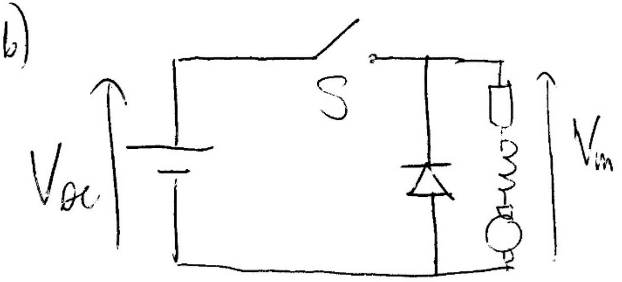
3B4 2018 crib Dr T Flack Dr T Long

1/ a) Advantages: high power density
: no field losses
: high efficiency.

Disadvantage: can't vary field strength so field weakening to attain higher speeds not possible.

Armature reaction. When DC motor is loaded currents flow in the armature winding which produce their own air gap field. This field reinforces the field flux over half a pole and cancels it over the other half-pole.

b)



Switch S (MOSFET) is switched on and off with duty cycle $\rho = T_{on}/T$. When on, $V_m = V_{DC}$, when off, freewheel diode conducts motor current so $V_m \sim 0$. $\therefore$ Mean value of V_m is ρV_{DC} and so armature voltage can be varied by varying ρ.

Taking $V_m \sim E = k\omega$ it is seen that varying V_m varies ω.

c) i) $R_a = 3\Omega$, $I_{rated} = 15A$

On open-circuit $V_{oc} = E = k\omega$

$\therefore \; 200 = k \times 1200 \times \frac{2\pi}{60}$

so $k = 1.59 \; Vs\,rad^{-1}$

$T = kI_a$ so $T_{rated} = 1.59 \times 15 = 23.9$ Nm

Max. speed will be when $\rho = 1$ so $V_m = V_{DC} = 500V$.

$V = E + I_a R_a$ so $E = 500 - 15 \times 3 = 455V$

$E = k\omega \Rightarrow 455 = 1.59\omega$ so $\omega = 286 \; rad\,s^{-1}$

$(= 2733$ rpm$)$

ii) $I_a = 7.5A$ for 50% rated torque

so $E = 500 - 3 \times 7.5 = 477.5V = 1.59\omega$

$\omega = 300 \; rad\,s^{-1}$ $(= 2868$ rpm$)$

$$P_{in} = V_m I_a = 500 \times 7.5 = 3.75\,kW$$

$$P_{out} = T\omega = 11.95 \times 300 = 3.585\,kW$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{3.585}{3.75} = 95.6\%$$

iii) Rated torque so $I_a = 15A$

500 rpm so $E_a = 1.59 \times 500 \times \frac{2\pi}{60} = 83.3V$

$$V_m = E_a + I_a R_a = 83.3 + 15 \times 3 = 128.3V$$

$$\therefore \rho = 128.3/500 = \underline{25.6\%}$$

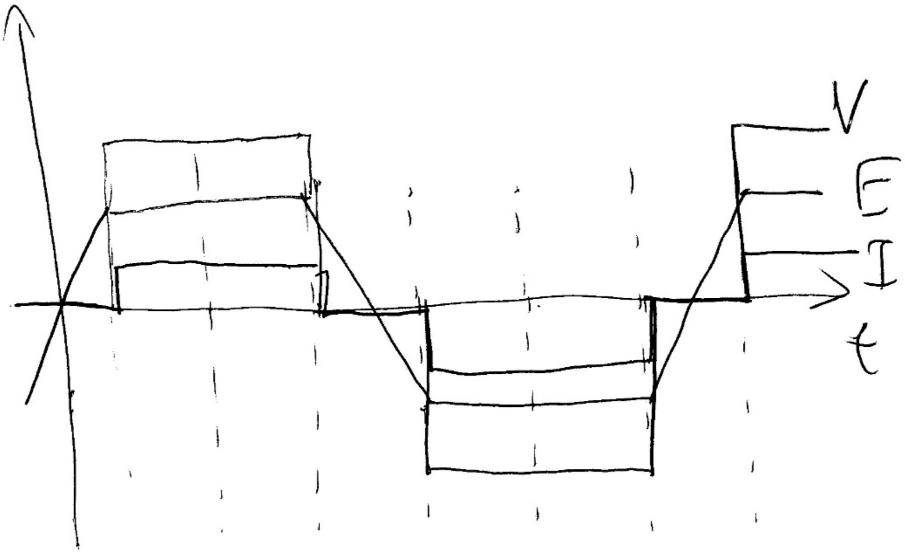
d) Electrical time constant: time taken for current to rise to $\overset{\times 63\%}{(V/R_a)}$ with armature locked (due to armature inductance, so L_a/R_a)

Electromechanical time constant: time taken for speed to rise to 63% of final speed (V/k).

