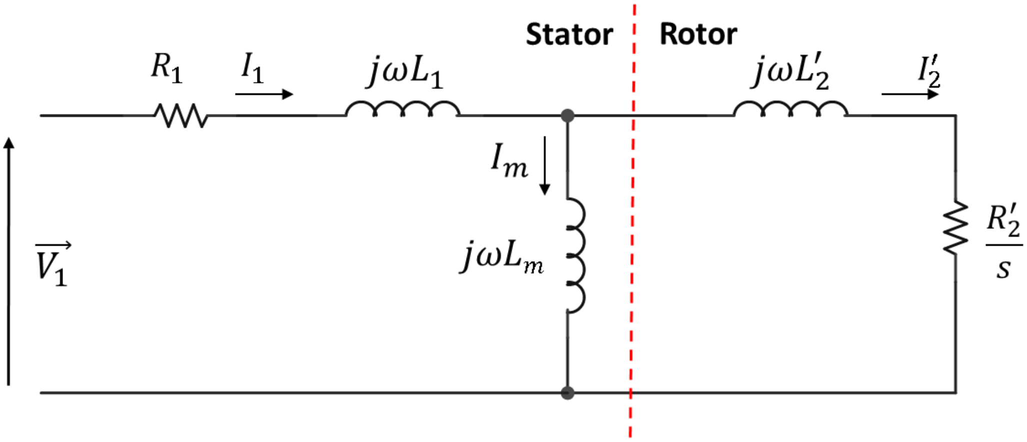
3B4 2024 Crib Q1 and Q2: Stephan Goetz Q3 and Q4 Tim Flack

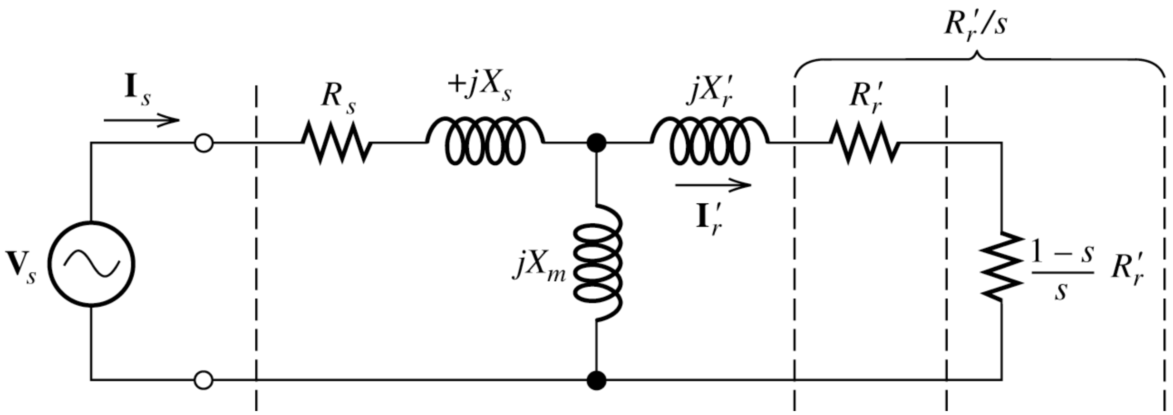
1 (a)



Splitting R_2'/s into two parts:

$R_2'/s = R_2' + (R_2'/s - R_2')$

=> constant part representing the losses: R_2';
power transfer from electrical to mechanical domain: $R_2'/s - R_2' = (1 - s)/s\ R_2'$



(b) Lecture 11

$$P_{\text{out}} = P_{\text{gap}} - P_{\text{loss}}$$

$$T\omega_r = T\omega_s - 3I_2'^2 R_2'$$

$$T = \frac{3I_2'^2 R_2'}{s\omega_s}$$

Neglecting the stator resistor R_1 and stator leakage inductor L_1, the EMF equals to the input voltage $\overrightarrow{V_1}$ (same voltage in form of vector):

$$I_2' = \frac{V_1}{\sqrt{\left(\frac{R_2'}{s}\right)^2 + X_2'^2}}$$

$$T = \frac{3V_1^2 R_2'}{s\omega_s\left(\left(\frac{R_2'}{s}\right)^2 + X_2'^2\right)}$$

