3B4 Solutions 2014

1 (a) (i) Using distributed windings spreads the windings more evenly around the periphery of the machine, allowing flexibility in the choice of the number of slots,making better use of the stator volumes..

Both distributed windings and short pitching reduce the harmonics in the air-gap field, thereby reducing losses. A short pitch also reduces the length of wire needed. [10%]

(ii) In the expression for the distribution factor K_d, the following are used:

β - the angle between slots (360/n_s where n_s is the number of slots)

m – the number of slots over which a phase is distributed.

To evaluate K_p, the short pitch factor, the angle of short pitching α is needed. This will be related to the number of slots by which a coil is short pitched and so will be a multiple of β. The overall winding factor K_w is $K_p . K_d$. [20%]

(b) (i) The flux in the air gap is given by $\phi_g = \hat{B}(w_s + w_t)\ell$ where $\hat{B}$ is the flux density over the tooth-slot combination, assumed to be sensibly constant, w_t and w_s are tooth and slot mouth widths respectively and ℓ is the length of the stator stack. For the tooth, the flux is $\phi_t = \hat{B}_t w_t \ell$

By conservation of flux and using $\hat{B} = \left(1 + \frac{w_s}{w_t}\right)\sqrt{2} B_{rms}$. Putting in numbers gives

$B_{rms} = 0.636 \quad T$ [10%]

(ii) There are 24 slots, 3 phases and 4 poles (2 pole-pairs), so there will be two coils per phase band:

where A, B and C are phases.

The total effective turns needed are $N_{eff} = \frac{E_{nms} p}{B_{nms} \ell d \omega}$

Putting in numbers gives $N_{eff} = 159$

$Nactual = N_{eff} / K_w$

$$K_w = k_d = \frac{\sin\left(\frac{2.15.2}{2}\right)}{2\sin\left(\frac{2.15}{2}\right)}$$

As the slot angle $\beta = 15^o$ hence $K_d = 0.966$

$N_{actual} = 165$

There are 4 coils per phase so round up to 42 turns per coil (only an integer number of turns is possible).

[35%]

(c) (i) The usual form is a lamination stack with rotor conductors in aluminium alloy formed by casting. [This is the usual technique for small machines. A cast copper rotor is a possibility and gives a lower resistance but the casting procedure is more difficult. It is not normally economical to fabricate a rotor by manual insertion of bars for a machine of this size, but could be done.] [5%]

(ii) Usually the number of rotor bars is less than the number of stator slots. Equal numbers lead to cogging. 20 slots is a reasonable number. 24 would not be acceptable. [5%]

(iii) Deep bars (Boucherot slot) or a double cage gives good starting torque by exploiting the skin effect. [At starting, current is confined to the outermost portion of the bar, giving a high effective resistance and hence a high starting torque. Once running, the rotor frequency falls to a low value, the current is spread throughout the bars and the low effective resistance leads to efficient running.] [5%]

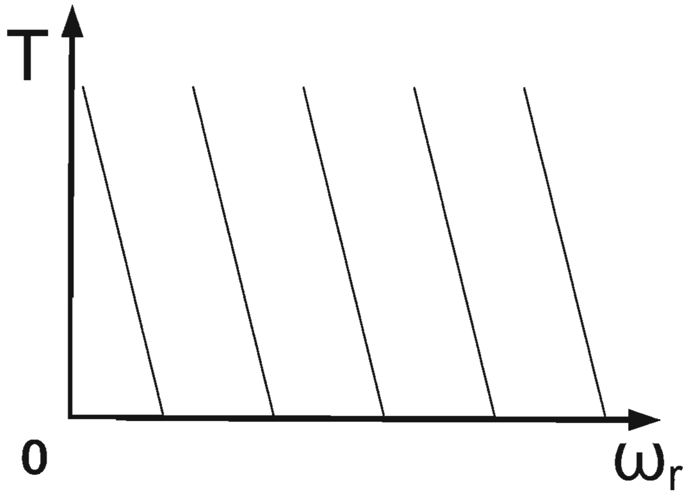
(iv) The rotor is better cooled because of rotation, although this cannot always be assumed in machines for variable speed drives. Also, as the bars are uninsulated, a higher temperature can be tolerated as there is not a concern about the degradation of conductor insulation. Against this, the resistivity of aluminium alloy conductors is higher than that of copper. For aluminium bars a current density of around 7 Amm^2 is generally typical. [10%]

