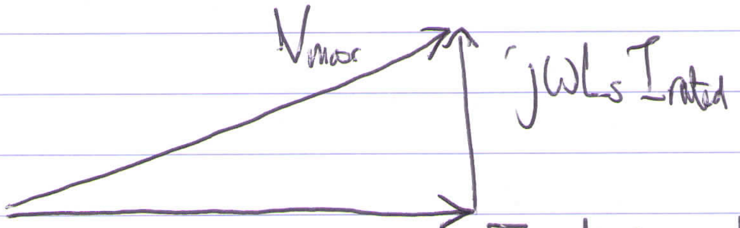
3B4 2022 Crib

Tim Flack
Teng Long

1\. a) The trapezoidal BLDCM has a concentrated stator winding in which conductors are wound around individual pole-pieces. The sinusoidal BLDCM uses a conventional three-phase winding, typically with multiple coils/phase, aiming to produce a sinusoidally-distributed air gap field. The rotors are of similar construction, both utilising surface-mounted permanent magnets attached to backing iron.

[10%]

b) (i) $T_{rated} = 3kI_{rated} = 3 \times 1.8 \times 150 = 810\ \text{Nm}.$



$$E = k\omega_r = \frac{k\omega}{p}$$

Rated speed corresponds to the maximum speed at which rated torque can be delivered $\therefore$ $I = I_{rated} = 150\ \text{A}$, $V = 415/\sqrt{3}$

$$V^2 = E^2 + (\omega L_s I_{rated})^2 = \left(\frac{k\omega}{p}\right)^2 + (\omega L_s I_{rated})^2$$

$$\left(\frac{415}{\sqrt{3}}\right)^2 = \omega^2\left(\left(\frac{1.8}{3}\right)^2 + (2.8\times10^{-3}\times150)^2\right)$$

$$\omega = 327\ \text{rad s}^{-1} \quad \text{so} \quad \omega_{rated} = \frac{\omega}{p} = 109\ \text{rad s}^{-1}\ (1041\ \text{rpm})$$

[10%]

(ii) $T = 70.7\%$ of rated torque $= 573\text{ Nm} = 3kI$

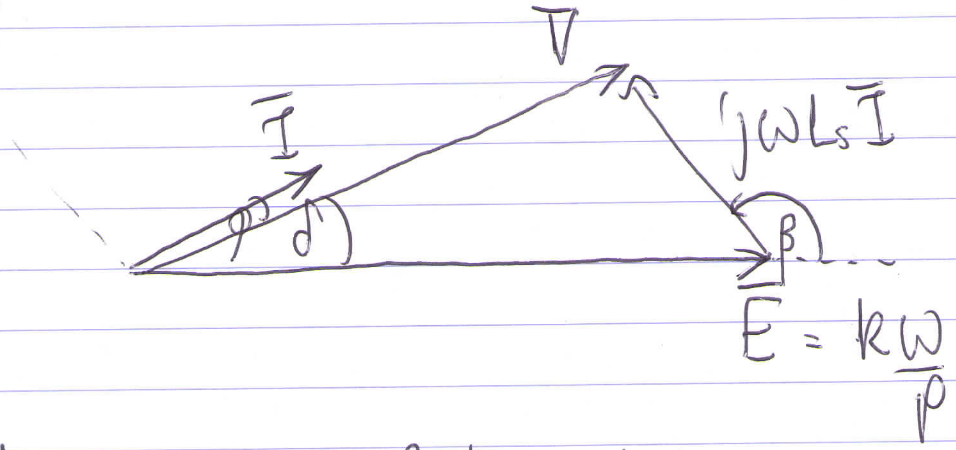
$\Rightarrow I = 106\text{ A}$

Same method as (i): $\left(\frac{415}{\sqrt{3}}\right)^2 = \omega^2 \left(\left(\frac{1.8}{3}\right)^2 + \left(2.8\times10^{-3}\times 106\right)^2\right)$

$\omega = 358\text{ rad s}^{-1}$ so $\omega = \frac{\omega}{p} = 119\text{ rads}^{-1}$ (1139 rpm)

[10%]

iii) Field-weakening involves injecting a component of stator current that will act to reduce the total airgap field, allowing greater speeds to be obtained at the expense of torque and efficiency.