Optionally/alternatively: Slip s close to 0 for normal operation => R_2'/s dominant in denominator

$$T = \left(\frac{3V_1^2}{\omega_s R_2'}\right)s = \left(\frac{3V_1^2}{\omega_s^2 R_2'}\right)(\omega_s - \omega_r)$$

(c)

Control parameters:
Torque increase: voltage (over back-emf)
Speed increase: frequency (and voltage following the back-emf)

Design parameters:
Torque maximum can be increased by reducing the rotor resistance

(d)

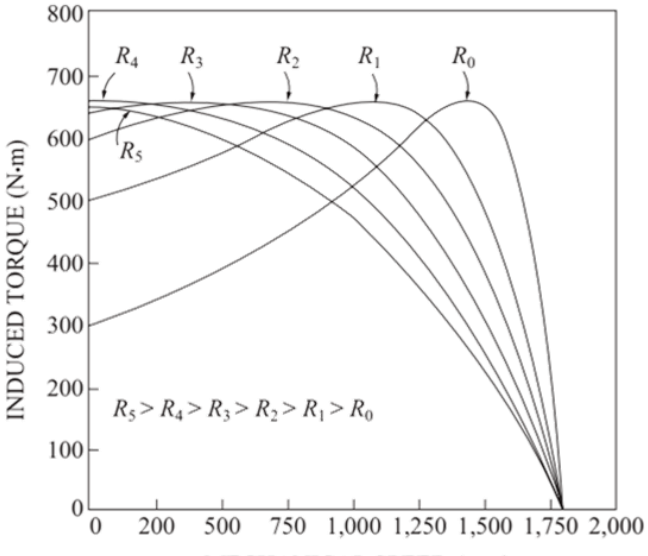
Copper resistance/aluminium resistance ~0.636

$$T_{\mathrm{max}} = \pm \frac{3V_1^2}{2\omega_s X_2'}$$

=> independent from the resistance for moderately low resistance

Corresponding slip:

$$s_{\mathrm{m}} = \pm \frac{R_2'}{X_2'}$$



(constant electrical frequency, decreasing slip)

=> slip for copper only ~64%
(or slip of aluminium rotor 57% higher)

However, max torque for higher resistance at lower speeds/ higher slip so that aluminium has on average higher starting torque.

(e)

(i) max speed at synchronous speed (practically no load/losses)

$n_{\text{max}} = f/p$ = 500 Hz/2 = 250 Hz = 15000 rpm

(ii) stand-still torque, starting torque,
stator resistance not necessarily negligible due to low frequency,
i.e., effective voltage on rotor side smaller
(about half of the stator voltage):

at low frequency: $R_1, R_2 \gg X_1, X_2$

$$V_2 \approx V_1 \cdot \frac{R_2 + jX_2}{R_1 + R_2 + jX_1 + jX_2} \approx V_1 \cdot \frac{R_2}{R_1 + R_2} = 25\ \mathrm{V}$$

$$T_{\mathrm{st}} = \frac{3V_2^2 R_2'}{\omega_s\left(R_2'^{\,2} + X_2'^{\,2}\right)} = 400\ \mathrm{Nm}$$

(iii)

$$T = \frac{3V_1^2 s}{\omega_s R_2'} = 25\ \mathrm{Nm}$$

$$\Rightarrow\ s = T \cdot \frac{\omega_s R_2'}{3V_1^2} \approx 0.02$$

$$\frac{100}{p}\,(1 - 0.02) = 49\ \mathrm{rps} = 2940\ \mathrm{rpm}$$

Note: small slip of 0.02 justifies use of approximate torque equation.

(iii) cont‘d

$$Z'_{\mathrm{rotor}} = \frac{R'_2}{s} + jX'_2 = 5.0\ \Omega + j0.01885\ \Omega$$

$$Z_{\mathrm{total}} = R_1 + jX_1 + \frac{jZ_{\mathrm{m}} \times jZ'_{\mathrm{rotor}}}{j(Z_{\mathrm{m}} + Z'_{\mathrm{rotor}})}$$

$$= (0.721 + j1.680)\ \Omega$$

$$|Z_{\mathrm{total}}| = \sqrt{0.721^2 + 1.688^2} = 1.83\ \Omega$$

$$I_1 = \frac{200}{\sqrt{3}|Z_{\mathrm{total}}|} = 63.1\ \mathrm{A}$$

2(a)

Concentrated windings

- do not overleap, typically even around a single tooth
- typically larger number of poles
  => slower speed for same electrical frequency

