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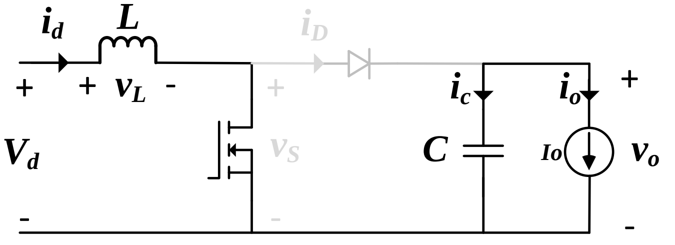
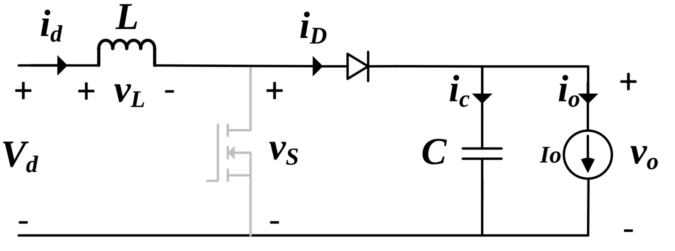
1(a)

S: ON for DT_s



S: OFF for (1-D)T_s



One transistor => two states

1(b)

Duty cycle $D = T_{on}/T = T_{on} f$

steady state assumption:

$$\int_0^T v_L = \int_0^{T_{on}} v_L \, \mathrm{d}t + \int_{T_{on}}^{T} v_L \, \mathrm{d}t = 0$$

$$V_{\text{in}} T_{\text{on}} + (V_{\text{in}} - V_{\text{out}}) T_{\text{off}} = 0$$

$$M(D) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{T}{T_{\text{off}}} = \frac{1}{1 - D}$$

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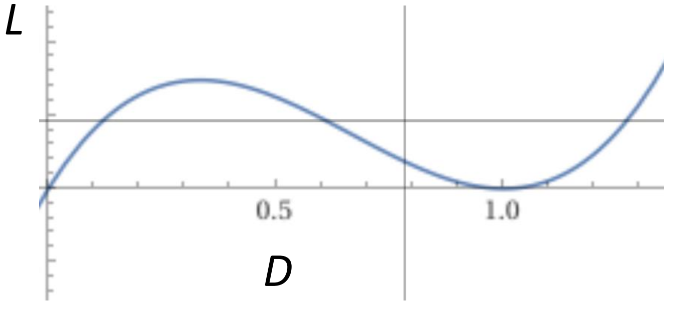
1(c)

Current does never cease: continuous conduction mode

V_{out} = 5 V
V_{in} = 2.3 V … 4.2 V
$1/(1 - D) = V_{out}/V_{in}$
$1 - D = V_{in}/V_{out}$
$D = 1 - V_{in}/V_{out}$
=> $D_{2.3} = 1 - 2.3\text{ V}/5\text{ V} = 0.54$
=> $D_{4.2} = 1 - 4.2\text{ V}/5\text{ V} = 0.16$

1(d)

Smallest Inductor: boundary conduction mode achieved everywhere
$T = 1/f = 10\ \mu s$
$20\text{ W} = V_{out}\, I_{out}$
=> $I_{out} = 4$ A
$R_{out,equiv} = 5\text{ V}/4\text{ A} = 1.25\ \Omega$
Boundary conduction mode:
$2\, L = RT\, D\, (1 - D)^2$
$L = ½\, RT\, D\, (1 - D)^2$



Find minimum via derivative:
$dL/dD = {}^1/_2\, RT\, (1 - 4D + 3D^2) = 0$
=> $D = {}^1/_3$ and 1; Max at ${}^1/_3$ and min at 1
=> $D = {}^1/_3$

$L(D={}^1/_3) = {}^1/_2\, RT\, {}^1/_3\, {}^4/_9 = {}^2/_{27}\, RT = {}^2/_{27}\, 12.5\ \mu s\text{ V/A} = 0.926\ \mu H$