Note that $\beta \neq 90°$ to achieve this.

For maximum speed, $I = I_{rated}$ so $\sin\beta = 0.707 \Rightarrow \beta = 135°$, $V = V_{max} = 415/\sqrt{3}$

Cosine rule: $V^2 = \left(\frac{k\omega}{p}\right)^2 + \left(\omega L_s I_{rated}\right)^2 - 2.\frac{k\omega}{p}.\omega L_s I_{rated}\cos 45°$

$$\left(\frac{415}{\sqrt{3}}\right)^2 = \left(\frac{1.8\omega}{3}\right)^2 + \left(2.8\times10^{-3}\times150\,\omega\right)^2 - 2\times\frac{1.8}{3}\times2.8\times10^{-3}\times150\times0.707\,\omega^2$$

$\omega = 564.7\text{ rads}^{-1}$, $\frac{\omega}{p} = 188\text{ rads}^{-1}$ (1798 rpm)

$P_{out} = T\omega = 573 \times 188 = 108\ kW$

Sine rule: $\dfrac{\sin\delta}{\omega L_s I} = \dfrac{\sin 45^\circ}{415/\sqrt{3}}$ giving $\delta = 44.4^\circ$

[30%]

c) i) $E_{ph} = \dfrac{30}{2} = 15 = k\omega_r = k \times 800 \times \dfrac{2\pi}{60}$

$k = 0.179\ V\,s\,rad^{-1}$ [5%]

ii) $T = 2kI = 2 \times 0.179 \times 5 = 1.79\ Nm$

Maximum line-line voltage $= 48V$, so $V_{ph\,max} = 24V$

$24 = E + 0.6 \times 5 \qquad E = 21V$

$\omega_{rated} = \dfrac{21}{0.179} = 117\ rad\,s^{-1}$ (1120 rpm) [10%]

iii) Current is 5 A for 2/3 cycle, 0 A for 1/3 cycle

$\therefore I_{rms} = \sqrt{\dfrac{2}{3} \times 5^2 + \dfrac{1}{3} \times 0^2} = 5\sqrt{\dfrac{2}{3}} = 4.08\ A$ [10%]

iv) 50% rated torque $\Rightarrow I = 2.5A$, $E = 24 - 0.6 \times 2.5$
$= 22.5V$

$\omega = \dfrac{E}{k} = \dfrac{22.5}{0.179} = 126\ rad\,s^{-1}$ (1200 rpm)

$P_{out} = T\omega = \dfrac{1.79}{2} \times 126 = 113\ W.$ $\quad P_{cu} = 2I^2R = 2 \times 2.5^2 \times 0.6 = 7.5\ W$

$P_{in} = \dfrac{113 + 7.5}{0.95} = 127\ W \qquad \eta = 113/127 = 89.1\%$ [15%]

2 (a) PM brushed dc motors have a fixed field, but are easy to control with a variable dc power supply (speed / torque). They are often found in toys such as electric cars / trains where a low voltage dc supply is available from batteries. Stepper motors require a more complex 2-phase drive, but by pulsing the windings sequentially, highly accurate open-loop position control is achieved. They are commonly found in 2-D and 3-D printers.

[10%]

b) i) $V_{ac} = 12 = e_a = k\phi\omega = k\phi \times 1000 \times \frac{2\pi}{60}$

