3B1 CRIB 2021

1 (a) $R = 80\,km$

$P_t = 16\,kW$, $G = 33\,dB = \times 1995$

Power density $= \frac{16\times10^3 \cdot 1995}{4\pi R^2} = \underline{3.97\times10^{-4}\ W/m^2}$

$\frac{E^2}{2\eta}$ = Power density $\therefore E = \underline{0.547\ V/m}$

where $\eta = 120\pi$

(b) The power delivered by an antenna to a matched load is given by P_r = effective aperture × incident power density

Gain is the ratio of the (peak) transmitted power density to the density from an isotropic antenna for the same power input – in the optimum direction

$1995 = 4\pi \frac{A_e}{\lambda^2}$ with $\lambda = \frac{3\times10^8}{2800\times10^6} = 0.107\,m$ $\therefore \underline{A_e = 1.822\,m^2}$

(G)

$A_e = 0.8\frac{\pi D^2}{4}$ $\therefore \underline{D = 1.70\,m}$

---

note: antenna eqn. $G = 4\pi A_e / \lambda_0^2$

(c)(i) $\lambda_0 = \frac{3\times10^8}{2800\times10^6} = \frac{c_0}{f} = 0.107\,m$

$\therefore A_e = \frac{G\lambda_0^2}{4\pi} = \frac{1995\times0.107^2}{4\pi} = 1.82\,m^2$

$P_{rec} = \frac{4.96\times10^{-3}\times1.82}{4\pi R^2}$ ← 80×10^3 $= 1.12\times10^{-13}\ W = \frac{v_r^2}{r}$ ← 50Ω

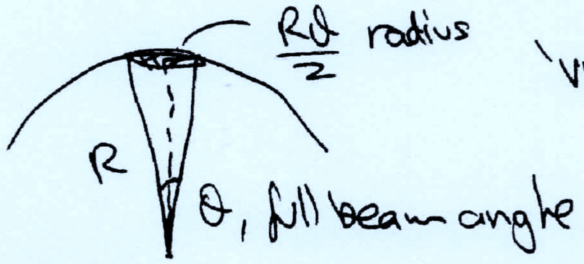
$\therefore \underline{v_r = 2.37\ \mu V_{rms}}$

(3.35 μV pk)

1(c)(ii) Reduce range by 10 $\Rightarrow$ power density $\times 10^4$

$\Rightarrow$ voltage $\times 100$ (to 23.7 μV_{rms}) $\Rightarrow$ 40dB

(d)



'viewed' area $= \pi\left(\frac{R\theta}{2}\right)^2$

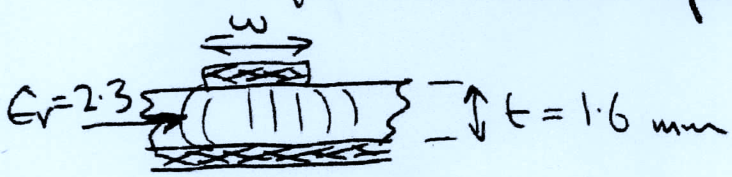
Total isotropic area $= 4\pi R^2$, $G = 1995 = \dfrac{4\pi R^2}{\pi\left(\frac{R\theta}{2}\right)^2}$

$$\Rightarrow \theta = \sqrt{\frac{16}{G}} = 0.0896 \text{ rads} \underset{\times\frac{180}{\pi}}{=} 5.13^\circ$$

(e) $v_m = f\lambda_m = \dfrac{1}{\sqrt{\mu_0\epsilon_0\epsilon_r}} = f\dfrac{\lambda_0}{\sqrt{\epsilon_r}} = \dfrac{3\times10^8}{\sqrt{2.3}} = 1.978\times10^8$ m/s

$\therefore \lambda_m = \dfrac{\lambda_0}{\sqrt{\epsilon_r}} = \dfrac{0.107}{\sqrt{2.3}} = 0.0706$ m