τ_e usually $\ll \tau_{em}$ so current rises to final value very quickly, so therefore does torque. τ_{em} is dictated mainly by J, the armature + load inertia. Effect of τ_e & τ_{em} can therefore account for transient behaviour.

Assessor's comments: Most candidates gave good answers to part (a) except for the part about armature reaction, which was often confused with armature power loss. Many candidates were able to sketch the DC chopper circuit, but few explained why varying the armature voltage results in speed control. Most candidates did well at the more numerical part (c) although a few were unable to use the open-circuit test information to find the torque/emf constant and struggled thereafter. Many good answers to part (d), with some candidates going as far as to derive expressions for the electrical and electromechanical time constants (not required but nice to see).

2/a



At any time, two phases conduct, one phase is floating. Hall effect sensors are used to detect the rotor position and hence the phase of the back-emf E. This enables the applied voltage V to be timed so that E & I are in phase (90° torque angle).

The main constructional difference concerns the stator. The trapezoidal BCDCM utilises a concentrated winding, in which conductors are wound around distinct pole-pieces. In sinusoidal BCDCMs, a conventional 3φ winding with multiple coils per phase wound into slots is used to produce a sinusoidal airgap field.

b) i) $1000\ \text{rpm} = \frac{2\pi}{60} \times 1000 = 104.7\ \text{rad}\,s^{-1}$

$E_{ph} = \frac{E_{line}}{2} = \frac{20}{2} = 10V = k\omega_r$ giving $k = 0.0955\ Vs\,\text{rad}^{-1}$

$T = 2kI$ so $T_{rated} = 2kI_{rated} = \underline{0.955\ Nm}$

Maximum line-line voltage $= 40V$ so maximum phase voltage $= 20V$

$\therefore 20 = E + I_a R_a = E + 5 \times 1$ so $E_{max} = 15V$ at rated torque.

$E = k\omega$ so $\omega = E/k = 157\ \text{rad}\,s^{-1} \approx \underline{1500\ \text{rpm}}$

ii) 25% rated torque means $I = 1.25A$

$E = 20 - 1.25 \times 1 = 18.75V$, $\omega = \frac{E}{k} = 196\ \text{rad}\,s^{-1}$ $(= 1875\ \text{rpm})$

c) i) $T_m = 2kI = \underline{0.764 \text{ Nm}}$ since $I = 4A$ fixed this torque is also fixed.

Load torque $T_L = k_T\omega$ such that $T_L = 0.4$ Nm when $\omega = \frac{800 \times 2\pi}{60}$

giving $k_T = 4.77 \times 10^{-3}$

Final speed of drive will be when $T_m = T_L$

$$\therefore 0.764 = 4.77 \times 10^{-3}\omega \quad \text{so } \omega = 160 \text{ rads}^{-1}$$