\+ low cost
\+ short end turns
\+ simple winding structure (and winding process)
\- larger harmonic content in rotating field
\- higher torque ripple (if not compensated with more intricate fractional slot winding schemes)
\- larger rotor losses (magnet loss, eddy currents)
\- (tends to lower winding factor)

Distributed windings

- loops (different strands and phases) overleap
- typically lower number of poles
  => faster speed for same electrical frequency

\- higher cost
\- longer end turns (loops need to be guided around each other)
\- complex winding structure (and winding/insertion process)
\+ lower harmonic content in rotating field
\+ lower torque ripple
\+ lower rotor losses (magnet loss, eddy current loss)
\+ (tends to higher winding factor, at least for low phase-band)

2(b)

Phase band: number of slots per pole per phase, essentially the spatial quantisation of the ideal spatially sinusoidal current distribution (and therefore (co-)sinusoidal air-gap flux distribution).

Higher phase band values allow a smoother, more sinusoidal air-gap flux, lower torque ripple, lower rotor loss (as harmonics appear as AC fields to rotor, but not the fundamental)

Problem of a high phase band: High space requirement (slots need to be cut out and may not leave enough space for teeth below saturation), winding factor plummets for higher phase-band values.

2(c)

Short-pitching: merge two poles into each other so that slots are not homogeneously filled with only one phase anymore. Thus, there will be slots that have some turns of one phase and some turns of another, visible as different layers if sequentially inserted.

Short-pitching allows smoothing the air-gap flux (more sinusoidal) without increasing the phase band (more slots) and also reduce the high-frequency effects in the stator winding in those slots (proximity effect and circulating eddy currents). However, short pitching often complicates manufacturing, requires careful guidance of loops around each other (clearly visible in bar windings), and reduces the winding factor.

2(d)

Magnetic loading: average radial flux density per pole

For high magnetic loading, the teeth near the maximum saturate. (Thus, magnetic saturation and maximum current in the winding limit it.)

High loading in the same machine typically indicates high stator current and therefore high torque. The speed has usually no influence.

(Also okay: In principle, the magnetic loading can also be increased on the d component of the current, which does not affect torque (if no reluctance effects are assumed) but shifts the back-emf so that the necessary voltage shifts for the same speed and feeding frequency.)

2(e)

Twice the air-gap length
- mutual inductance reduced (linearly up to a certain point, i.e., by 50%)
- effective excitation and therefore back-emf halfed in synchronous and dc machines
- reduction of air-gap flux, approx. 50% for same current
- harder excitation and higher loss in induction machines (as excited through stator)
- above a certain level, drastic increase of leakage flux
- smoother airgap flux (lower rotor loss)

Affected parameter in induction machine equivalent circuit:
- mutual inductance, 50%

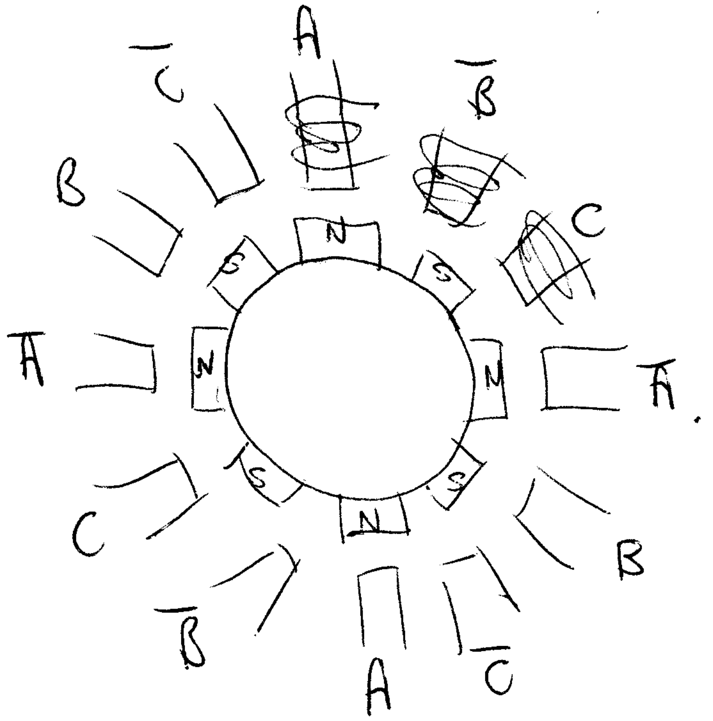
Affected parameters in synchronous machine equivalent circuit:
- mutual inductance, 50%
- in case of permanent magnets or if current-constant electrical excitation: back-emf
  for electrical excitation ~50%, for permanent magnets often approximated similarly but strictly speaking more complicated