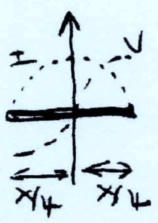
Hence half-wave resonant patch length $= 0.0353$ m (3.53 cm)



Effective track width (incl. fringing) $= w + 2t$ $\therefore C = (w+2t)\times\dfrac{\epsilon_0\epsilon_r}{t}$ (capacitance/m)

$Z_0 = \dfrac{\sqrt{L}}{\sqrt{C}}$ and $v_m = \dfrac{1}{\sqrt{L}\sqrt{C}}$ $\therefore Z_0 = \dfrac{1}{v_m C}$

$$\therefore 50 = \frac{1.6}{1.978\times10^8 \cdot 8.854\times10^{-12}\times 2.3\,(w+2\overset{1.6}{t})}$$



$\therefore 0.201(w+3.2) = 1.6$ $\therefore w = 4.76$ mm

Patch impedance is $\overset{=0}{\text{min.}}$ @ centre and $\overset{=\infty}{\text{max.}}$ @ edge due to V and I standing wave $\lambda/2$: V is sine, I is cosine $\therefore \frac{V}{I}$ is tan (approx.)

2(a) $f = 1060\ \text{MHz}$, $Z_0 = \frac{\sqrt{L}}{\sqrt{C}}$ and $v_c = \frac{1}{\sqrt{L}\sqrt{C}} = f\lambda_c$

$\therefore v_c = \frac{1}{Z_0 C} = 2.05 \times 10^8\ \text{m/s}$ for $Z_0 = 75\Omega$,

$C = 65\ \text{pF/m}$

$\therefore \lambda_c = \frac{v_c}{f} = \underline{0.194\text{m}}$ $L = 3.66 \times 10^{-7}\ \text{H/m}$

(b) Plot point 'A' @ $0.68\angle 85^\circ = 0.4 + j : \times 75 = \underline{30 + j75\ \Omega}$

Rotate to compensate for cable effects a/c/w by

$\frac{0.12}{0.194} = 0.619\ \lambda \quad -0.5\lambda$ whole turn $\Rightarrow 0.119\ \lambda$

$\therefore 0.132\lambda - 0.119\lambda = 0.013\ \lambda$, point 'B'

Impedance @ B $= 0.19 + j0.08 : \underline{\times 75 = 14.3 + j6\ \Omega}$

(c) For matching we need a cable length which presents an impedance at the cable end $= 1 + jX$ on $Re = 1$ circle @ point 'C' on the Smith Chart.

$\therefore$ electrical length $= 0.185\lambda - 0.013\lambda = 0.172\ \lambda + \frac{n\lambda}{2}$

with $\lambda = 0.194\ \text{m}$ (194 mm) from part (a) then

the physical length $= (33 + 97n)$ mm eg: $l = 130$ mm for $n = 1$

(extra 10mm from original)

point 'C' $= 1 + 1.87j$ so series capacitor needs impedance

$= -1.87j \times 75\ \Omega = -j140\Omega$

$\therefore \frac{1}{2\pi f C} = 140 \Rightarrow \underline{C_{series} = 1.07\ \text{pF}}$