$k\phi = 0.115 \ \text{V s rad}^{-1}$

$T_{rated} = k\phi \, i_{rated} = 0.115 \times 3 = 0.344 \ \text{Nm}$

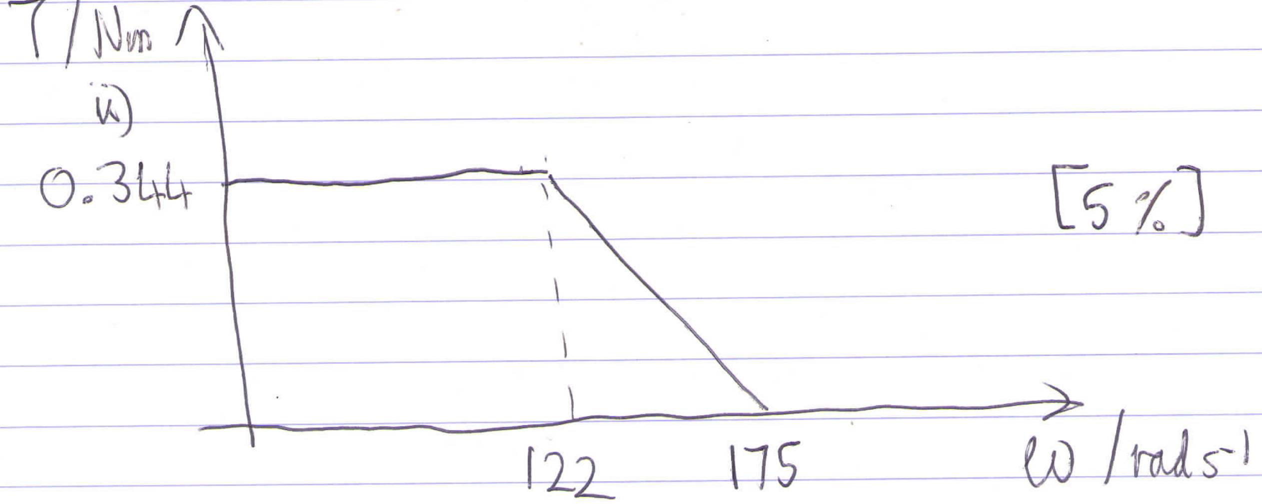
$20 - i_{a\,rated} R_a = e_{a\,rated} = k\phi \, \omega_{rated}$

$20 - 3 \times 2 = 14 = 0.115 \, \omega_{rated} = 122 \ \text{rad s}^{-1} \ (1167 \ \text{rpm})$

Maximum speed corresponds to $T_L = 0 \ \therefore \ i_a = 0, \ e_a = V_a$

$20 = k\phi \, \omega_{max} \qquad \omega_{max} = \frac{20}{0.115} = 175 \ \text{rad s}^{-1} \ (1667 \ \text{rpm})$

(ii)



[5%]

c) $1400 \text{ rpm} = 1400 \times \frac{2\pi}{60} = 147 \text{ rad s}^{-1}$.

Therefore we need to break the calculation down into two parts:

① $\omega = 0 > \omega_{rated}$, where $T = T_{rated} = 0.344$

② $\omega_{rated} < \omega \leqslant 147 \text{ rad s}^{-1}$, where T reduces linearly to zero

① $T = 0.344 = J \frac{d\omega}{dt}$ so $\frac{d\omega}{dt} = 6.88 \text{ rad s}^{-2}$

so $\omega = 6.88t$ from $\omega = 0 \rightarrow 122 \text{ rad s}^{-1}$.

Time taken to accelerate from 0 to 122 rad s^{-1} $= \frac{122}{6.88} = 17.7 \text{ s}$

② Equation for the torque is $T = \frac{k\phi V_a}{r_a} - \frac{(k\phi)^2}{r_a}\omega$

giving $1.15 - 6.613 \times 10^{-3} \omega$

(Checks: this gives 0.344 at $\omega = 122$ and 0 at $\omega = 175$)