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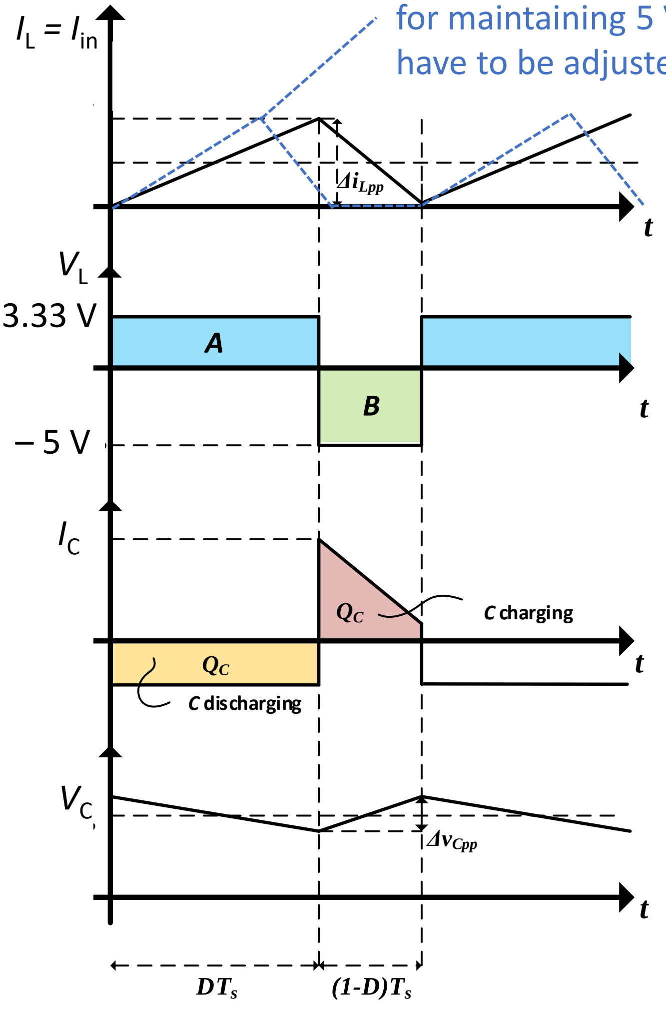
1(e)

Minimum inductance of L = 0.926 µH and $D = {}^{1}/_{3}$
input voltage $V_{in} = (1 - D)\ V_{out} = {}^{2}/_{3}\ 5\ V = {}^{10}/_{3}\ V = 3.33\ V$

smaller inductance => steeper current
for maintaining 5 V, the duty cycle would have to be adjusted



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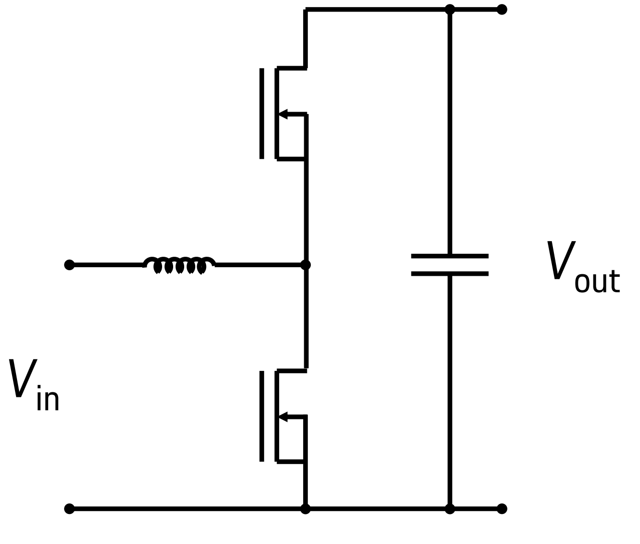
1(f)

Bidirectionality => diode to be replaced by transistor
Additional transistor is to be operated complementary to the first one; when one transistor is on, the other one is off; additionally, a dead time is required so that when one transistor is turned off, there is a gap on the order of maybe 100 ns to 1 µs dependent on the devices to allow the transistors to fully turn off before the other one is actively turned on.
If implemented as field-effect transistor (as is standard these days), the transistor does not have a pn forward voltage compared to a diode and can therefore increase efficiency. The transistor also does not have issues such as reverse recovery and can typically switch faster.

A lower battery voltage than output voltage uses a boost converter, which has a lower ripple current on the battery side than a buck converter. As the ripple would be reactive current supplied by the battery, it would generate extra heating on the battery side and can increase degradation. Boost converters reduce the problem and shift the ripple problem to the output side.