f: $1060 \times 10^6\ \text{Hz}$

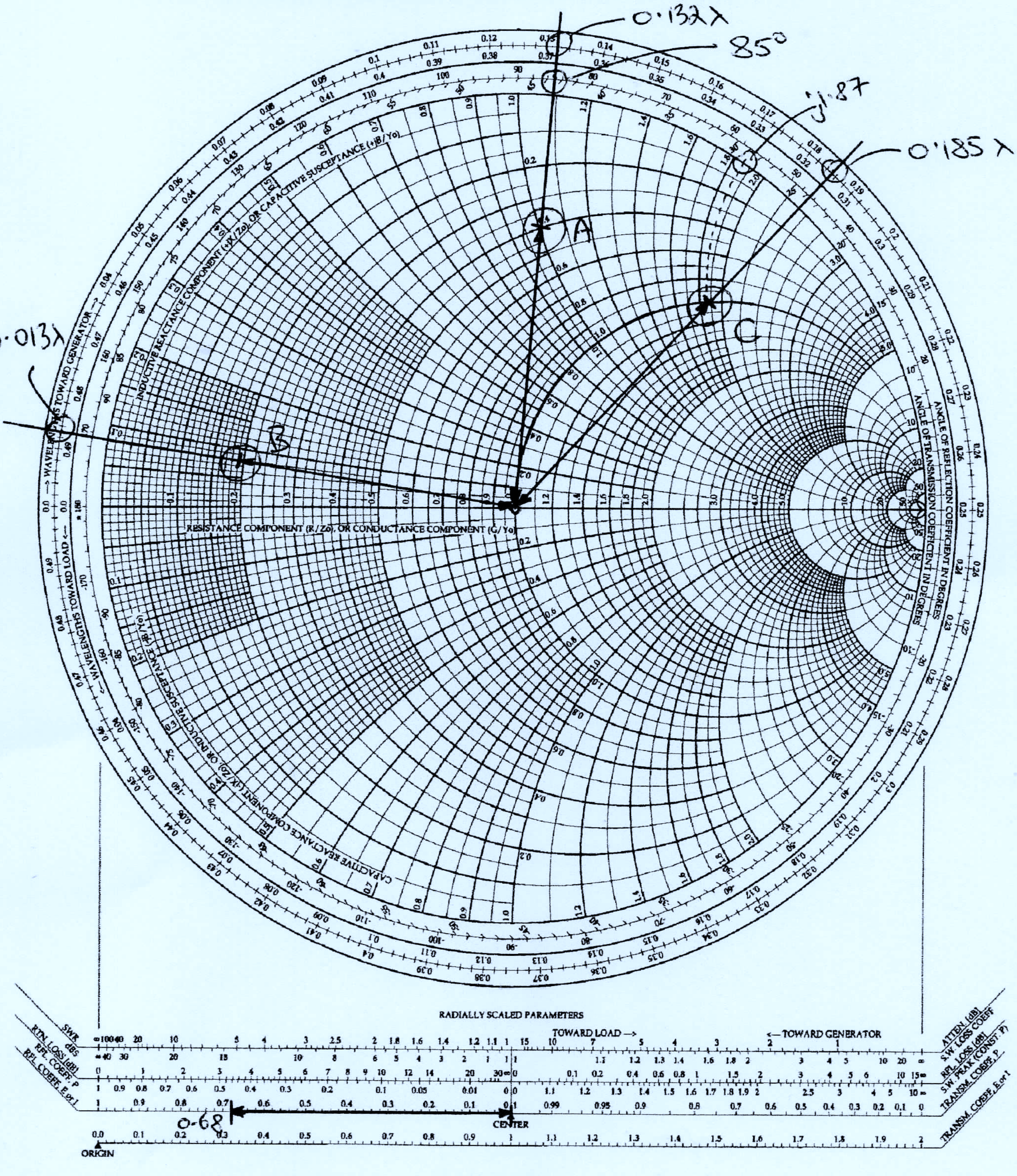
4/9

CRIB

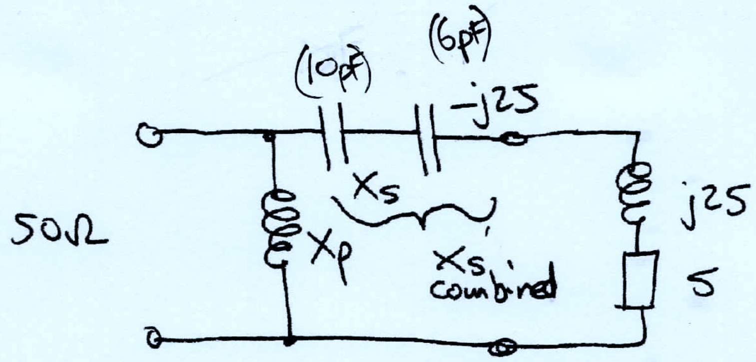
EGT2

ENGINEERING TRIPOS PART IIA

Tuesday 27 April 2021, Module 3B1, Smith Chart for reference in Question 2



2(d)