2(f)

Faster rotational speed for same motor causes a larger back-emf. Furthermore, the same winding inductance (i.e., same winding with same space requirement) acts as larger reactance. As a consequence, the machine may require a higher voltage. If a sufficient insulation level can be provided, the electrical power increases and so does the mechanical power. For the same torque and same current, the power increases. The constant current ensures that the magnetic loading does not increase (which would require more iron cross section and therefore space). Increasing the (functional) insulation voltage level typically only increases the thin (level of 100 µm) insulation layers on the wires and is almost negligible to a current increase, which certainly requires larger slots for more copper and more iron for guiding the increased flux.
Thus, in short: more speed increases the power density almost proportionally and keeps the magnetic utilisation constant.

The limit is typically the rotor (in several dimensions):
- mechanical integrity (glued-on magnets coming off through centrifugal forces, thin lamination pegs break, etc.)
- increased rotor loss and thermal limit (insulating polymers or permanent magnets) in synchronous and dc machines
- unequal thermal expansion of overheating rotor (particularly in induction machines with their short air gaps and exploitation of massive short-term overload capability).

3/a)



The trapezoidal BLDCM has separate stator poles around which the phase windings are wound, such that the number of rotor and stator poles are different to avoid cogging.

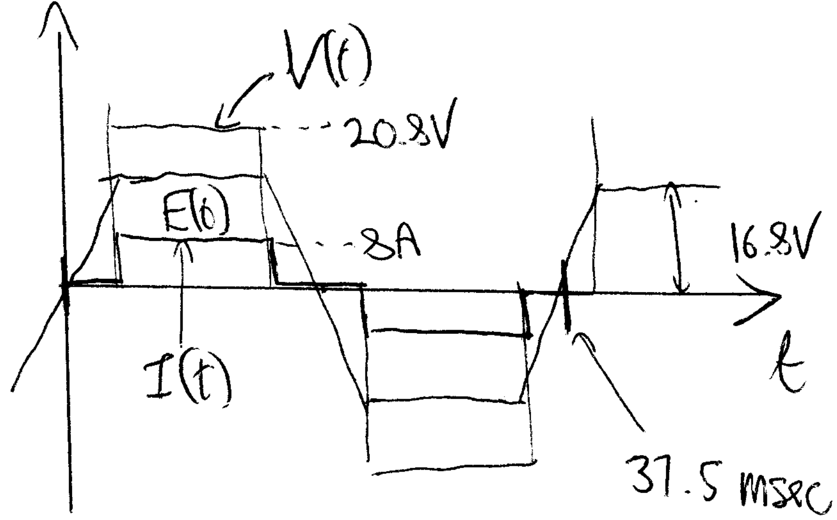
The sinusoidal BLDCM has a convention three-phase winding, distributed amongst many stator slots. [15%]

b) i) One electrical cycle ≡ 1/4 revolution of the rotor. This takes

$$T = \frac{1/4}{400/60} = 37.5\,\text{msec.}$$

Peak back-emf $E = k\omega_r = 0.4 \times 400 \times \frac{2\pi}{60} = 16.8V$

$$V = E + IR = 16.8 + 8 \times 0.5 = 20.8V$$



[15%]

ii) $T_r = 2kI_r = 2\times0.4\times8 = 6.4 \text{ Nm} = J\dfrac{\Delta\omega}{\Delta t}$

$\Delta\omega = 1000\times\dfrac{2\pi}{60}$, $J = \overset{0.1}{\cancel{0.005}}\ \text{kg m s}^{-2}$ so $\underline{\Delta t = 1.6\ \text{s}}$

$P_{loss} = 2I^2R = 2\times8^2\times0.5 = 64\ \text{W}$

so energy lost $= 64\times1.6 = 102.4\ \text{J}$

$\Delta K.E = \dfrac{1}{2}J\omega^2 = 548.3\ \text{J}$

so energy into motor $= 102.4 + 548.3 = 650.7\ \text{J}$

Inverter 95% efficient so energy from supply $= \dfrac{650.7}{0.95} = 685\ \text{J}$