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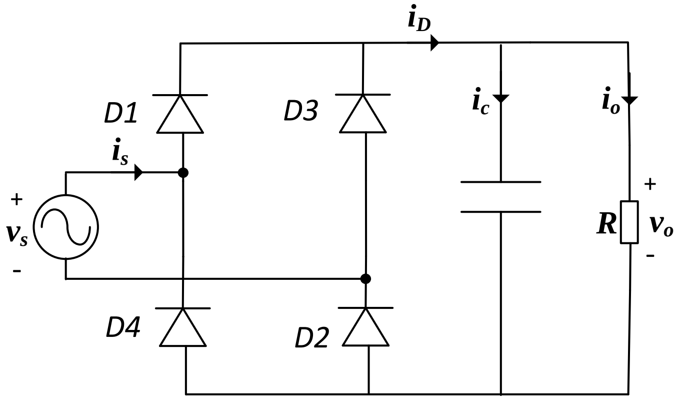
2(a)



2(b)

$$\Delta V_o = \sqrt{2}V_s - \sqrt{2}V_s e^{\frac{-t}{\tau}} = \sqrt{2}V_s - \sqrt{2}V_s e^{\frac{-\pi}{2\pi f_s RC}}$$

$$\approx \sqrt{2}V_s - \sqrt{2}V_s \left(1 - \frac{1}{2f_s RC}\right) = \frac{\sqrt{2}V_s}{2f_s RC}$$

Assumptions

1. Linear discharge of the capacitor under assumption that $RC \gg \frac{\pi}{\omega_s}$
2. The discharge occurs over the whole half cycle, i.e., charging is effectively instantaneous

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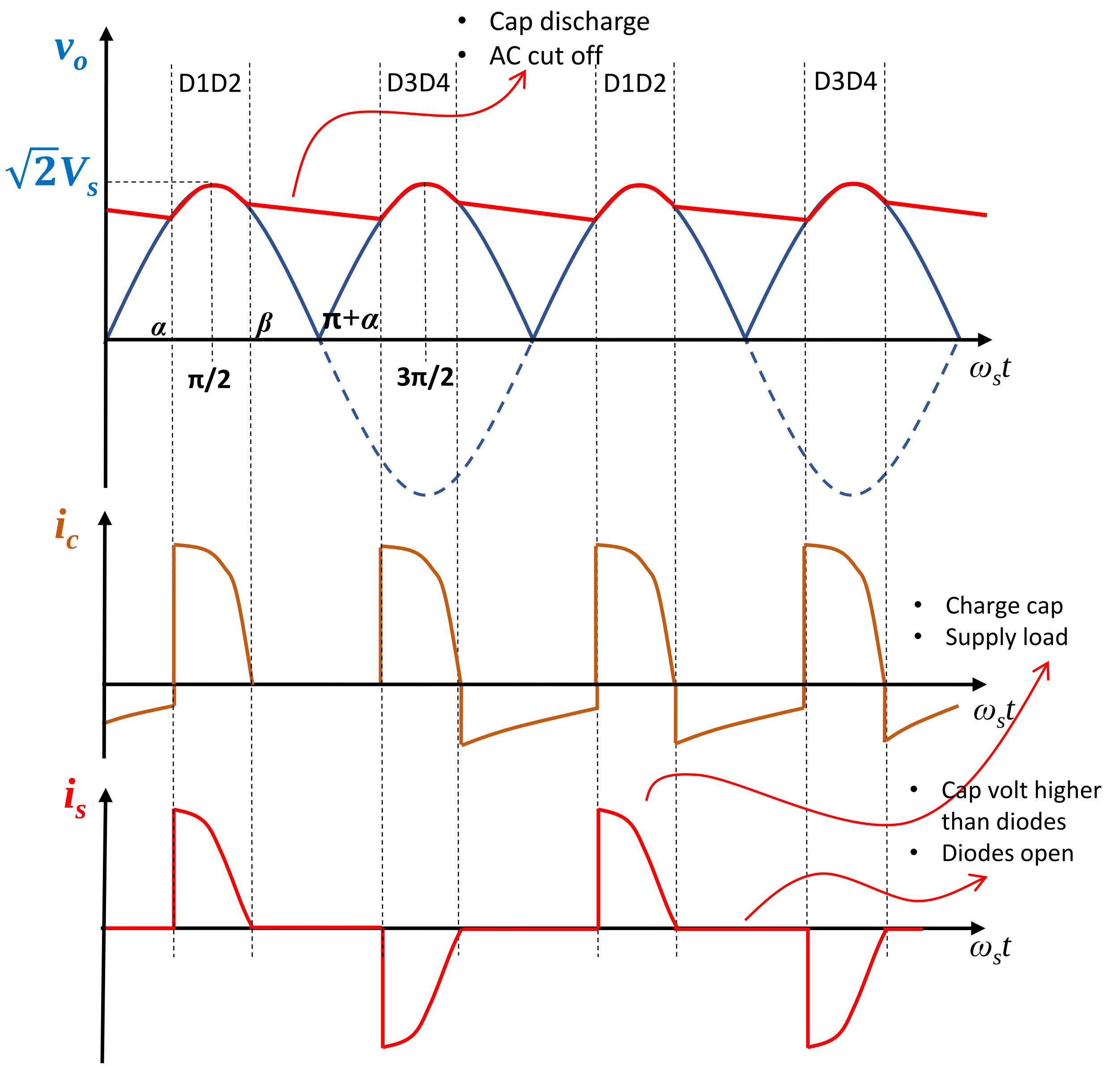
2(c)