Matching eqn.

$$Q = \frac{R_{hi}}{X_p} = \frac{X_s}{R_{lo}} = \sqrt{\frac{R_{hi}}{R_{lo}} - 1}$$

Cancel $j25$ with series cap. of $-j25$ (will subsume this into X_s later to just have 2 components X_s and X_p).

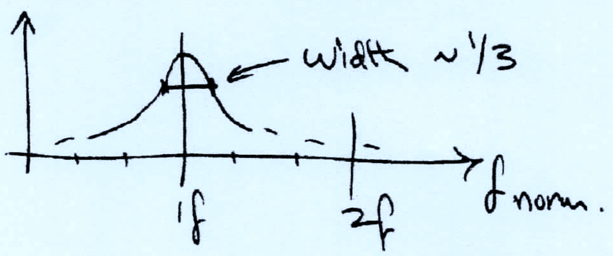
Then $R_{hi} = 50\,\Omega$, $R_{lo} = 5\,\Omega$ $\therefore Q = 3$

$X_p = j16.7\,\Omega$ and $X_s = -j15\,\Omega$ @ 1060 MHz

So for $X_s' = -j40\,\Omega = \frac{1}{2\pi f C}$ $\therefore C = 3.75\,pF$

and for X_p: $16.7 = 2\pi f L$ $\therefore L = 2.51\,nH$

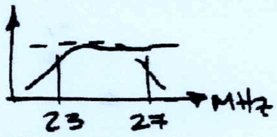
The Q factor is fairly low at 3, but still quite a peak.

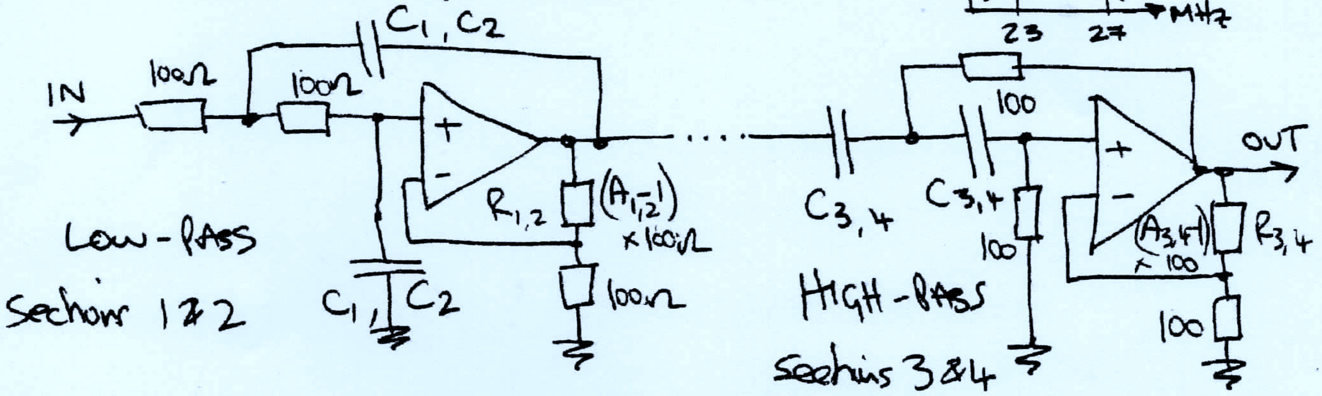


so @ 2f, the response will be less and the matching not so good.

