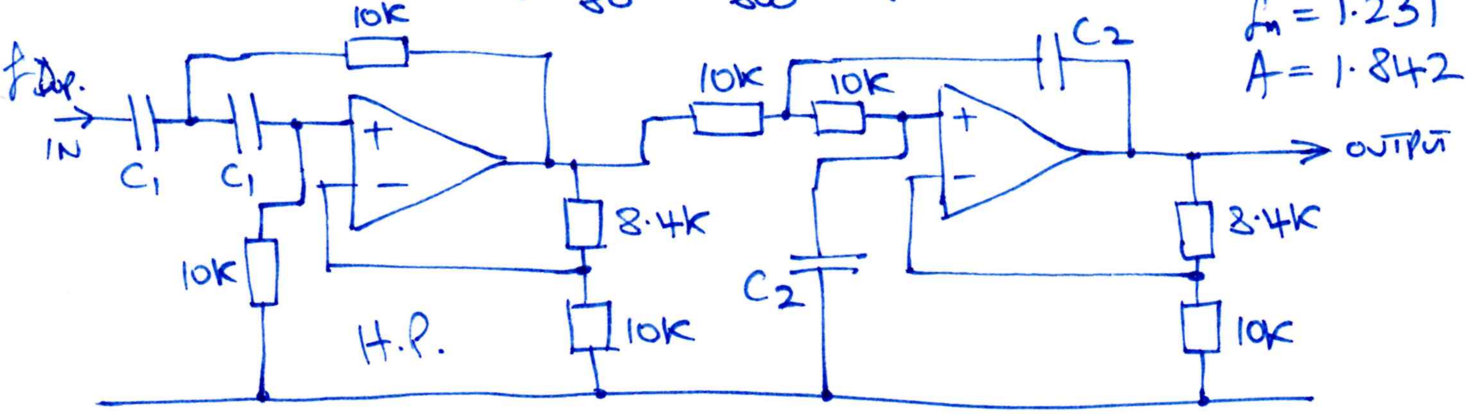
∴ $f_{Dop.} = 80 - 800$ Hz

choose Cheby. for sharp cut-off high-pass & low-pass for bandpass filter:



$f_n = 1.231$

$A = 1.842$



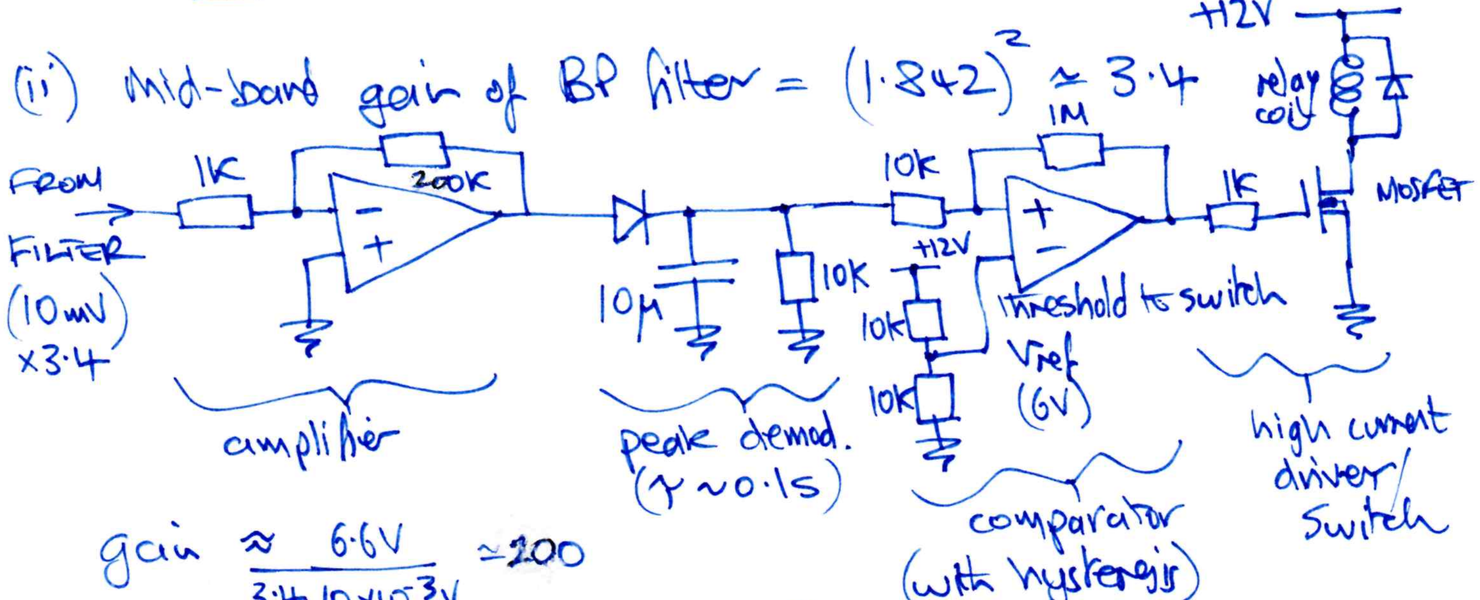
H.P. $80 = \frac{f_n}{2\pi R C_1}$ ← 1.231, R ← 10K