$$= \underline{1528 \text{ rpm}}$$

ii) $$T_m - T_L = J\frac{d\omega}{dt}$$

$$T_m - k_T\omega = J\frac{d\omega}{dt}$$

$$\int_0 \frac{J\,d\omega}{T_m - k_T\omega} = \int_0 dt$$

$$-\frac{J}{k_T}\ln\frac{(T_m - k_T\omega)}{C} = t$$

$t=0,\ \omega=0 \quad \therefore C = T_m$

$$\ln\left(\frac{T_m - k_T\omega}{T_m}\right) = \frac{-k_T t}{J}$$

$$\frac{T_m - k_T\omega}{T_m} = e^{\frac{-k_T t}{J}}$$

$$\frac{T_m}{k_T}\left(1 - e^{\frac{-k_T t}{J}}\right) = \omega$$

iii) 63% means one time constant so $\frac{k_T t}{J} = 1$

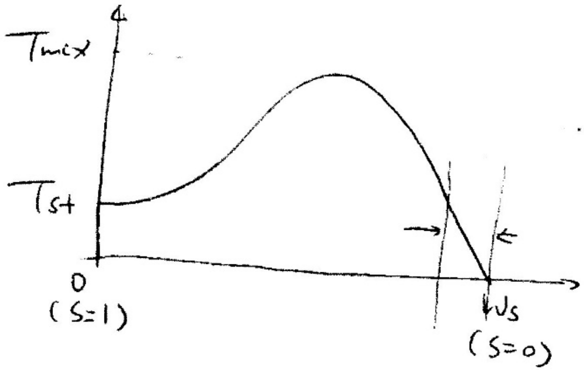
$$t = \frac{J}{k_T} = \frac{0.02}{4.77\times10^{-3}} = \underline{4.19s}$$

Assessor’s comments: Many excellent answers to part (a), with candidates showing a good understanding of the principles of operation of this sort of motor. Part (b) was more mixed, with many candidates mixing up line and phase quantities, leading to incorrect answers. Part (c) was supposed to be more challenging, but there were many excellent answers to this, which required candidates to consider the mechanical transient behaviour of the drive.

Q3 (a) $T\omega_r = T\omega_s - 3I_2^2R_2$

$sT\omega_s = 3I_2^2R_2$, $T = \dfrac{3I_2^2R_2}{s\omega_s}$, $\omega_s = \dfrac{2\pi f_1}{P}$ ← pole-pairs

$|I_2|^2 = \dfrac{V_1^2}{(\frac{R_2}{s})^2 + X_2^2}$, $T = \dfrac{3V_1^2R_2}{s\omega_s\left[(\frac{R_2}{s})^2 + X_2^2\right]}$



. normal operation region
S is close to zero

i) $T_{st} = \dfrac{V_1^2R_2}{\omega_s(R_2^2 + X_2^2)}$

ii) $\dfrac{dT}{ds} = 0.$

$$\frac{d}{ds}\left(\frac{3V_1^2R_2}{s\omega_s\left((\frac{R_2}{s})^2 + X_2^2\right)}\right) = \frac{3V_1^2R_2}{\omega_s}\left(X_2^2 - \frac{R_2^2}{s^2}\right) = 0$$

$X_2^2 = \dfrac{R_2^2}{s^2}$, $s = \pm\dfrac{R_2}{X_2}$

as $0 \le s \le 1$, when $s = \dfrac{R_2}{X_2}$, $T = T_{max} = \dfrac{3V_1^2}{2X_2}$

iii) The maximum torque T_{max} equals to starting torque T_{st}, to obtain the possible maximum torque at starting.

As $s = 1$ when starting, then $s = \dfrac{R_2}{X_2} = 1$, $R_2 = X_2$.

Adjust R_2 to equal to X_2, which is possible if wound rotor

b) i) S needs to be very small, normally between 0 and 0.1

ii) As s is small, $\frac{R_2}{s} >> X_2$ then ignore X_2

~~As~~ Hence R_1, X_1, X_2 all neglected.

then, $$T = \frac{3V_1^2 R_2}{s\omega_s\left(\left(\frac{R_2}{s}\right)^2 + X_2^2\right)} \approx \underline{\frac{3V_1^2 s}{\omega_s R_2^2}} = \frac{3V_1^2(\omega_s - \omega_r)}{\omega_s^2 R^2}.$$

iii) $$\frac{dT}{d\omega_r} = -\frac{3V_1^2}{\omega_s^2 R_2}$$

The motor is normally ~~operated~~ operating at the linear region

hence, $\frac{dT}{d\omega r}$ must be a constant. $\frac{V_1^2}{\omega_s^2}$ is a constant, $\frac{V_1}{\omega_s}$ is a contst.

iv) $$T = \frac{3k^2 s\omega_s}{R_2^2}, \quad k = \frac{V_1^2}{\omega_s^2}.$$

to have same torque when adjusting speed,

$$\underline{\frac{V_1^2}{\omega_s^2} = \frac{V_1'^2}{\omega_s'^2}},$$

V) $$T = \frac{3V_1^2 s}{\omega_s R_2^2}, \quad P = T\cdot\omega_s = \frac{3V_1^2(\omega_s - \omega_r)}{\omega_s R_2^2}$$