Signal amplitude likely to drop by a factor of several times.

3(a) Use chebyshev filter for sharp cut-off — 23 MHz HP, 27 MHz LP. Cascade high-pass (HP) and low-pass (LP) sections for band pass 23–27 MHz:





Low-pass: $27 \times 10^6 = \dfrac{1}{2\pi f_n RC_{1,2}}$ — High pass: $23 \times 10^6 = \dfrac{f_n}{2\pi RC_{3,4}}$

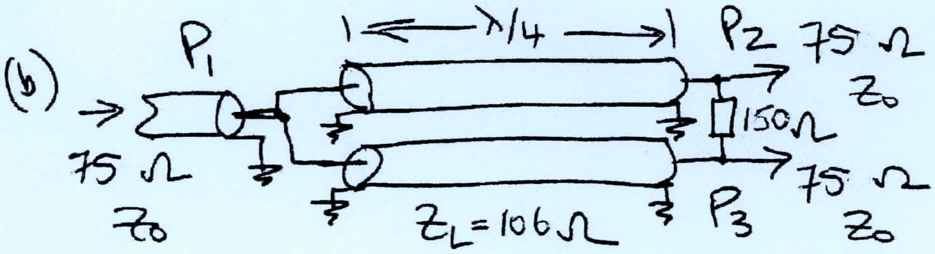
$A_1 = 1.582 \quad \therefore R_1 = 58.2\ \Omega$ — $A_3 = A_1 \quad \therefore R_3 = 58\ \Omega$

$R = 100\Omega \quad C_1 = 99\text{pF}$ — $C_3 = 41\text{pF}$

$A_2 = 2.660 \quad \therefore R_2 = 166\ \Omega$ — $A_4 = A_2 \quad \therefore R_4 = 166\Omega$

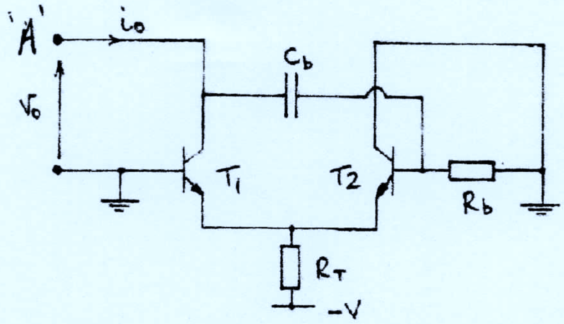
$C_2 = 57\text{pF}$ — $C_4 = 71\text{pF}$

(b)

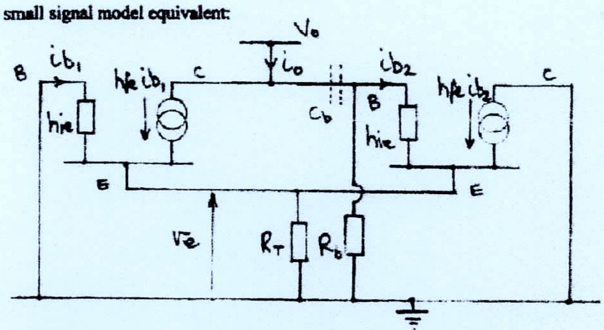


Wilkinson coupler uses a pair of $\lambda/4$ lines with char. impedance $Z_L = \sqrt{2}\, Z_0$. The $\lambda/4$ lengths of line transform P_2 and P_3 impedances of 75 Ω to $\left(\frac{106}{75}\right)^2 \cdot 75 = 150\Omega$ each. These are in parallel at port 1 to make $Z_0 = 75\ \Omega$ match. So power input to port 1 splits equally to P_2 and P_3. Any power back into P_2 or P_3 has a phase delay of $\lambda/2 = 180°$ and cancels direct signal from 150Ω between P_2 and P_3 – so they are mutually isolated (to 30dB). P1 split to P2, P3 is ideally lossless, as 100Ω carries no current.