The capacitor is only recharged when the capacitor voltage falls below the input voltage, i.e., when the absolute value of the sinusoidal mains voltage exceeds the capacitor voltage. That leads to very brief current surges around the maximum and minimum of the sine on the mains side, which stresses components and generates high-frequency mains currents.

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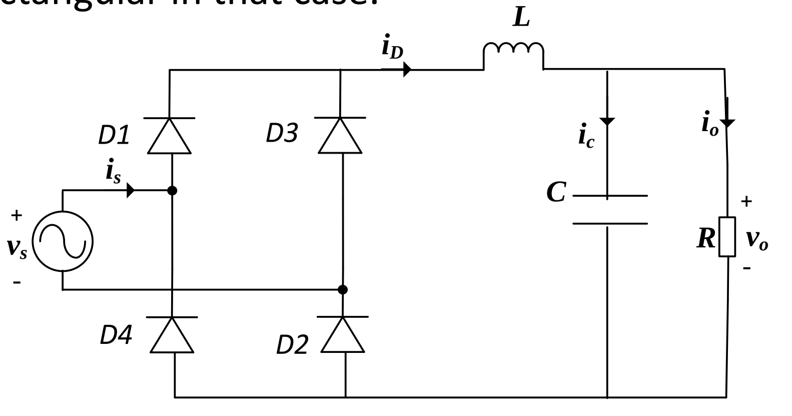
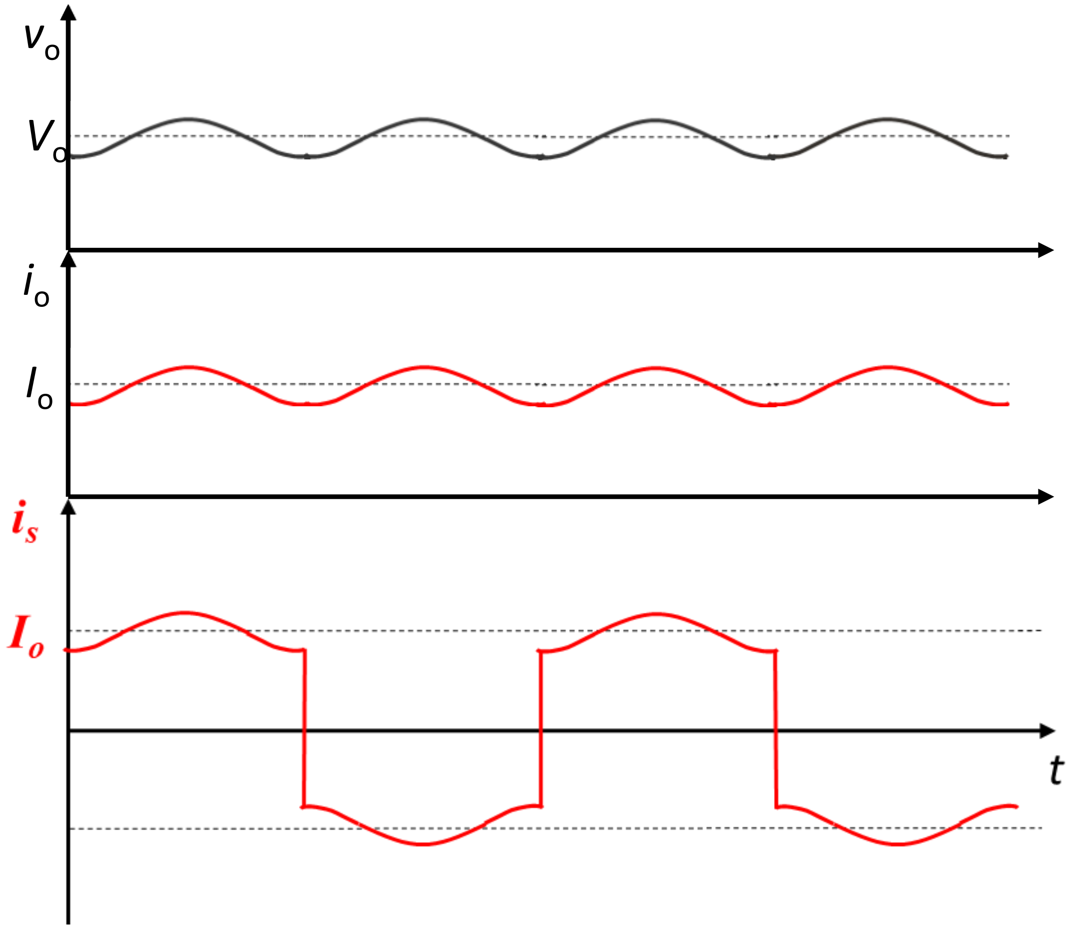
2(d)