$\eta = \dfrac{548.3}{685} = 80.0\%$

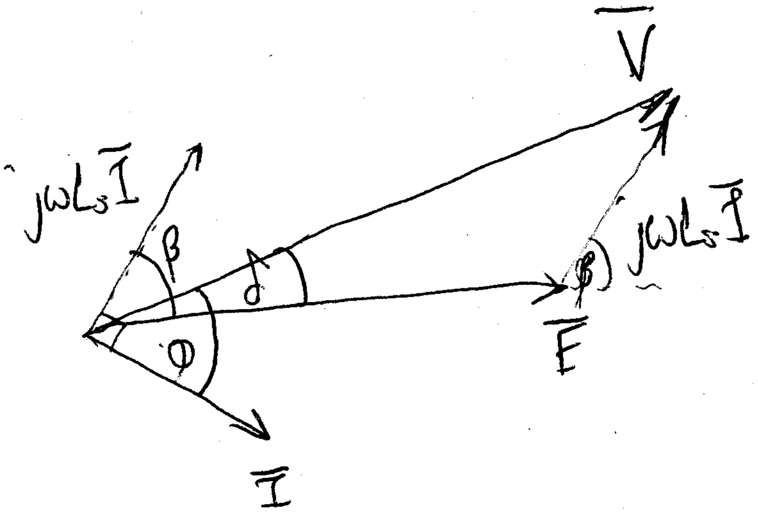
[15%]

iii) Hall effect sensors detect the rotor position, and the applied phase voltage occurs 30 electrical degrees after alignment of rotor pole with phase. This enables switching instants to be determined accurately.

Sensorless drives detect zero crossing of back emf in the isolated phase. This is hard to do at low speeds (back emf too small, prone to noise) and under stop-start conditions. Thus sensored drive is preferred for such applications. [15%]

c)

i)



Start from data book expression for P: $P = \frac{3VE\sin\delta}{\omega L_s} = T\omega_s = \frac{T\omega}{p}$

$E = k\omega_s = \frac{k\omega}{p} \Rightarrow T = \frac{3Vk\sin\delta}{\omega L_s}$

Sub. $V\sin\delta = \omega L_s I\sin\beta \Rightarrow T = 3kI\sin\beta$

β = torque angle

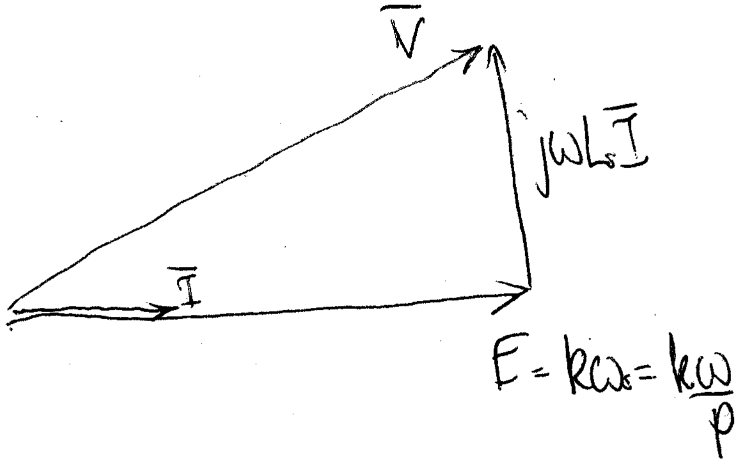
I = phase current

k = phase emf constant.

[10%]

ii) $T_r = 3kI_r = 3\times 2\times 35 = \underline{210\ \text{Nm}}$

Rated speed is maximum speed at which rated torque can be delivered, limited by maximum inverter voltage ie 400V



$V = V_{max} = 400V$ (Δ-connected)

$400^2 = (\omega L_s I_r)^2 + \left(\frac{k\omega}{p}\right)^2 = \omega^2\left((0.01\times35)^2 + \left(\frac{2}{3}\right)^2\right)$