3(c)



The small signal model equivalent:



① $i_{b1} = \frac{-V_e}{h_{ie}}$, ② $i_{b2} = \frac{V_o - V_e}{h_{ie}}$, ③ $V_e \approx R_T h_{fe}(i_{b1}+i_{b2})$,

④ $i_o = h_{fe} i_{b1} + i_{b2} + V_o/R_b$ subst. ① and ② into ③:-

$$V_e = -R_T h_{fe} \frac{V_e}{h_{ie}} + R_T h_{fe} \frac{V_o}{h_{ie}} - R_T h_{fe} \frac{V_e}{h_{ie}}$$

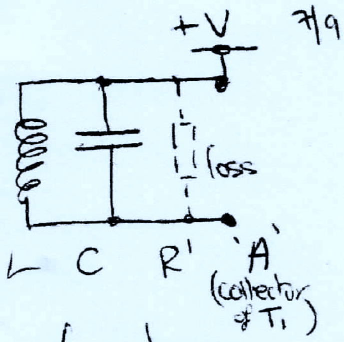
$$\therefore V_e \left(\overset{\text{small cf. as } R_T \sim h_{ie}}{\cancel{1}} + \frac{2R_T h_{fe}}{h_{ie}}\right) = \frac{R_T h_{fe} V_o}{h_{ie}} \quad \therefore V_e \approx \frac{V_o}{2} \quad ⑤$$

subst. ① and ② into ④ and subs. for V_e using ⑤

$$\therefore i_o = -\frac{h_{fe} V_o}{2h_{ie}} + \overset{\text{small cf.}}{\cancel{\frac{V_o}{2h_{ie}}}} + \frac{V_o}{R_b}$$

$$\therefore i_o = V_o\left(\frac{1}{R_b} - \frac{h_{fe}}{2h_{ie}}\right) \text{ and as } \frac{h_{ie}}{h_{fe}} = r_e,$$

$$Z_o = \frac{V_o}{i_o} = \left(\frac{1}{R_b} + \frac{1}{-2r_e}\right)^{-1} \Rightarrow Z_o = R_b \| -2r_e$$



$$f = \frac{1}{2\pi\sqrt{LC}} = 2800 \times 10^6 \text{ Hz}$$

with $L = 1.5 \times 10^{-9}$

$\Rightarrow C = 2.15\text{ pf}$

with $Q = 25$ and $L = 1.5\text{ nH}$

$$R' = \omega L Q = 2\pi f L Q = 660\Omega$$

to compensate with -ve resistance in parallel with 50Ω load

$\therefore R_b \| -2r_e < -46.5\Omega$

Aim for say -35Ω, so that the LC tank circuit is unstable and oscillations are sustained. $r_e = \frac{kT/q}{I_C}$ from Ebers-Moll

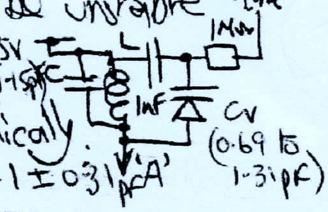
$r_e \approx \frac{0.025}{I_C}$ $\therefore$ for $r_e = 15\Omega$, $I_C = 1.66\text{mA} \rightarrow$ say 2mA.

Choose R_b and $R_T = 1\text{k}\Omega$, then $I_E \simeq \frac{5-0.65}{2 \times 1000} = 2.2\text{mA}$ ok ✓

Then $Z_o = 1\text{k}\Omega \| -2\times15\Omega \simeq -31\Omega$ o.k. should be unstable

