1

(a) Please name at least three magnetic principles to generate torque in an electrical machine. Name at least four electrical machine types and assign them to the torque generation principle that dominates in that device. There are machine designs that mix more than one of these principles. Please name at least one such electrical machine design and briefly explain how the two principles of torque generation operation work. Furthermore, please name one key difference between these two torque-generating mechanisms.

The three main magnetic principles used to generate torque:

- Magnetic pole–pole attraction/repulsion (principle: equal poles attract, unequal ones repel each other)
- Magnetic reluctance (principle: energy minimisation, magnetically well conducting materials are attracted as the same magnetic flux can flow at lower energy/magnetic field, paper clip containing ferritic steel (i.e., iron in its allotropic ferrite structure) attracted by a magnet, no matter which pole is presented).
- Eddy-current torque (an electrically conductive feels a torque if it moves faster or slower than a moving magnetic pole; practically an eddy-current break relative to a moving magnetic field; parts of the electrically conductive object (e.g., rotor) leaving an area with high magnetic flux density (e.g., a pole of the rotating stator field) experience an induced current counteracting the flux reduction (Lenz's rule), which leads to attraction, while parts of the electrically conducive object entering an area with high magnetic flux density experiences an induced current reducing the flux, which leads to repulsion)

Electrical machine types:

- DC motor (brushed and brushless): mostly pole–pole attraction
- Universal motor: pole–pole attraction
- Synchronous machine: mostly pole–pole attraction, sometimes mixed with reluctance torque (e.g., in electric cars), rarer mixed with eddy-current torque (e.g., in large generators as or damper cage as vibrations as motion relative to the synchronous speed is suppressed or in so-called line-start motors to speed them up as induction motors until they reach almost the synchronous speed at which the synchronous torque would automatically take over)
- Induction machines: eddy-current torque
- Reluctance machine: obviously reluctance torque

Machine types with at least two mixed mechanisms of torque generation:

- Synchronous Line-start motor: Typically mix of permanent-magnet synchronous machine with induction machine, i.e., a squirrel cage integrated into a rotor with permanent magnets. As indicated above, start-up procedure when connected to the three-phase AC mains solely as induction machine through eddy-current torque (explanation of mechanism see above). As soon as the machine approaches its synchronous speed, the synchronous torque (in case of proper design) takes over and the slip is eliminated (explanation of torque mechanism see above). A key difference between the two torque mechanisms: pole–pole attraction is synchronous, eddy-current torque needs a slip.

- Most modern electric vehicle motors with buried permanent magnets: The permanent magnets are arranged such that they generate magnetic well conductive paths through the rotor and purely conductive paths (through the permanent magnet!). These differences can be used to generate torque from reluctance, thus that the stator flux "wants" to flow through magnetically well conductive paths and therefore attracts those. Both pole–pole attraction and reluctance torque are both synchronous mechanisms. Often, they are designed such that they have their maxima at different angles between rotor and stator poles. Key difference: pole–pole attraction only works poles of different polarity (N and S) so that a stator pole only attracts every other rotor pole. Reluctance torque, however, attracts every well conductive path, i.e., every rotor pole.

(b)

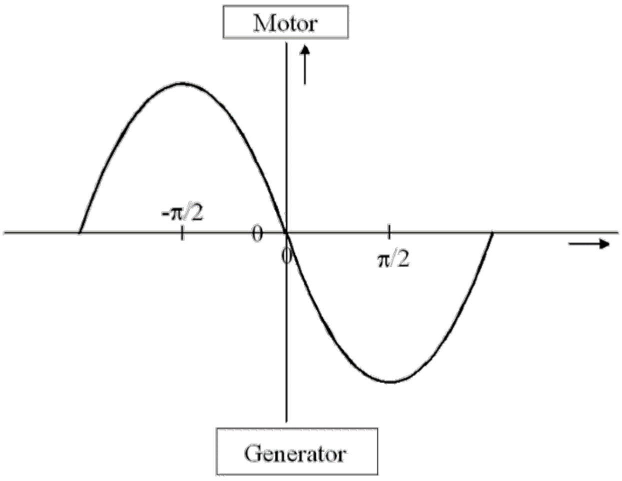
Latest brushless machines use three ways to increase the power in the same volume:

- Elimination of mechanical commutator, which typically contributes to a large share of the overall armature resistance and also limits speed;
- Use of latest rare-earth permanent magnetic, which correspond to exceptionally high excitation in an electrically excited machine;
- Substantial increase of the rotor speed from a few thousand rotations per minute in most brushed DC motors to even 100,000 rpm.