Add inductor between rectifier and capacitor, which makes the current more "inert" so that it suppresses the spikes and rather generates relatively smooth currents. For very large inductors, the current i_D is practically constant. The mains current becomes near-rectangular in that case.





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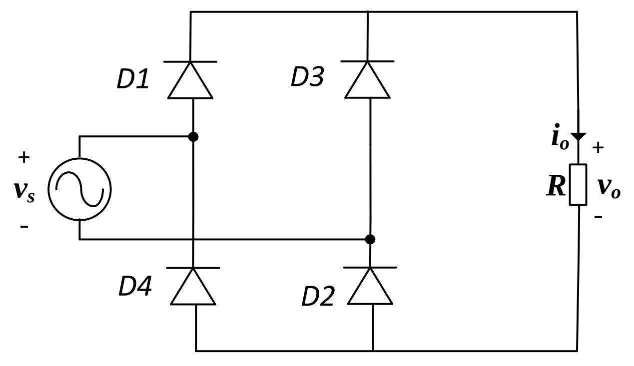
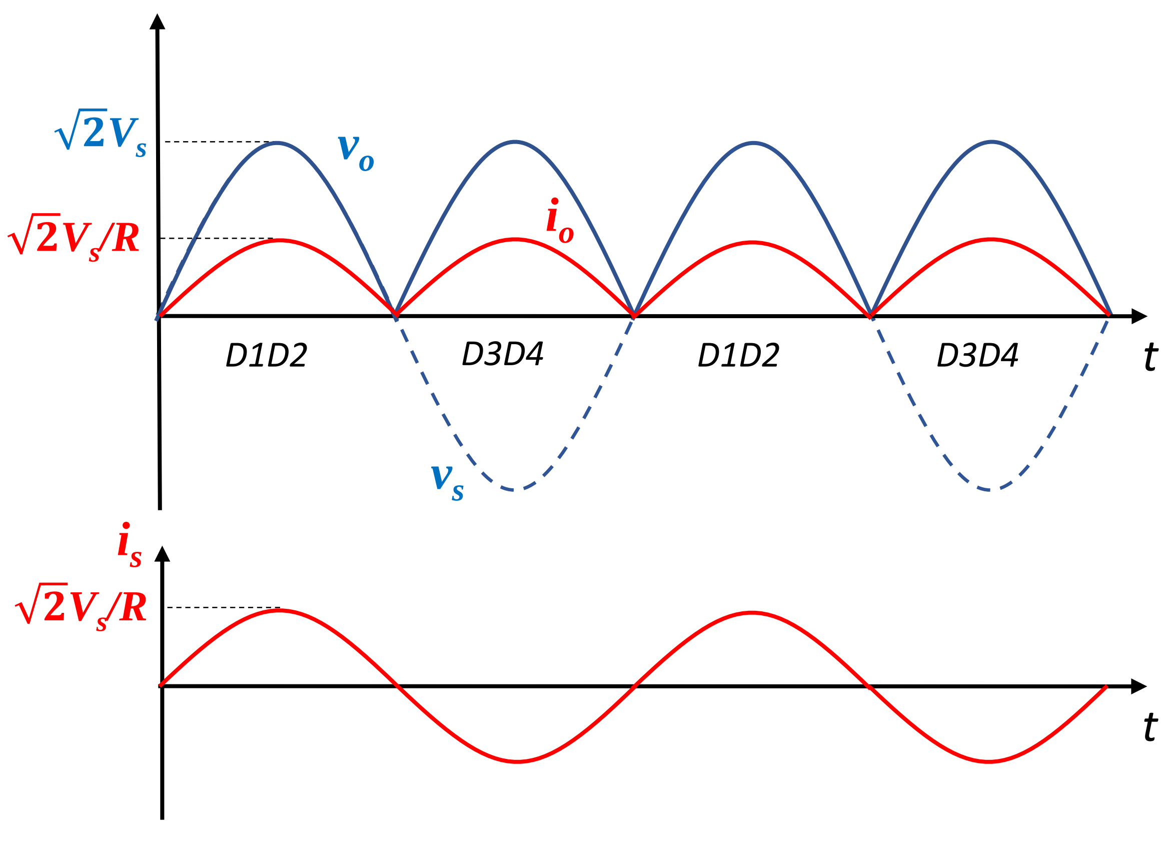
2(d)