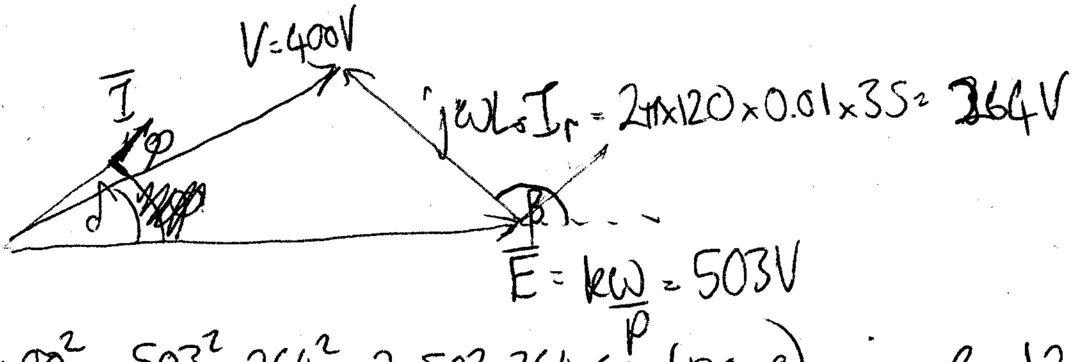
giving $\omega = 531\ rad\,s^{-1}$ (= 85 Hz so not limited by inverter maximum frequency of 120 Hz)

$\therefore \omega_r = \frac{\omega}{p} = \frac{531}{3} = 177\ rad\,s^{-1}$ ($= \underline{1690\ rpm}$)

Rated power = Rated torque × rated speed = $210 \times 177 = \underline{37.2\ kW}$ [10%]

iii) Max. speed limited by inverter = $\frac{60\times120}{p} = 2400\ rpm$

Now need field weakening



Cosine rule: $400^2 = 503^2 + 264^2 - 2\times503\times264\cos(180-\beta)$ giving $\underline{\beta = 128^\circ}$

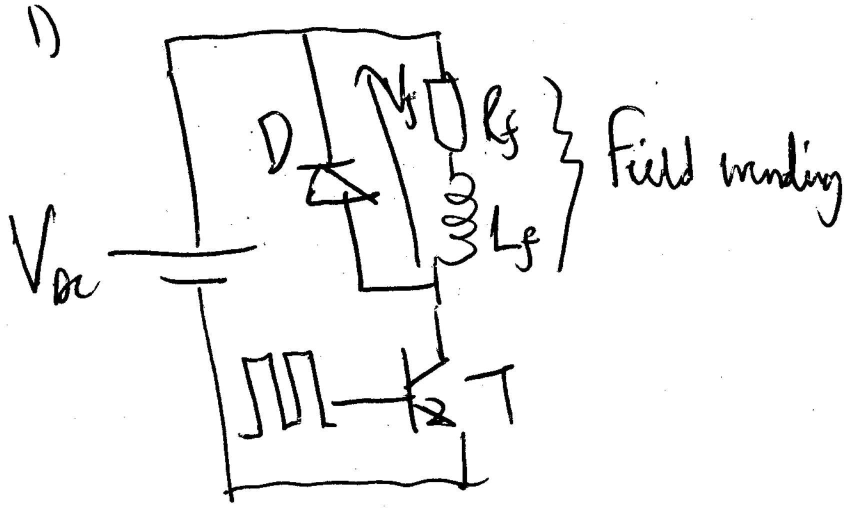
Sine rule: $\frac{\sin\delta}{264} = \frac{\sin(180-\beta)}{400}$ gives $\delta = \underline{31.4^\circ}$

$\phi + \delta + 90^\circ = \beta$ giving $\underline{\phi = 6.6^\circ}$ $T = 3kI_r\sin\beta = \underline{165\ Nm}$ [20%]

4/ a) Field-wound brushed DC motors are very easy to control, requiring only a variable voltage DC power supply for the field and the armature (point of interest: series/shunt motors only require one DC power supply, but are less controllable). They also offer a wide torque-speed range through the ability to field-weaken. Replacing with PM means no field winding losses ⇒ better efficiency.