(c)

Whereas increasing the torque of a motor needs (over a wide range) a proportional increase in current and therefore more cross section of the winding to carry the electrical current and of the teeth and stator back-iron to carry the additional flux, i.e. overall more space, speed promises a relatively easy way to increase the power of an electrical machine. The increase of the power density typically easily overcompensates any gearing needed to adjust the speed to the process to be driven. However, the increase of speed faces mechanical limits (without mechanical commutators, which are the major speed limiter in brushed DC machines, the mechanical consistency of the rotor is typically a hard limit) and electromagnetic loss limits, typically dominantly on the rotor in permanent magnet machines (AC effects, such as skin/proximity effects and eddy current loss in the lamination and the magnets).

(d)



For pure pole–pole attraction, ideally ~ sin(β) [NB If reluctance torque is mixed into it, a sin(2β) component is added as every pole (not just every other one) generates attraction). The sine shape is stretched in y direction (max increases linearly) if the voltage is increased.

The machine is typically operated between maximum (motor) and minimum (generator). If the max. torque is exceeded, the rotor slips and slides to the next pole pair (the sin curve continuous there). If power electronics run the motor and tries to keep it always at its ideal angle, the machine is kept close to the maximum and gives the motor just enough voltage to achieve the necessary torque close to the maximum of the curve.

(e)

A fully magnetised high-performance hard magnet has hardly any softmagnetic moments left, all atomic dipoles are aligned so that μ_r --> 1. Ferrites have µr in the hundreds, good silicon steel up to thousands. [Though not asked: Ferrites saturate in the hundreds of mT, good silicon steel can exceed 1 T].

Diamagnetism is associated with $\mu_r < 1$, which means that the material counteracts external magnetic fields and practically “pushes” them to the outside

2

(a)

$k_V = 1900\ V^{-1}\ min^{-1}$

max torque $T_{max} = {}^{1}/_{2\pi}\ 1/k_V\ I_{max} = {}^{1}/_{2\pi}\ 1/1900 \times V\ min \times 60\ s/min \times 200\ A = 1.0\ Nm$

(b)

Equivalent electrical circuit: current source (representing loss power) feeding a parallel connection of a capacitor (representing thermal capacitance) and resistor (representing power dissipation). The voltage across them (representing the temperature) must not exceed a certain limit.

$\theta(t)/R_{dissipation} + C_{thcap} \times d\theta(t)/dt = P_{loss}$ for both cases

For continuous heat dissipation, the thermal capacitance does not play a role as the temperature change $d\theta/dt=0 \Rightarrow P_{cont} = \theta/R_{dissipation} = c \times (200\ A)^2 = c \times 40{,}000\ A^2$, i.e., resistive loss proportional to the squared current with a constant c

During the short-term overload, both dissipation and capacitance play a role. The dissipation depends on the temperature difference to the environment. Although the temperature is not known, its profile is: it increases from the ambient temperature (no dissipation) linearly to the max temperature (same as for continuous operation)
=> On average, it has the temperature in the middle and the power dissipation is approximately half of the continuous one.

Integrate the differential equation for the 10 s solution:

$\int \theta(t)/R_{dissipation}\ dt + C_{thcap} \times \theta_{max} - \theta_{ambient} = c \times (450\ A)^2 \times 10\ s$
$\frac{1}{2}\ c \times (200\ A)^2\ 10\ s + C_{thcap} \times \theta_{max} - \theta_{ambient} = c \times (450\ A)^2 \times 10\ s$

$C_{thcap} \times \Delta\theta = c \times 10\ s \times [(450\ A)^2 - \frac{1}{2}\ (200\ A)^2] = c \times 1.8 \times 10^6\ A^2 s$

30 s:
$\frac{1}{2}\ c \times (200\ A)^2\ 30\ s + c \times 1.8 \times 10^6\ A^2 s = c \times I_{30\ s}^2 \times 30\ s$
$\frac{1}{2}\ (200\ A)^2\ 30\ s + 1.8 \times 10^6\ A^2 s = I_{30\ s}^2 \times 30\ s$
$\Rightarrow I_{30\ s}^2 = \frac{1}{2}\ (200\ A)^2 + 1.8 \times 10^6\ A^2 s/30\ s$
$\Rightarrow I_{30\ s} = 283\ A$

60 s:
$\Rightarrow I_{60\ s}^2 = \frac{1}{2}\ (200\ A)^2 + 1.8 \times 10^6\ A^2 s/60\ s$
$\Rightarrow I_{60\ s} = 224\ A$

In case we ignore dissipation entirely during first 10 seconds:
$C_{thcap} \times \Delta\theta = c \times 10\ s \times (450\ A)^2 = 2.0 \times 10^6\ A^2s$ (10% deviation)