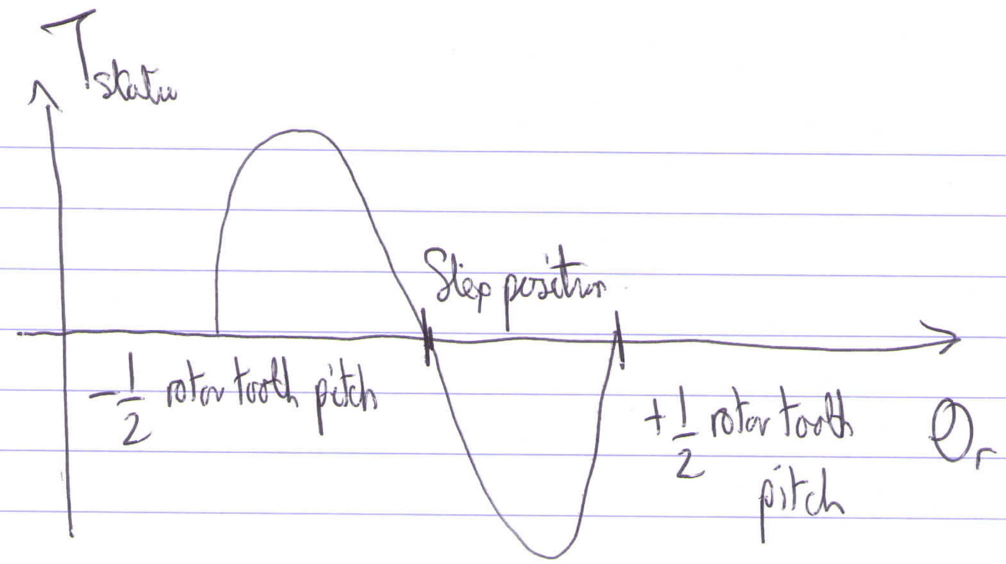
$T = J \frac{d\omega}{dt}$ so $J \int_{122}^{147} \frac{d\omega}{1.15 - 6.613 \times 10^{-3} \omega} = \int_0^{t_1} dt$

$$\frac{0.05}{6.613 \times 10^{-3}} \int_{122}^{147} \frac{d\omega}{175 - \omega} = t_1$$

$$7.56 \ln \frac{53}{28} = t_1 = 4.82 \text{ s}$$

$\therefore$ Total time $= 17.7 + 4.82 = 22.5 \text{ s}$ [25%]

d) i)



With no torque the stepper motor will align itself to minimise the stored magnetic energy i.e. at a stable equilibrium. With torque applied, the rotor will move so that the motor torque is equal to the applied torque. This will take it away from a full step position, and the small angle moved through $\Delta\theta$ is the angular position error.

[10%]

ii) $T = \hat{T} \sin 50\theta \qquad \hat{T} = \frac{1.5}{2} \times 0.25 = 0.1875 \text{ Nm}$

$\therefore 0.1 = 0.1875 \sin 50\theta$ so $\theta = 0.64°$ i.e. around 1/3 of the full step size of $1.8°$.

[10%]

iii) $5\cancel{0}$ rpm $\equiv \cancel{50}\,5/60$ rps $\equiv \frac{\cancel{5}\,50}{600} \times 200$ switchings/second

$\therefore f_s = \frac{\cancel{20}\,50}{3}$ Hz $= \frac{1}{2\pi}\sqrt{\frac{50 \times \cancel{0.1875}\,0.25}{J}}$ giving $J = \cancel{5.9 \times 10^{-4}}\; 1.14 \times 10^{-3}$ kg m^2

Stepper motors have speeds that should be avoided so that resonance is not excited, which can cause missed steps.

Two ways to avoid: accelerate through quickly, use microstepping.

[20%]

Q3. a) Specific magnetic loading: Average flux density over one pole pitch.

Specific electric loading: Total effective current averaged around the air gap.