to have the same power when adjusting speed

$$\underline{\frac{V_1^2}{\omega_s} = \frac{V_1'^2}{\omega_s'}},$$

c i) Unloaded, $T \approx 0$

$$N_{rmax} = \frac{150 \times 60}{4} = 2250 \text{ r/min}$$

ii) $$T = \frac{3V_1^2(\omega_s - \omega_r)}{\omega_s^2 R_2^2} = 136\ \text{Nm}$$

$$\frac{3\times\left(\frac{415}{\sqrt{3}}\right)^2\left(2\pi\times\frac{50}{4} - \omega_r\right)}{\left(2\pi\times\frac{50}{4}\right)^2 \times 1.2^2} = 136, \Rightarrow \omega_r = 71.5\ \text{rad/s}$$

$$N_r = \frac{71.5}{2\pi}\times 60 = \underline{683}\ \text{r/min}$$

$$S = \frac{\frac{50}{4} - \frac{71.5}{2\pi}}{\frac{50}{4}} = 0.0896$$

$$I_2^2\frac{R_2}{S} = \frac{1}{3}\cdot T\cdot\omega_r$$

$$I_2 = \sqrt{\frac{136\times 71.5\times 0.0896}{3\times 1.2}} = 15.6A.$$

$$I_m = \frac{415/\sqrt{3}}{j100} = -j2.4\ A$$

$$I_1 = I_2 + I_m = 15.6 - j2.4 = 15.8\angle{-8.7^\circ}\ A$$

$$\underline{|I_1| = 15.8A}$$

iii) As rated magnetising current, flux is unchanged, $T \propto I_2$

half of torque means half of I_2, $I_2 = \frac{15.6}{2} = 7.8A$

As $I_m = -j2.4A$, $I_1 = I_2 + I_m = 7.8 - j2.4A$

At 1Hz, $X_1 \ll R_1$, hence $V_1 = V_m + R_1 I_1$

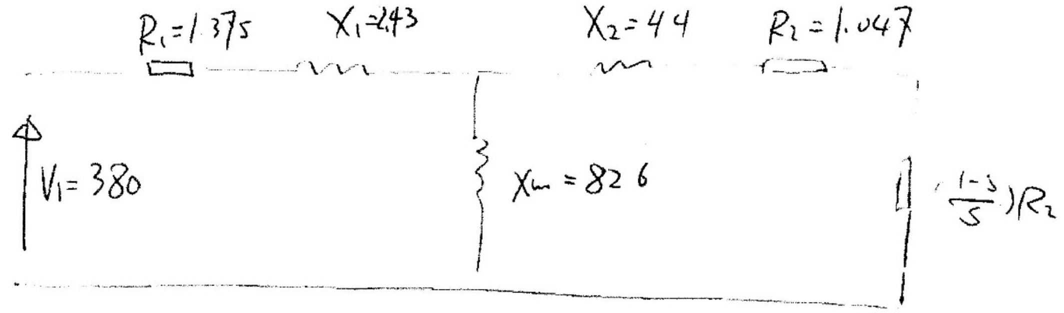
$$V_m = I_m\cdot\frac{X_m}{50} = I_m\cdot\left(X_m\times\frac{1}{50}\right) = -j2.4\cdot j2 = 4.8V$$

$$V_1 = 4.8 + (7.8 - j2.4)\times 1.5 = 16.3\angle{-12.8^\circ}\ V$$

$$V_{boost} = V_1 - V_m = 16.3\angle{-12.8^\circ} - 4.8\angle{0^\circ} = 11.7\angle{-18^\circ}\ V$$

Assessor's comments: Most of candidates attempted this question were able to answer most of the Part a. Candidates have shown good understandings of torque-speed characteristics and were able to sketch the curve. About three quarter of candidates were able to apply slip to zero for finding the starting torque and about and two third of candidates were also able to apply derivative of the torque to find out the slip for the maximum torque. Most of candidates found difficult to calculate the boost voltage. The observation is the basics of AC circuit taught in IA and IB were not been fully understood and students were struggling to using vectors to solve the circuit correctly and quickly and many scripts of this part were in bad presentation, indicating candidates' uncertainty and lack of confidence of solving such questions.. Only about 3 students have got the final answer correct or close to be correct.