2 (a) The desirable part of the torque-speed characteristic of an induction motor is that with the steep negative slope. Slip is zero at synchronous speed which is related to supply frequency. Thus an inverter which generates a variable voltage, variable frequency output enables the useful range of the torque speed curve to be translated to a wide range of actual speeds. To maintain the correct magnetization of the machine, the applied voltage needs to vary with frequency to give so-called constant volts per Hertz operation. [20%]

(b) (i) The first assumption is that the stator impedance, $R_1 + j\omega L_1$, is small compared to the parallel combination of magnetizing reactance and the referred rotor impedance for operation as described in (a). Therefore the supply voltage appears in full across the referred rotor impedance.

The second assumption is that the referred rotor reactance is small compared to the referred rotor resistance – again when operated as in (a). [10%]

(ii)



[15%]

Parallel and straight!

c (i) To find rated torque make the assumption that the stator drop is small.

$$I_1 = I_m + I_2 = \frac{400}{\sqrt{3}}\frac{1}{j108} + I_2$$

$$I_2 = \sqrt{g^2 - 2.14^2} = 8.74A$$

assuming that I_2 and I_m are in quadrature.

$$T = \frac{3V_1{}^2 s}{\omega_s R_2}$$

Using $V_1/\omega = \dfrac{V_1}{p\omega_s} = K = \dfrac{\frac{400}{\sqrt{3}}}{2\pi 50} = 0.735$

Where $\omega = 2\pi f$ (mains frequency) and p is the number of pole-pairs.

$$T = \frac{3K^2 ps\omega}{R_2}$$

$$I_2 = V_1/R_2/S = \frac{s\omega K}{R_2}$$

$$s\omega = \frac{8.74 \times 1.6}{0.735} = 19.02$$

Hence $T = \frac{0.735^2 \times 2 \times 19.02 \times 3}{1.6} = 38.5\ Nm$ [15%]

(ii) At half rated torque, $s\omega = 19.02/2 = 9.51$

At standstill s=1 so ω=9.51 rads/s. Supply frequency is then 1.51 Hz. [15%]

(iii) $V_1 = K\omega$

$= 0.735.9.51 = 6.99$ V

so $I_1 = 6.99 \big/ R_{2/s} - j\frac{400}{\sqrt{3}}\frac{1}{108}$

$= 4.37 - j2.14$

As the voltage across the rotor and magnetizing branches is small, the stator drop cannot be ignored – a voltage boost is needed. Current I_1, has magnitude 4.87 A. A simple approximation, which assumes this is in phase with the rotor branch current, adds 1.8x4.87=8.76 V, giving 15.7 phase or 27.3 V line. Phasor addition gives 15.35 V phase or 26.6 V line. [Note: only R_1 should be used.] [20%]

(iv) Energy is returned to the DC link. [Unless the drive is of the four quadrant type with a bi-directional converter, this energy cannot be returned to the mains if mains connected. Small amounts of energy can be absorbed by the losses in the drive. Larger amounts must be dissipated to avoid damage from an excessive rise in DC link voltage.] An IGBT controlled dynamic braking resistor is a common solution. [5%]

3 (a) (i) The root definition is to do with the shape of the emf waveform induced in each motor phase. However the motors may be used interchangeably. The drives are different in that sinusoidal drives use sinewaves and a fixed orientation between current and emf waveforms. [10%]

(ii) Trapezoidal uses a fixed current switched into any pair of windings.

Sinewave drives work by interaction of nice sinewaves of current and flux, and to obtain a useful torque and smooth performance they must have the same pole numbers. Trapezoidal just depends on waveforms (windings and magnets) with portions aligned, so there is no need for matched pole numbers and these are explicitly avoided where concentrated windings are used to avoid it locking up magnetically (cogging). [20%]

(iii) Sinusoidal → machine tool cutter or you get waves in the cut. Trapezoidal – lower weight with inertial loads → good for vehicle propulsion, door window winders etc.

(b) (i) Ideal trapezoidal means we have nice waveforms as in part (a) above i.e. magnitude of current in phase with magnitude of voltage (2 phases on). The inductance only affects the edges of the current waveform

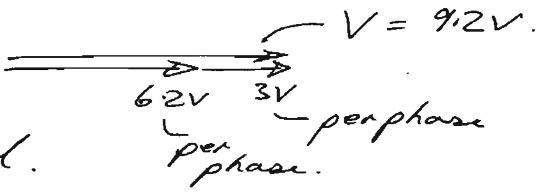
Find E_a.