L.P. $800 = \frac{1}{2\pi f_n R C_2}$

∴ $\underline{C_1 = 245\text{ nF}}$

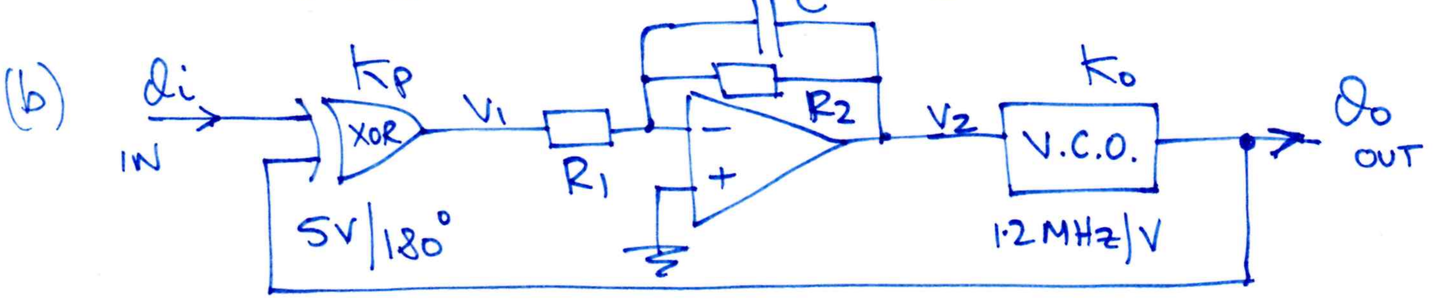
∴ $\underline{C_2 = 16.2\text{ nF}}$

(ii) Mid-band gain of BP filter $= (1.842)^2 \simeq 3.4$



gain $\approx \frac{6.6V}{3.4.10\times10^{-3}V} \simeq 200$

(b)



$C \| R_2 : \quad \frac{R_2/jwC}{R_2 + 1/jwC} = \frac{R_2}{1 + jwCR_2}$

4(b) contd. $\theta_0 \equiv e^{j(\omega t+\theta_0)}$ $\therefore \frac{d\theta_0}{dt} = j\omega\theta_0 , \frac{d^2\theta_0}{dt^2} = -\omega^2\theta_0$

Phase detector : $V_1 = K_p(\theta_0 - \theta_i)$ ——— (1)

V.C.O. : $\frac{d\theta_0}{dt} = j\omega\theta_0 = K_0 V_2$ ——— (2)

Filter : $\frac{V_2}{V_1} = \frac{-R_2}{R_1(1+j\omega CR_2)}$ ——— (3)

Sub. for V_2 from (3) into (2): Sub. for V_1 from (1):

$$j\omega\theta_0 = \frac{-K_0V_1R_2}{R_1(1+j\omega CR_2)} = \frac{-K_0K_p(\theta_0-\theta_i)R_2}{R_1(1+j\omega CR_2)}$$

$$\therefore j\omega\theta_0R_1(1+j\omega CR_2) + K_0K_p\theta_0R_2 = K_0K_pR_2\theta_i$$

$$\therefore \frac{j\omega\theta_0R_1}{K_0K_pR_2} - \frac{\omega^2\theta_0R_1\cancel{R_2}C}{K_0K_p\cancel{R_2}} + \theta_0 = \theta_i$$

$$\therefore \ddot{\theta}_0\frac{R_1C}{K_0K_p} + \dot{\theta}_0\frac{R_1}{K_0K_pR_2} + \theta_0 = \theta_i$$

$\frac{\ddot{\theta}}{\omega_n^2} + \dot{\theta}\frac{2\zeta}{\omega_n} + \theta = x$ cf. Mechanics Data Bk pg 6

$\therefore \omega_n^2 = \frac{K_0K_p}{R_1C}$ and $\frac{2\zeta}{\omega_n} = \frac{R_1}{K_0K_pR_2}$. Set $\underline{R_1 = 10k\Omega}$

$K_0 = 1.2MHz/V = 2\pi.1.2\times10^6 = 7.54\times10^6$ rad $s^{-1}V^{-1}$

$K_p = 5V/180° = 5V/\pi\,rad = 1.59$ V rad^{-1}

$$\therefore \omega_n^2 = (20\times10^3.2\pi)^2 = 157.9\times10^8 = \frac{7.54\times10^6.1.59}{10^4C}$$

$$\Rightarrow \underline{C = 76nF}$$

for 15% overshoot, $\zeta = 0.5$, $\omega_n = 126\times10^3$ rad/s

$$\therefore \underline{R_2} = \frac{R_1\omega_n}{2\zeta K_0K_p} = \frac{10^4.126\times10^3}{2.0.5.7.54\times10^6.1.59} = \underline{105\Omega}$$