4. a) i)



ii) $S = \dfrac{\frac{50\times60}{2} - 1458}{\frac{50\times60}{2}} = 0.028$

$Z_{in} = jX_m // (\frac{R_2}{s} + jX_2) + R_1 + jX_1$

$= \dfrac{jX_m \cdot (\frac{R_2}{s} + jX_2)}{j(X_m + X_2) + \frac{R_2}{s}} + R_1 + jX_1$

$= \dfrac{j82.6 \cdot (j4.4 + \frac{1.047}{0.028})}{j(82.6+4.4) + \frac{1.047}{0.028}} + 1.375 + j2.43$

$= \dfrac{82.6\angle90^\circ \cdot 37.7\angle6.7^\circ}{94.7\angle66.7^\circ} + 2.8\angle60.5^\circ$

$= 32.9\angle30^\circ + 2.8\angle60.5^\circ$

$= 35.3\angle32.3^\circ$

$Z_{in} = 35.3\angle32.3^\circ$

$p.f = \cos32.3^\circ = 0.845$ lagging

iii) $I_1 = \dfrac{V_1}{Z_{in}} = \dfrac{380\angle0^\circ}{35.3\angle32.3^\circ} = 10.8\angle-32.3^\circ A$

$P_{in} = 3 \cdot U_1 I_1 \cos32.3^\circ = 3\times380\angle0^\circ \cdot 10.8\angle-32.3^\circ \cdot 0.845 = 10404 W$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{9600}{10404} \times 100\% = 92.3\%$$

iii) $I_1 = 10.8\angle 32.3^\circ$

$$V_m = V_1 - I_1 (R_1 + jX_1)$$

$$= 380\angle 0^\circ - 10.8\angle{-32.3^\circ} \cdot (1.375 + j2.43)$$

$$= 380\angle 0^\circ - 30.2\angle 28.2^\circ$$

$$= 353.7\angle{-2.3^\circ}\ V$$

$$I_2 = \frac{V_m}{X_2 + \frac{R_2}{s}} = \frac{353.7\angle{-2.3^\circ}}{37.4 + j4.4} = 9.4\angle{-9^\circ}\ A$$

$$P_{loss1} = 3I_1^2 R_1 = 3 \times (10.8)^2 \times 1.375 = 481 W$$

$$P_{loss2} = 3I_2^2 R_2 = 3 \times (9.4)^2 \times 1.047 = 278 W$$

$$P_{loss\,elect} = 481 + 278 = 759$$

$$P_{loss\,mec} = 10404 - 9600 - 759 = 45 W$$

b). i) $p = c\frac{d\theta_m}{dt} + k\theta_m$, θ_m is the temperature rise above ambient

$$\theta_m = \frac{p}{k}(1 - e^{-t/\tau}), \quad \tau = \frac{c}{k}$$

hence, $\theta = \frac{p}{k}(1 - e^{\frac{t}{\tau}}) + \theta_0$

ii) $\tau = \frac{c}{k} = \frac{5000\ J/kg}{10 W/K} = 500 s$

$p = 10404 - 9600 = 804 W$

$$\theta = \frac{P}{k}(1 - e^{-t/\tau}) + \theta_0$$

$$= \frac{804}{10}(1 - e^{-\frac{100}{500}}) + 40 = 54.5°C$$

iii) As the motor is cooled to 50°C, the initial temperature of the motor is 50°C, i.e. $\theta_0 = 50°C$

$$\theta = \theta_0 + (\frac{P}{k} - \theta_0)(1 - e^{-t/\tau}), \quad \frac{P}{k} = 80.4$$

the peak temperature occurs at the end of each cycle, $t = 100s$

$$\theta_{max} = 50 + (80.4 - 50)(1 - e^{-0.2})$$

$$= 55.5°C$$

iv) The peak temperature exceeds the maximum allowed temperature, hence additional cooling measures such as fan needs to be fit to increase the dissipation co-efficient.

Assessor's comments: This is a popular question. In Part a, about a quarter of candidates attempted this question were able to correctly answer the power factor of the induction machine and about a third of candidates were able to correctly calculate the efficiency. The observation is that candidates were not very clear on the physical meaning of magnetising current of the induction machine and not with full competence of solving vectors of the circuit, causing incorrect answer of power factor. Some candidates were not clear on power flow (loss breakdown) of the induction, causing the incorrect answer of efficiency.

In Part b, the temperature equation derivation was answered well and candidates have shown good understandings of temperature characteristics when machine operated at different duty cycles. For some candidates, the numerical answers were not correct, which was mainly due to the lack of competence of finding the power flow (loss breakdown), which was observed in Part a too.