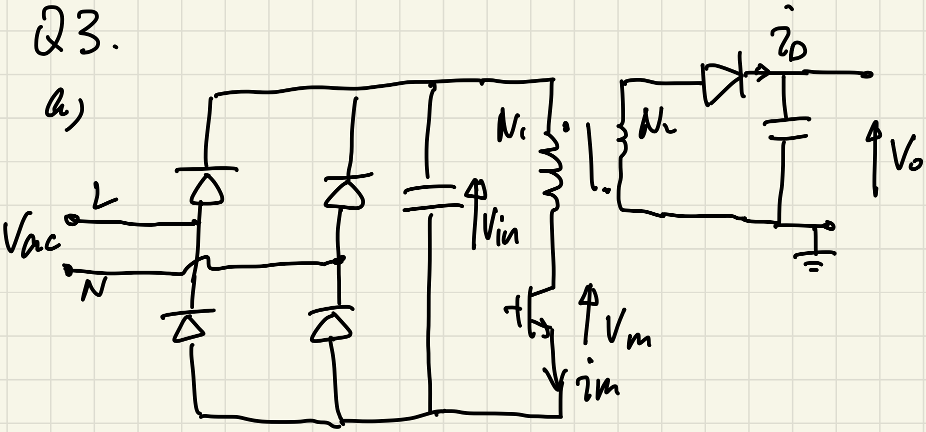
If a resistor is connected right to the rectifier output (with ideal diodes) without any capacitor or inductor, the rectifier would generate a voltage proportional to abs(sin(ωt)). The current would have the same profile. Voltage and current on the mains side would be sinusoidal without phase shift or larger distortion.





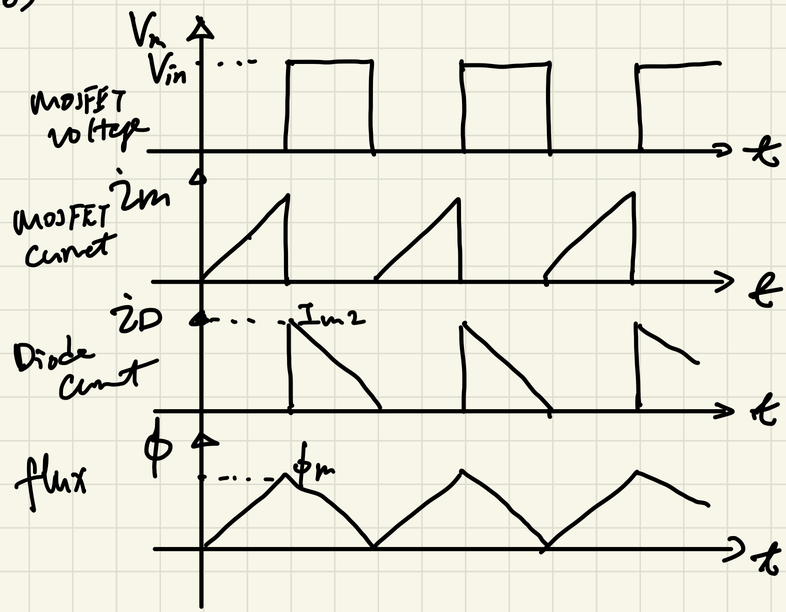
Q3.

a)



The flyback provides galvanic isolation which allows grounding at the output side.

b)



c) $V_{in} = 230 \times \sqrt{2} = 325V.$

$V_{in} = n_1 \frac{d\phi_m}{dt}$, $B_m = 0.36 \times 75\% = 0.27T$

$\phi_m = B_m \cdot A = 0.27 \times 0.25 \times 10^{-4} = 0.0675 \times 10^{-4} Wb$

$$n_1 = \frac{V_{in} \cdot DT}{\phi_m} = \frac{325 \times 0.5 \times \frac{1}{500 \times 10^3}}{0.0675 \times 10^{-4}} = 48.148$$

$n_1 = 48$

If $D = 0.5$, $\frac{n_1}{n_2} = \frac{325}{15}$, $n_2 = \frac{15 \times 48}{325} = 2.22$

$n_2 = 2$ or 3.