(c)
(i)

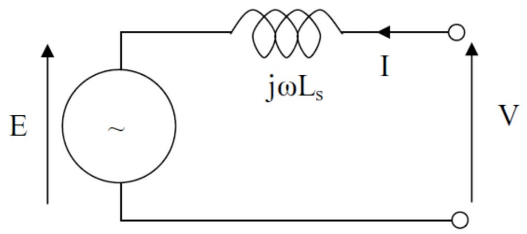
$N_{max} = 1900\ rpm/V \times 48\ V = 91{,}200\ rpm$

Speed falls with the torque due to the voltage drop at the internal resistances (e.g., of the winding)!

$N_{max}(T) = k_V \times (48\ V - R_{int}\ I) = k_V \times (48\ V - R_{int}\ 2\pi\ k_v\ T)$

=> linear decrease

(ii)



Injecting a current with the right phase into the winding inductance can generate a voltage drop so that V < E. This current has to have a 90° phase offset relative to the voltage E so that $V = E - j\omega L \times (-jI) = E - \omega LI < E$

The torque-generating current would be in phase with E so that the overall current has a torque-generating and a voltage-reducing (also called field-weakening) component.

(d)

Potential sensors:

- Encoder: provides position information only in discrete steps (but this step can be very fine); except for so-called absolute encoders, usually relative position information until a full turn (or the next sync mark) has passed
- Resolver: provides absolute position information, also for a motor standing still
- Hall sensors: only very rough information, depending on the number of sensors only few discrete steps (typically just three, one between each pair of stator phases so that only 60° information); in between interpolation possible but not ideal

Sensorless operation of the power electronics

- Most frequently through measuring the back-emf at the phase terminals. Works mostly with trapezoidal control (or stopping sinusoidal injection from time to time) and instead of setting active zero voltage at the specific phase, leaving the respective phase floating (or at 0 A) so that the back-EMF becomes detectable at the terminal. The zero-crossing of the back-emf indicates the position of the rotor. For three phases, this measurement mode is handed from phase to phase, and these zero crossings provide 60° information about the rotor position, similar to Hall sensors.
- More advanced position estimators usually measure the position-dependent stator inductance, normally through injecting a higher frequency current component that does not contribute to torque generation. The measured inductance is a superposition of the two extreme values (so-called d and q values at the d and q rotor positions respectively), from which the position in between.

Q3

1. (a) The voltage of a three-phase motor is limited by the airgap flux density (saturation, iron losses) and the ability of the winding insulation to withstand the peak electric field. The phase current of a three-phase motor is limited by the heat produced due to I^2R losses in the winding (overheating, reduced efficiency). Hence it is the volt-amps of the motor that is limited, and the volt-amp rating is given by the product of the maximum rms phase current and the maximum rms phase voltage and the number of phases.

Starting from S = 3VI, taking V and E as equal and substituting for I in terms of specific electric loading, J, and V in terms of specific magnetic loading, B, gives the result.

Note that the expression for E involves B_{rms} and so B_{rms} must be expressed in terms of B using $B_{rms}=\pi B/(2\sqrt{2})$.

d: airgap diameter; l: axial length; ω: angular supply frequency; p: pole-pairs; B: specific magnetic loading; J: specific electric loading.

(b) (i) Rated output power = 100 kW and full-load efficiency is 92%, so input power at full load is 100/0.92 = 108.7 kW. The input power factor is 0.8 and so the volt-amp rating of the motor is 108.7/0.8 = 136 kVA = S.

$B=0.5$ T, $J=20$ kAm^{-1}, $\omega/p=2\pi\times50/5 = 62.8$ rads^{-1} and $d=4l$. Substituting into the expression for S:

$136000=\pi^2\times4l^3\times62.8\times0.5\times20000/\sqrt{2}$ giving $l^3 = 0.00776$ and so $l=0.198$ m and $d=4l=0.792$ m.

(ii) $k_w = \cos(p\alpha/2)\times\sin(mnp\beta/2)/(m\sin p\beta/2)$, number of slots $N_S = 60$ and so $\beta=360/60 = 6^0$. Single layer winding so it must be fully-pitched and so $\alpha=0$. 60 slots means 30 coils (for a single layer winding) and so 10 coils per phase, so since $p=5$, $m=2$ giving $mnp\beta/2=30^0$ and $p\beta/2=15^0$ giving:

$k_w=\cos(0)\times\sin30/(2\times\sin(15)=0.966$.