$T\omega = 2EI_a$ (2 phases)

$0{\cdot}706 \times \frac{500\times2\pi}{60} = 2E_{a_{rated}} \times 3$

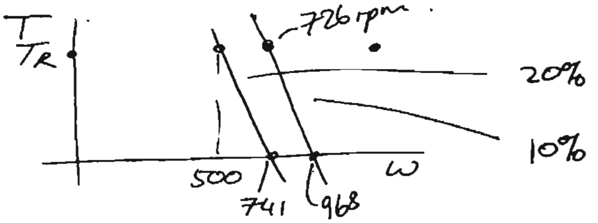
$E_{a_{rated}} = 6{\cdot}2V$ per phase.

$IR = 6V$ (2 phases)

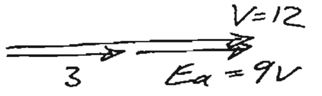


No load.

$\frac{9{\cdot}2}{6{\cdot}2} \times 500 = 741$ rpm $T \propto I$.



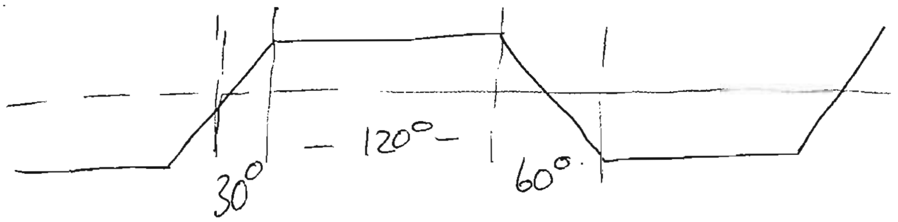
$E_a = 2E_{a_{rated}} = 12{\cdot}4$ V per phase. but only 24V drive!



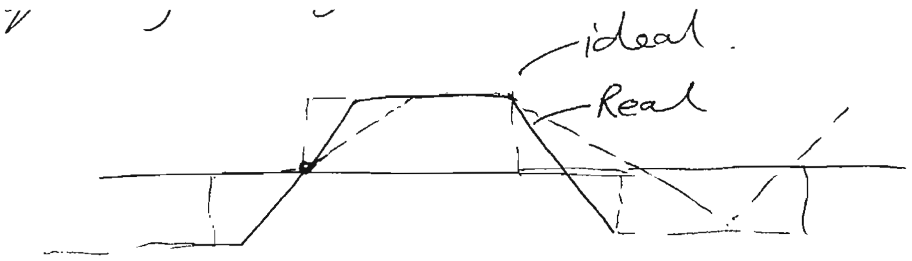
$\frac{9}{6{\cdot}2} \times 500 = 726$ rpm

No load; 968 rpm

3 (c) Back emf sensing uses a fixed 30° from the cross over to change the conduction pair of phases.

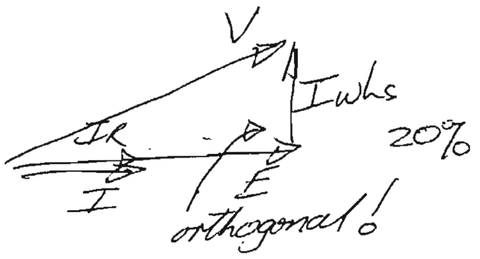


But at high speeds, say 24 V motor the current may not rise anywhere quickly enough, due to the unductance, and the shape in part (a) above is slowly lost.



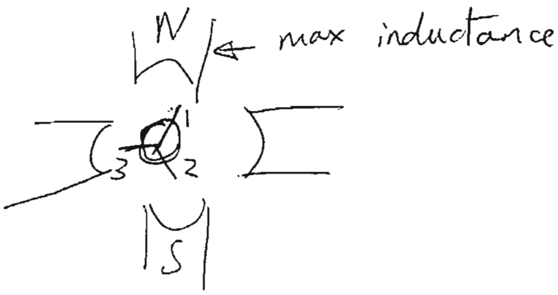
So the phase condition is lost AND current magnitude is lost.

The current becomes more sinusoidal anyway, but the phase is wrong. A good sinusoidal drive (probably Hall effect sensing) will get the correct phase.