b) Specific magnetic loading: $\bar{B}$

Specific electric loading: $\bar{J}$

Machine apparent power: S

$$S = \frac{P}{\eta\% \cdot P.f} = \frac{\pi}{\sqrt{2}} \frac{\omega}{P} \bar{B} \cdot \bar{J} \cdot Vol, \quad Vol \text{ is the volume of rotor.}$$

$$\frac{50 \times 10^3}{0.8 \times 0.8} = \frac{\pi}{\sqrt{2}} \times \frac{6000 \times 2\pi}{60} \times 0.5 \times 30000 \cdot Vol$$

$$Vol = 3.73 \times 10^{-3} \, m^3 \quad (3.73 L)$$

c) Assume the stator resistance and leakage inductance are neglected,

$$V_{ph} = E = ld \cdot \left(\frac{\omega}{P}\right) \cdot N_{ph} \cdot k_d \cdot k_p \cdot B_{rms}$$

$$l = \frac{Vol}{\pi \left(\frac{d}{2}\right)^2} = \frac{3.73 \times 10^{-3}}{\pi \cdot \left(\frac{240 \times 10^{-3}}{2}\right)^2} = 0.0825 m$$

$$B_{rms} = \frac{\bar{B} \pi}{2\sqrt{2}} = \frac{0.5 \times 3.14}{2 \times 1.414} = 0.56 T$$

$$k_d = \frac{\sin\left(\frac{m \beta p}{2}\right)}{m \sin\left(\frac{\beta p}{2}\right)}, \quad m = \frac{48}{3 \times 8} = 2, \quad \beta = \frac{360°}{48} = 7.5°$$

$$k_d = \frac{\sin\left(\frac{2\times 7.5^\circ\times 8}{2}\right)}{2\sin\left(\frac{7.5^\circ\times 8}{2}\right)} = 0.866$$

$$k_p = \cos\left(\frac{\alpha p}{2}\right), \quad \alpha = \beta = 7.5^\circ, \quad k_p = \cos\left(\frac{7.5^\circ\times 8}{2}\right) = 0.866$$

$$260 = 0.0825 \times 0.24 \times 200\pi \times 0.866 \times 0.866 \times 0.56 \times N_{ph}$$

$$N_{ph} = 48.8 \text{ turns.}$$

There are $\frac{48}{3} = 16$ slots per phase. therefore the turns per phase is ideally integer of 16, yield:

$$N_{ph} \approx 48.$$

Therefore, 3 turns per slot.

Q4 a) i)

ii)

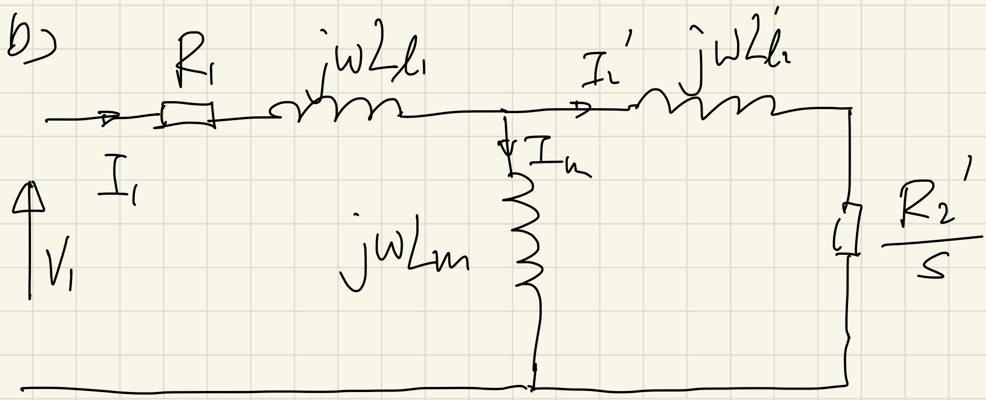
iii)

① Torque is linearly proportional to speed

② A much wider and uniformed adjustbility of speed can be obtained

③ High efficency, no additional resistive loss.

b)



R_1 : stator resistance

L_{l_1} : stator leakage inductance

L_m : magnetising inductance

L_{l_2}' : rotor leakage inductance referred to stator

R_2' : rotor resistance referred to stator.

s : slip

ω : synchronous angular frequency.