If $n_2 = 2$, D has to be larger than 0.5 for 15V output, which is not acceptable.

Therefore, $n_2 = 3$.

When $n_2 = 3$, $V_2 = \left(\frac{D}{1-D}\right) \cdot \frac{n_2}{n_1} \cdot V_1$ $\frac{D}{1-D} = 0.7384$

$15 = \left(\frac{D}{1-D}\right) \cdot \frac{3}{48} \cdot 325$

$D = 0.425$

d) At the secondary side of transformer.

$$\int_0^T I_o\,dt = \int_{(1-D)T}^{T} i_{m2}\,dt$$, i_{m2} is the magnetizing current at 2nd side.

$$I_o \cdot T = \frac{1}{2} \cdot I_{m2}\,(1-D) \cdot T,$$

$$I_{m2} = \frac{2 I_o}{1-D} = \frac{2 \times 2}{1-0.425} = 7A$$

$$L_{m2} = \frac{V_{out} \cdot (1-D)T}{I_{m2}} = \frac{15 \times (1-0.425) \times \frac{1}{500 \times 10^3}}{7} = 2.46\mu H$$

$$L_{m2} = \frac{n_2^2 \mu_0 \mu_r A}{l_m}, \quad \mu_r = \frac{2.46 \times 10^{-6} \times 2 \times 10^{-2}}{1.26 \times 10^{-6} \times 0.25 \times 10^{-4} \times 9}$$

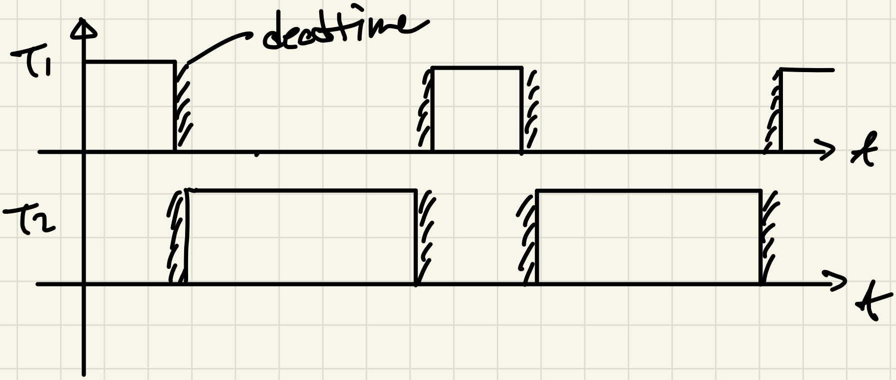
$$= 173.5.$$

e) Critically continous flux means no DC flux in the core. Therefore the maximum flux density of the core is smaller than that with an DC offset.

Most of high frequency magnetic cores have limited maximum flux density, removing DC offset is perferable

Q4.

a)



Deadtime is used to have both high side (T_1) and low side (T_2) MOSFETs at the off state. This deadtime acts like a time buffer to avoid incident short through when T_1 and T_2 have states being switched.

b) $$I_{Lo} = \frac{P_{in}}{V_{in}} = \frac{250}{V_o \cdot (1-D)} = \frac{250}{100 \times 0.25} = 10A$$