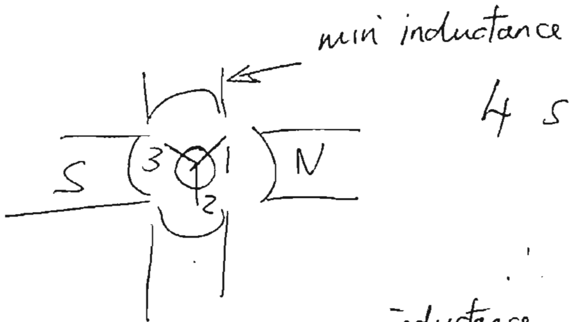


[20%]

4 a i/



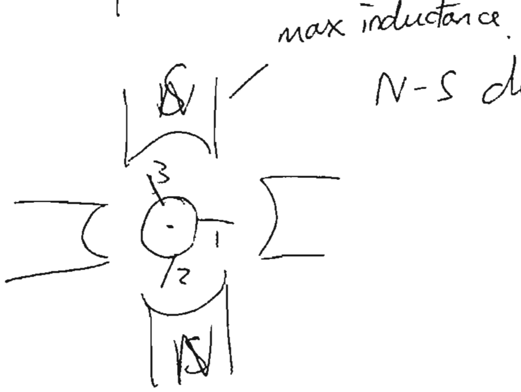
30°



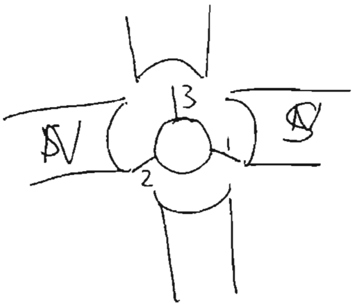
4 steps for 120°

∴ 30° step angle

60°

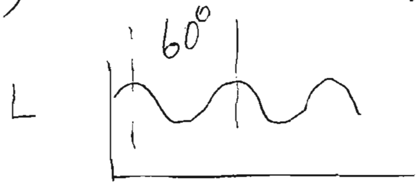


N-S direction either way!



4 20%

ii)



$L = L_0 + L_a \sin 6\omega t$

$T = \frac{1}{2} I^2 \frac{dL}{d\theta}$

4 a ii) cont.

$$\omega = \frac{100{,}000}{60}\times 2\pi \qquad \text{Power} = T\omega$$

$$T = \frac{160\times 60}{100{,}000\times 2\pi} = 15.3\ mNm$$

$$= \frac{15^2}{2}\frac{dLa}{d\theta}\sin 6\theta = L_a\frac{15^2}{2}.6 \approx 15.3\,m$$

$Lz = 22.7\ \mu H$ [20%]

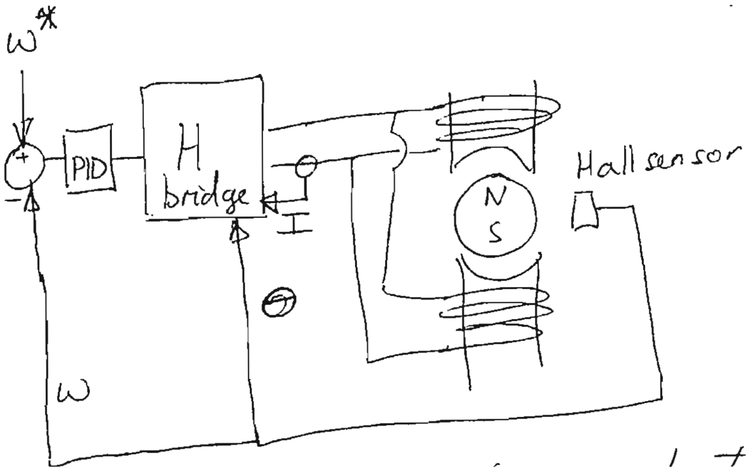
4 b i)

BLDC needs expensive magnets

BLDC is more efficient

BLDC is quieter

BLDC needs something special to keep rotor together. [20%]



i) Asummetric so it starts
ii) Hall sensor for timing of H bridge
iii) H bridge so reversible current
iv) Low inductance (high frequency needed) [30%]

4 (c) 1600 W is a lot!
$\cong 10\times$ more rotor volume / machine volume, but thata a large increase.

So make the volume work harder so 4 pole stator (better magnetic path), retaining simple construction. [10%]