[15%]

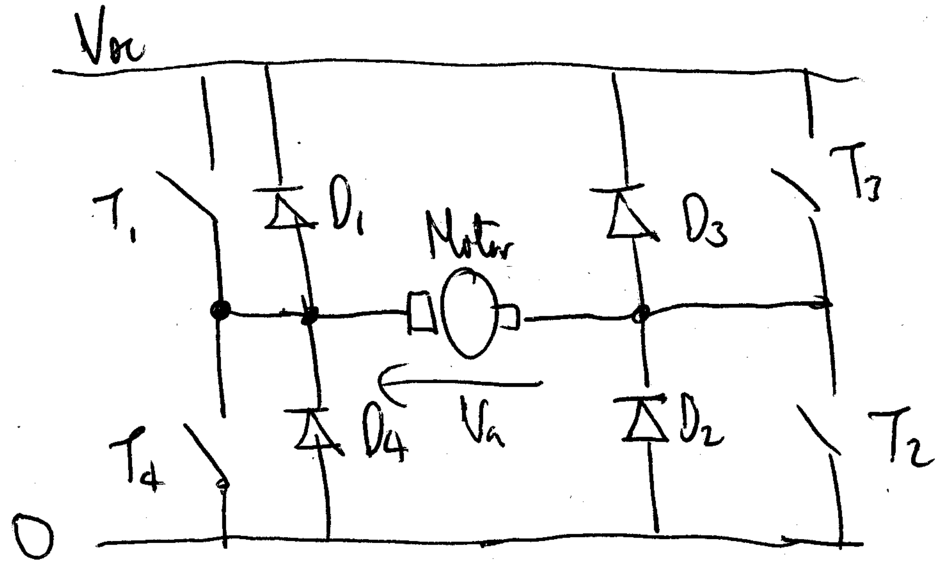
b) i)



With T ON, V_{DC} volts is across the field winding. T OFF, field winding current freewheels through D, so $V_f \approx 0$. Thus, by controlling $T_{ON} : T_{OFF}$ the mean DC voltage, and hence mean field current is controlled.

[10%]

ii)



Transistors $T_1 \ldots T_4$ shown as switches. For forward motoring need +ve armature voltage so T_1 and T_2 switched as a pair in duty ratio ρ to vary magnitude of armature voltage. T_3 and T_4 are OFF. When T_1 and T_2 are OFF, motor current free wheels through D_4 and D_3 so $V_a = -V_{oc}$. To reverse motor, V_a needs to be negative, so T_3 and T_4 switched as a pair, T_1 and T_2 OFF. Otherwise same operation.

[10%]

c) i) From open-circuit test at 1000 rpm at rated flux, find $(k\phi)_{rated}$:

$$E_a = V_{oc} = 25 = (k\phi)_{rated}\,\omega \quad \text{giving } (k\phi)_{rated} = 0.239 \text{ Vs rad}^{-1}$$

T_{rated} when $(k\phi) = (k\phi)_{rated}$ and $I_a = I_{a\,rated} = 20\text{A}$

$\therefore T_{rated} = (k\phi)_{rated} I_{a\,rated} = 0.239 \times 20 = \underline{4.77 \text{ Nm}}$

Rated speed is the maximum speed at which rated torque can be delivered and is limited by $V_{a\,max} = 50\text{V}$

$$V_{a\,max} = E_a + I_a R_a = (k\phi)_{rated}\,\omega_{rated} + 20 \times 0.5 \quad \text{giving } \omega_{rated} = 168 \text{ rad s}^{-1}$$

$(\approx 1600 \text{ rpm})$.

$I_f = 1\text{A}$ so $V_f = R_f I_f = 40 \times 1 = 40\text{V} = \rho \times 50$

$\therefore \underline{\rho = 0.8}$

[15%]

ii) For maximum speed $(k\Phi)$ should be minimum needed to give 10% rated torque ie $(k\Phi) = 0.1 \times 0.239 = 0.0239$. Also, $I_a = 20\,A$ and $V_a = 50V$

$$V_a = k\Phi\omega + I_a R_a \Rightarrow 50 = 0.0239\omega + 20 \times 0.5$$

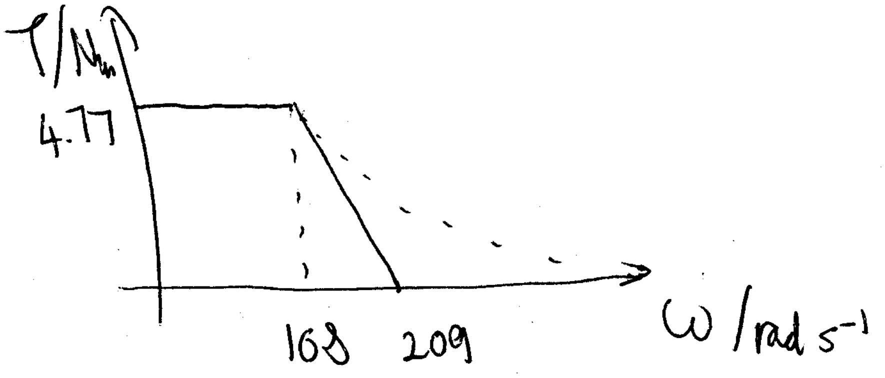
$\omega = 1674$ rads^{-1} ($\approx 16\,000$ rpm).