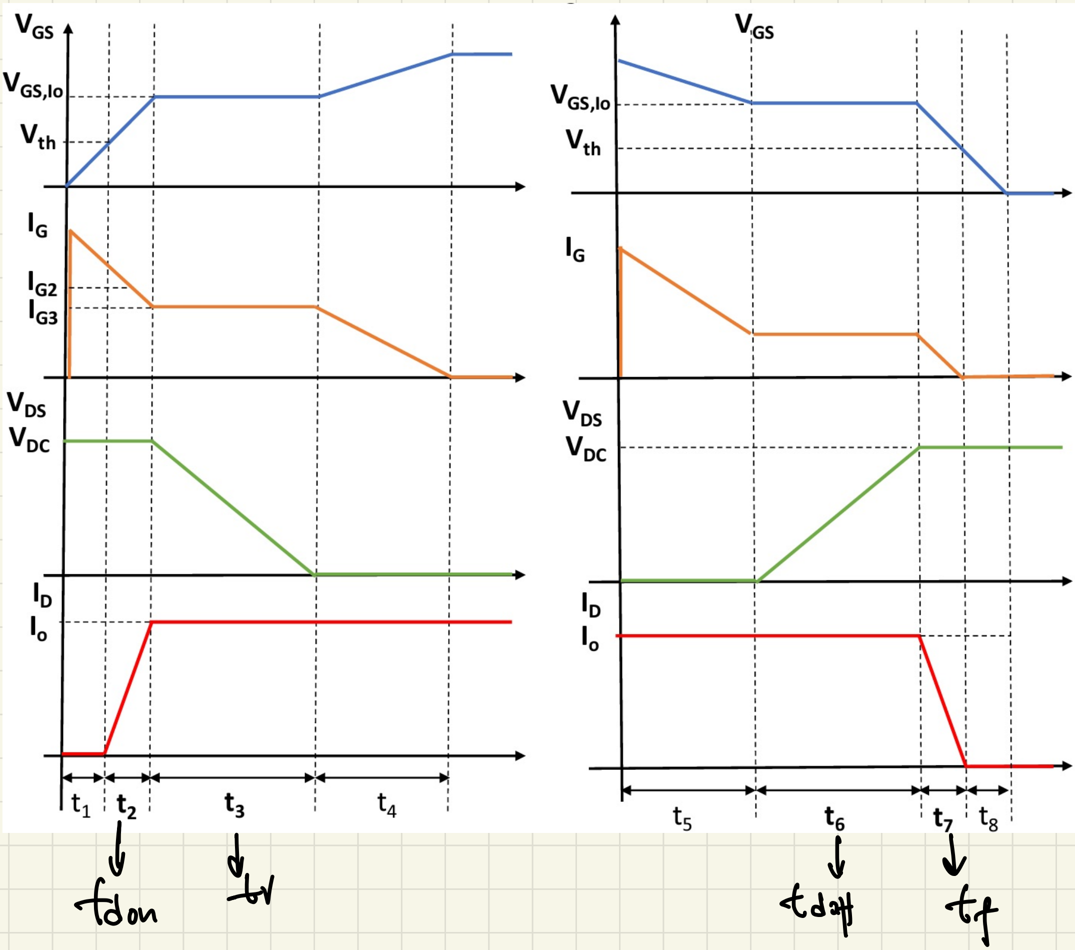
$$\Delta I_L = 10 \times 40\% = 4$$

$$P_{cond\,T_2} = R_{don} \cdot \left(\sqrt{I_{Lo}^2 D + \frac{\Delta I_L^2 D}{12}}\right)^2$$

$$= 50 \times 10^{-3} \times \left(10^2 \times 0.75 + \frac{4^2 \times 0.75}{12}\right)$$

$$= 3.8W$$

c)

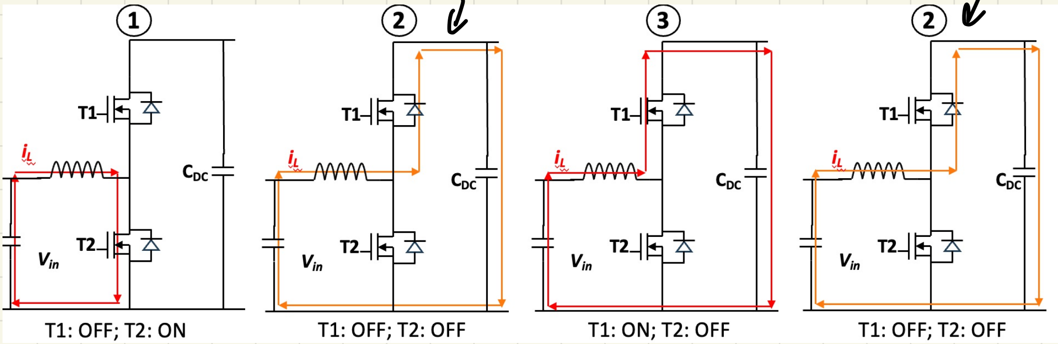


d) $$P_{swT_2} = \left(\frac{V_o \cdot I_L}{2}\right)\cdot\left(\frac{t_{don} + t_r}{T}\right)\cdot 2$$

$$= \left(\frac{100 \times 10}{2}\right) \times \frac{(14+16)\times 10^{-9}}{1/(100\times 10^{3})} \times 2 = 3W$$

e)

deadtime   deadtime.



During the deadtime, the inductor current needs to commutate before T_1 is switched on. This commutating current is conducted by the body diode of T_1. This means that T_1 has nearly zero voltage before being switched on, as the body diode forward bias voltage is much smaller than V_o. This gives much smaller switching loss of T_1 than T_2.