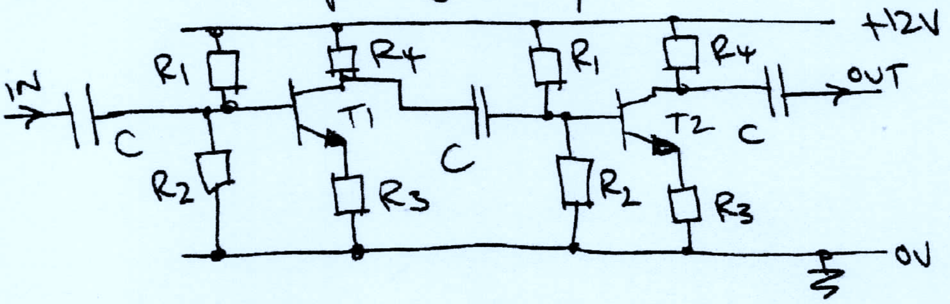
Take oscillation output across R_b i.e. ~~base~~ of T_2

Add varactor diode in parallel with LC for tuning electronically.



For $2.8 \pm 0.2\text{GHz}$, $C_{total} = 2.15 \pm 0.31\text{pf}$ $\therefore$ choose eg: $C = 1.15\text{pF}$ and $C_v = 1 \pm 0.31\text{pF}$

4(a) 35dB power gain requires 2 transistors @ ~20dB each.



R_1 & R_2 provide ~~base~~ dc bias

R_3 is -ve feedback to set gain and improve bias stability

R_4 is collector load resistor: sets output impedance

C are d.c. blocking coupling capacitors

35 dB power gain $= \times 10^{35/20} = \times 56.2$ linear gain

also needs to compensate for 2 extra coupling losses between stages $\therefore$ o/cct. linear gain $= 56.2 \times 2 \times 2 = \times 225$,

spread equally $= \times 15$ ($\times 7.5$ loaded in cct)

$R_4 = 50\Omega$ output imped. $R_3 = \frac{R_4}{15} = 3.3\Omega$ (say 3Ω)

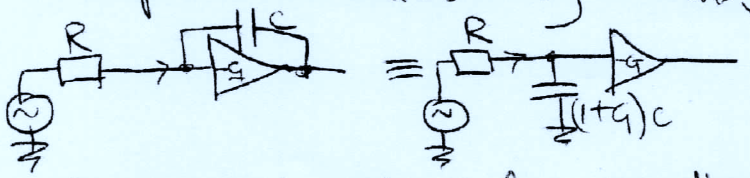
Choose $R_2 = 1.5 \times 50 = 75\Omega$. Set ~~base~~ @ 1.5V dc.

$\therefore R_1 = 510\Omega$ $\left(V_b = \frac{75}{585} \times 12 = 1.54\text{V} \ \checkmark \text{o.k.}\right)$

$I_c = \frac{6}{50} = 0.12\text{A}$ $\therefore r_e = \frac{0.025}{I_c} = 0.21\Omega$ $[0.15\Omega]$

and $\frac{V_e}{V_b} \simeq \frac{3}{3.21} = 0.93$ (emitter gain), $C = 1\text{nF}$ each

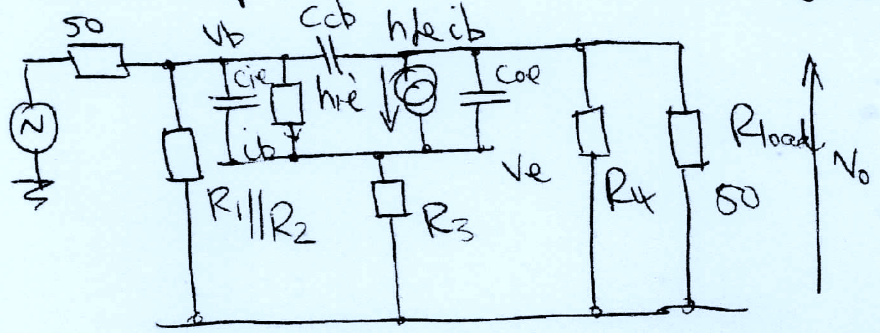
(b) Miller Effect: input impedance to inverting amplifier is reduced by capacitance between output and input - value of capacitance appears larger by factor of $(1+G)$. This can reduce the input cct. bandwidth by increasing RC time constant, where



and $f_{-3dB} = \frac{1}{2\pi RC(1+G)}$

R is source impedance from preceding stage. Can mitigate by using cascode or differential amp. cct. (see notes).