V_1 : phase voltage

I_1 : stator phase current

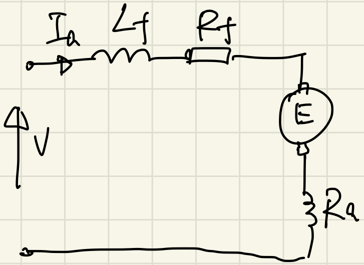
I_m : magnetising current per phase

I_2' : rotor phase current referred to stator.

$$I_m = \frac{V_1}{\omega L_m}, \quad \phi = I_m L_m$$

The air gap flux ϕ should be kept constant therefore I_m is constant. Therefore $\frac{V_1}{\omega}$ is constant. ω is synchronous speed. The inverter or drive voltage has frequency as $p \cdot \omega$ therefore the inverter or drive voltage/frequency should be constant.

c) i)



$R_f + R_a = R$

When operating at D.C

$E = K_c I_a \omega_r$

$V = R I_a + E$

$$\omega_{r1} = \frac{60}{60} \times 2\pi = 2\pi, \quad \omega_{r2} = \frac{150}{60} \times 2\pi = 3\pi$$

$$E_1 = 2 \times 2\pi \times K_c = 4\pi K_c, \quad E_2 = 3 \times 3\pi \times K_c = 9\pi K_c$$

$$\begin{cases} 7 = 2R + 4\pi K_c \\ 14.3 = 3R + 9\pi K_c \end{cases} \Rightarrow \begin{cases} K_c = 0.4 \\ R = 1 \end{cases}$$

When operating at A.C

$$T_{ave} = \frac{1}{2} K_c (\hat{I}_{aac})^2 = \frac{1}{2} \times 0.4 \times (8\sqrt{2})^2 = 25.6 \text{ Nm}$$

ii) $P_{out} = T_{ave} \cdot \omega_r = 25.6 \times \frac{270}{60} \times 2\pi = 723\,W.$

$P_{loss} = I_{aac}^2 R = 8^2 \times 1 = 64\,W$

$S = VI = 110 \times 8 = 880\,VA.$

$$P.f = \frac{P_{out} + P_{loss}}{S} = \frac{723+64}{880} = 0.89.$$

$$\eta\% = \frac{P_{out}}{P_{out}+P_{loss}} = \frac{723}{723+64} = 91.9\%$$

d) $$\frac{40-20}{70-20} = e^{-\left(\frac{(12-10)\times 60}{\tau}\right)} \Rightarrow \tau = 131\ \text{sec.}$$

$$P_{loss} = \frac{1000 \times 0.5}{0.9} \times 0.1 = 56\,W$$

$$70 = \frac{56}{K} + \left(20 - \frac{56}{K}\right) e^{-\left(\frac{60\times 10}{131}\right)}$$

$K = 0.794\ W/K$

$\tau = C/K, \quad C = \tau \cdot K = 131 \times 0.794 = 104\ J/K$

Examiners' comments

Q1 Sinusoidal and trapezoidal Brushless DC motors: 49 Attempts, Mean 12.8/20

All candidates attempted this question, and there were many excellent attempts. The most common errors concerned mixing up mechanical and electrical angular frequencies, and calculating quantities from phasor diagrams when analysing operation under field-weakened conditions in (b)(iii). Some candidates confused rated and maximum speed at various points.

Q2 Brushed DC motors and stepper motors: 45 Attempts, Mean 14.1/20

A popular questions with many very good attempts. A common error in (b)(ii) was taking the torque characteristic beyond rated speed as a constant power one rather than linear. Very few candidates succeeded with (c), although many understood the need to consider the two parts of the torque-speed curve separately. Part (d) achieved many excellent answers, with only part (iii) causing problems, to do with mixing up frequencies.

Q3: Induction motor design: 14 Attempts, Mean 12.64/20

Most students answered Part (a) and (b) well. However, a common mistake of forgetting the power factor for the apparent power in Part (c) has been observed. Most of students could list the essential equations but not able to use the right parameters to answer Part (d).

Q4 Induction and universal motors, and duty cycle analysis: 39 Attempts, Mean 10.2/20

The Part (a) and (b) have been answered well. Only a very few students answered the Part (c) right. The common mistake of the AC universal machine torque has been observed. Common mistake of finding the actual loss when half load of Part (d) has also been observed.