$E=V=6.6\text{kV}/\sqrt{3}=3.81$ kV $=l\times(\omega/p)\times d\times N_{ph}\times k_w\times B_{rms}$ with $B_{rms}=\pi B/(2\sqrt{2})=0.555$ T. Putting in the numbers and solving for N_{ph} gives N_{ph} = 722.5 and hence 72.3 turns per coil. This should be rounded up to the nearest integer so that the specific magnetic loading is slightly under the desired value, so N_{coil} = 73 and N_{ph} = 730.

(iii) Assuming that all the flux crossing a slot pitch enters the tooth then:

$B_t w_t=(w_s+w_t)B\pi/2$ where w_s and w_t are the slot width and tooth width respectively. Rearranging:

$B_t=(1+w_s/w_t)\,B\pi/2$ and solving for w_s/w_t with $B_t=1.57$ gives $w_s/w_t=1$ and so $w_s=w_t$.

Also, $w_s+w_t=\pi d/60$ = 41.4 mm and so $w_s=w_t=20.7$ mm

For the stator core thickness, the principle is that all of the pole flux must split in half, travel round the core and then back through the teeth one pole-pitch later. Thus, the peak core flux, φ_c, is given by:

$B_c yl=0.5\times Bl\pi d/(2p)$ where y is the core thickness and B_c is the peak core flux density which is to be 1.4 T. Solving for y gives 44.4 mm.

(iv) The stator slot must be deep enough so that its area is big enough to accommodate the required total conductor area. First find the rated phase current:

$S=3V_{ph}I_{ph}$ gives $136000=\sqrt{3}\times6600\times I_{ph}$ resulting in $I_{ph}=11.9$ A.

With the rms current density limited to 6Amm^{-2} the conductor cross-sectional area is $11.9/6=1.98$ mm^2 and so the total conductor area is $N_{coil}\times1.98=144.5$ mm^2. The slot area required to accommodate this is 144.5/0.7 because of the 70% slot fill factor giving a total slot area of 206 mm^2.

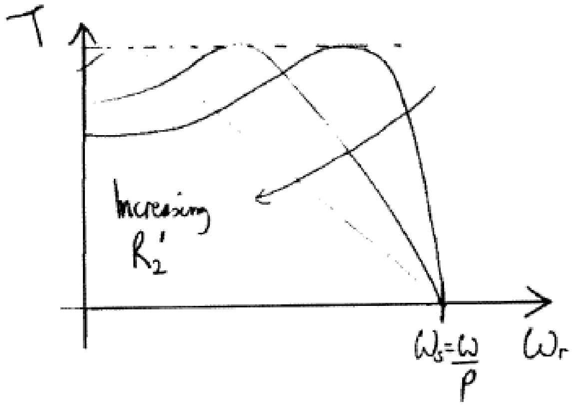
Assuming rectangular stator slots, this area is given by w_sd_s where d_s is the slot depth and w_s was found earlier to be 20.7 mm. Thus d_s = 9.98 mm.

(v) Airgap radius =d/2=0.792/2=0.396 m=396 mm. Total motor radius r_m=d/2+ds+y=396+9.98+44.4=450mm.

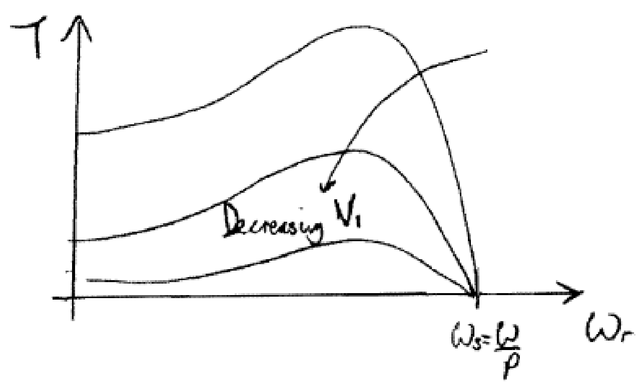
Motor volume is $=\pi r_m^2 l=0.126$ m^3

Q4

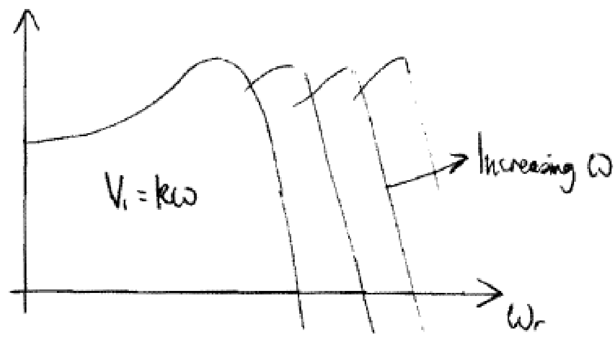
2(a) (i) Torque-speed characteristics for varying R_2.