4(c) Consider input cct. to T_1 : small sig. model:



From before $V_e = 0.93\,V_b$, $r_e = 0.21\,\Omega$, $f_t = 15\,GHz$

$C_{ie} = \dfrac{1}{2\pi f_t r_e} = 50.5\,pF$ but emitter feedback

reduces effect of this to $(1-0.93) \times 50.5 = 3.54\,pF$

C_{cb} is multiplied by Miller effect to: $\left(1 + \dfrac{50||50}{3+0.21}\right) \times 0.15 = 1.32\,pF$

$(\times 8.78)$

So input resistances are $50 || 75 || 510 || \dfrac{h_{fe} r_e}{(1-0.93)}$ $= 750\,\Omega$

$\therefore R' = 27.3\,\Omega$

$C' = 3.54 + 1.32 = 4.86\,pF$ and f_{-3dB} per stage

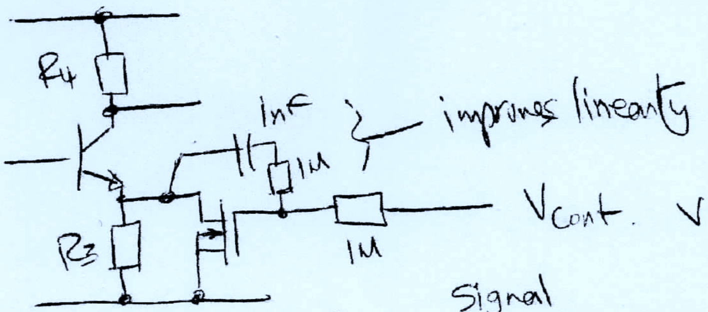
is given by $f_{-3dB} \simeq \dfrac{1}{2\pi R'C'} = \underline{1.20\,GHz}$ actually this

will be $-6dB$ as 2 stages, so operating @ $1.03\,GHz$ is ok.

(d)(i) We would need a variable gain-amplifier : can shunt

R3's with a FET to vary resistance electronically

$\Rightarrow$ increase gain for smaller signals



(ii) Range 100:1 = Power $100^2:1$ = Signal Voltages 100:1 = 40 dB range

If 30dB compensation, we are left with 10dB variation = $\sqrt{10}$ on

Signal amplitudes $= 3.16$

Examiner's comments

Q1 Antennas & microstrip

A very popular and straightforward question, well-answered by most candidates. Most candidates knew the antenna term definitions and the antenna equation, and applied it correctly to calculate signal magnitudes although a number of attempts did not take the isotropic back scatter into account correctly. The beam angle was generally estimated correctly.

Q2 Smith chart and impedance matching

A less popular question; attempted by about 40% of the cohort. Most attempts at determining the input impedance values were correct – with some latitude allowed for reading values of the screen, however, several attempts at compensating for the cable length moved around the chart in the wrong direction. The matching circuit was well attempted in many cases.

Q3 VCVS filters, power divider and oscillator

This question was attempted by almost all candidates and quite well answered. The VCVS filter section was quite straightforward and well attempted in most cases, with a correct choice of filter type and values. The power divider section was less well done, although it is in the notes, many attempts did not cover the details well. The negative impedance oscillator was generally well attempted.

Q4 RF amplifier

A fairly popular question with good attempts on the whole. The 2-stage amplifier design was well answered, although the gain was sometimes incorrect by a factor of 2 either way. The frequency response was also quite well attempted in many cases, although the unloaded gain was occasionally considered rather than the loaded value. The variable gain section at the end attracted a number of attempts of variable quality.