Since $I_f = 10\%$ of $I_{f\,rated}$, $\rho = 0.1 \times 0.8 = \underline{0.08}$ [10%]

d) Now $(k\Phi)$ fixed to $(k\Phi)_{rated} = 0.239$, $T_{rated} = 4.77$ Nm as before, $\omega_{rated} = 168$ rads^{-1} as before

However, maximum speed now found as $V_a = k\Phi\omega$ ie $I_a = 0$, so $T = 0$.

$\omega_{max} = 209$ rad s^{-1}.



The previous field-wound motor was capable of performing as above by fixing I_f to 1A. But, it can also deliver much greater speeds by field weakening. If these are needed then the replacement motor will not be suitable [20%]

Only possibility is to use a much larger armature voltage of ~ 500V to give the speed range.

e) The electromechanical time constant is a measure of how long it takes for a DC motor to accelerate to $V_a/k\phi$ rad s^{-1} when unloaded. Thus it is a measure of responsiveness to a step demand in speed.

Start from $V_a = E_a + I_a R_a$, $E_a = k\phi\omega = k'\omega$, $T = k\phi I_a = k' I_a$

and $T = J\frac{d\omega}{dt}$

$$V_a = k'\omega + I_a R_a \quad \text{so} \quad I_a = \frac{V_a - k'\omega}{R_a} \quad \text{and} \quad T = k' I_a = \frac{k' V_a}{R_a} - \frac{k'^2 \omega}{R_a}$$

$$= J\frac{d\omega}{dt}$$

$$\therefore V_a = \frac{R_a J}{k'}\frac{d\omega}{dt} + k'\omega$$

Taking Laplace transforms: $\omega(s) = \dfrac{V_a(s)}{k'\left(s\frac{R_a J}{k'^2} + 1\right)}$

so $\tau_{em} = \dfrac{R_a J}{k'^2}$

[20%]

Examiners' comments

Q1 Three-phase induction motor drives: 51 attempts, mean 62.7%

The first question was popular with students, likely because it widely followed the conventional calculation style. In the middle (1(d)), the question contained a sub-question requiring understanding and a bit transfer. In the calculations, a typical problem was that students forgot factors (pole pairs etc.) or converting units. Sub-question (d), however, challenged some students as it required some deeper understanding of the behaviour of induction machines and how the various design factors influence operation. The last part of (e) (iii) included a mistake in the question so that the motor could not reach an operating point with 300 Nm torque with the given terminal voltage and frequency. This mistake was spotted during the marking process and not during the exam itself. Hence this part of the question, worth only 20% of the total, was marked based on the candidates' approach and steps they took, and made no difference to the overall outcome. The question was updated in the post-exam versions of the exam paper and crib, to serve as a teaching revision aid for future students.

Q2 Motor design: 27 attempts, mean 64.7%

The second question was picked by fewer students (27). The focus of the second question was on knowledge in the beginning and more understanding as well as a bit of transfer closer to the end. This stronger focus on knowledge and understanding may contrast with previous years. Some of the students struggled and mixed up concepts or did not understand the reasons for certain design choices, while others solved the questions without issues.

Q3 Brushless DC motors: 62 attempts, mean 52.9%

A very popular question attempted by all but 2 candidates. Many excellent answers were received. Common errors were mixing up the number of stator poles with rotor poles when calculating the electrical supply frequency, and calculating the drive energy under the assumption of constant supply voltage, not constant torque (and therefore current) as required by the question. Very few candidates were able to complete c) (iii), with many not realising that it is the maximum inverter frequency that limits the maximum drive speed.

Q4 DC motor drives: 50 attempts, mean 60.6%

Many very good attempts at this question, the most common mistake was to misunderstand the difference between field and armature current, and use the wrong current in the calculations. Also in c) (ii) not realising that maximum speed is obtained by reducing the field current to the minimum value required to give 10% of rated torque.