(ii) Torque-speed characteristics for varying V_1.



(iii) Torque-speed characteristics VVVF speed control.



Advantages of VVVF control: the motor is always operating on the steep part of the torque-speed characteristic, so efficient; wide speed range possible at high efficiency; maximum available torque is unaffected unlike variable voltage control; motor always fully-fluxed which means it is operating efficiently and making good use of the magnetic materials.

(b) Start from the databook expression for torque:

$T = 3I_2^2R_2/s\omega_s$

$I_2^2 = V_1^2/((R_2/s)^2 + \omega^2L_2^2) = s^2V_1^2/(R_2^2 + s^2\omega^2L_2^2)$ assuming that the voltage across the rotor branch, E, is equal to the stator applied voltage, V_1. For small slip, $s\omega L_2 \ll R_2$ and may be ignored (this corresponds to the steep part of the torque-speed characteristic as required) giving $I_2 = sV_1/R_2$

Substituting into the torque equation gives:

$T = 3V_1^2 s/(\omega_s R_2)$ as required.

The slope of the steep part of the torque-speed characteristic is found by differentiating the above torque expression wrt ω_r. The easiest way to do this is to use the chain rule, so:
$dT/d\omega_r = dT/ds \times ds/d\omega_r$ where $s = (\omega_s - \omega_r)/\omega_s$ and so $ds/d\omega_r = -1/\omega_s$

$dT/ds = 3V_1^2/(\omega_s R_2)$ and so $dT/d\omega_r = -3V_1^2/(\omega_s^2 R_2)$

Substituting in $\omega_s = \omega/p$ gives $dT/d\omega_r = -3p^2V_1^2/(\omega_s^2 R_2)$

For VVVF control with $V1 = k\omega$ this slope becomes $-3k^2p^2/R_2$ and is therefore fixed.

(c) (i) The unloaded speed at 50 Hz of an 8 pole motor is $60f/p = 3000/4 = 750$ rpm. Therefore the maximum unloaded speed of the drive corresponds to the inverter operating at its maximum frequency of 150 Hz, giving $N_{max} = 750 \times 150/50 = 2250$ rpm.

To find the rated torque, ignore $R_1 + jX_1$ and jX_2. This is justified because the motor will be operating at a small value of slip.

$I_1 = 415\sqrt{3}/jX_m + I_2 = -j2.40 + I_2$

Since $R_2/s >> X_2$, I_2 is approximately in phase with V_1 and so at the stator current limit of 15 A:

$I_2 = \sqrt{(15^2 - 2.4^2)} = 14.8$ A (which shows that $I_1 = I_2$ is a reasonable approximation for an induction motor under full-load conditions)

$k = V_b/\omega_b = 415/\sqrt{3}/(2\pi \times 50) = 0.764$

$I_2 = V_1/(R_2/s) = V_1 s\omega/(\omega R_2) = s\omega k/R_2 = 14.8$ so $s\omega = 14.8 \times 1.2/0.764 = 23.3$ and this is fixed for rated torque and rated rotor current.

$T_{rated} = 3pk^2 s\omega/R_2 = 3 \times 4 \times 0.764^2 \times 23.3/1.2 = 136$ Nm

V_1 is at its maximum when $f = 50$ Hz and delivering rated torque. The slip at this frequency is given by $2\pi \times 50s = 23.3$ so $s = 0.0742$.

Maximum speed at which rated torque can be delivered is $(1 - s) \times 60f/p = (1 - 0.0742) \times 750 =$ 694 rpm.

(ii) $f = 1$ Hz so voltage across magnetizing reactance for rated flux is $(1/50) \times 240 = 4.8$ V. At rated magnetizing current torque is proportional to I_2, and so for 50% of rated torque, $I_2 = 14.8/2 = 7.4$ A.

Thus, total stator current is $(7.4 - j2.4)$ A and this will produce a voltage drop across R_1 (no need to include X_1 since its value will be negligible compared to R_1 at $f = 1$ Hz) of:

$V_{R1} = 1.5 \times (7.4 - j2.4) = 11.1 - j3.6$

and so $V_1 = 4.8 + 11.1 - j3.6 = 15.9 - j3.6$ and the magnitude of V_1 is 16.3 V. Voltage boost is the difference between V_1 and the voltage required to produce rated flux ie the voltage across the magnetizing reactance, which is 4.8 V. Therefore $V_{boost} = 16.3 - 4.8 = 11.5$ V.