3B4 crib 2021 **Tim Flack and Teng Long**

QUESTION 1

(a)

Advantages: improved efficiency (no field winding losses); high power/torque density; more reliable (no commutator/brush wear; no mechanical switching → less maintenance)

Disadvantages: increased complexity of control (need sensors); can't vary field strength so field weakening for higher speeds is not possible

(b)

Drive for all-electric vehicle needs high power/torque density, minimal torque ripple and be highly controllable (torque, speed); no brushes (vs field-wound counterpart).

Trapezoidal BLDCMs are suitable for many applications, such as white goods, computers, etc. Control strategies are less complicated (reduced hardware requirements → cheaper); sensorless control is possible, whereas rotor position sensing is needed for the sinusoidal BLDCM.

(c)(i)

2000 rpm = $2\pi/60*2000$ = 209.4 rad/s

$E_{ph} = E_{line}/2 = 24/2 = 12$ V $= k\omega_r \rightarrow k = 0.0573$ Vs/rad

Rated torque, $T_{rated} = 2kI_{rated} = 2*0.0573*4 = 0.458$ Nm

Max. line-line voltage = 48 V so max. phase voltage = 24 V

$24 = E + I_aR_a = E + 4*1.5 \rightarrow E_{max} = 18$ V at rated torque

$E = k\omega$ so $\omega_{max} = E_{max}/k = 18/0.0573 = 314.14$ rad/s $\rightarrow n_{rated} = 3000$ rpm

(c)(ii)

50% rated torque → $I = 0.5*4 = 2$ A

$V_{ph} = E + I_aR_a \rightarrow E = 24 – 2*1.5 = 21$ V $\rightarrow \omega = E/k = 21/0.0573 = 366.5$ rad/s $\rightarrow n = 3500$ rpm

10% max. speed: $V_{ph} = k\omega + I_aR_a = 0.0573*36.65 + 2*1.5 = 6.15$ V → duty cycle = 0.256

50% max. speed: $V_{ph} = k\omega + I_aR_a = 0.0573*183.25 + 2*1.5 =18.75$ V → duty cycle = 0.781

(d)(i)

$T_m = 2kI = 2*0.0573*3 = 0.344$ Nm

$T_f = k_f\omega_r^2$ and @ 1500 rpm = 0.15 Nm $\rightarrow k_f = 0.15/(1500*2\pi/60)\text{^}2 = 6.08 \times 10^{-6}$

At final speed, inertia has no effect since ω_r = const. $\rightarrow T_m = T_f$

$0.344 = 6.08 \times 10^{-6}*\omega_r^2 \rightarrow \omega_r = 237.86$ rad/s → n = 2271 rpm (d)(ii)

$$T_m - T_f = J\frac{d\omega_r}{dt}$$

$$T_m - k_f\omega_r^2 = J\frac{d\omega_r}{dt}$$

$$\int dt = J\int\frac{d\omega_r}{T_m - k_f\omega_r^2} = \frac{J}{k_f}\int\frac{d\omega_r}{a^2-\omega_r^2} \quad \text{where } a^2 = \frac{T_m}{k_f}$$

$$a = \sqrt{\frac{0.344}{6.08\times10^{-6}}} = 237.86 \text{ rad/s}$$

$$\frac{k_f t}{J} = \int\frac{d\omega_r}{a^2-\omega_r^2} = \frac{1}{2a}\int\frac{1}{a+\omega_r} + \frac{1}{a-\omega_r}\,d\omega_r$$

$$= \frac{1}{2a}\ln\left[\frac{a+\omega_r}{a-\omega_r}\right]$$

$$= \frac{-1}{2a}\ln\left[\frac{a-\omega_r}{a+\omega_r}\right]$$

$$\Rightarrow \frac{a-\omega_r}{a+\omega_r} = e^{-\frac{2ak_f}{J}t} = e^{-t/\tau}, \quad \tau = \frac{J}{2ak_f} = 6.915$$

$$a - \omega_r = (a+\omega_r)e^{-t/\tau}$$

$$\omega_r(1+e^{-t/\tau}) = a(1-e^{-t/\tau})$$

$$\omega_r = \frac{a(1-e^{-t/\tau})}{1+e^{-t/\tau}} = a\frac{\cancel{e^{-t/2\tau}}(e^{t/2\tau}-e^{-t/2\tau})}{\cancel{e^{-t/2\tau}}(e^{t/2\tau}+e^{-t/2\tau})} = a\tanh(t/2\tau)$$

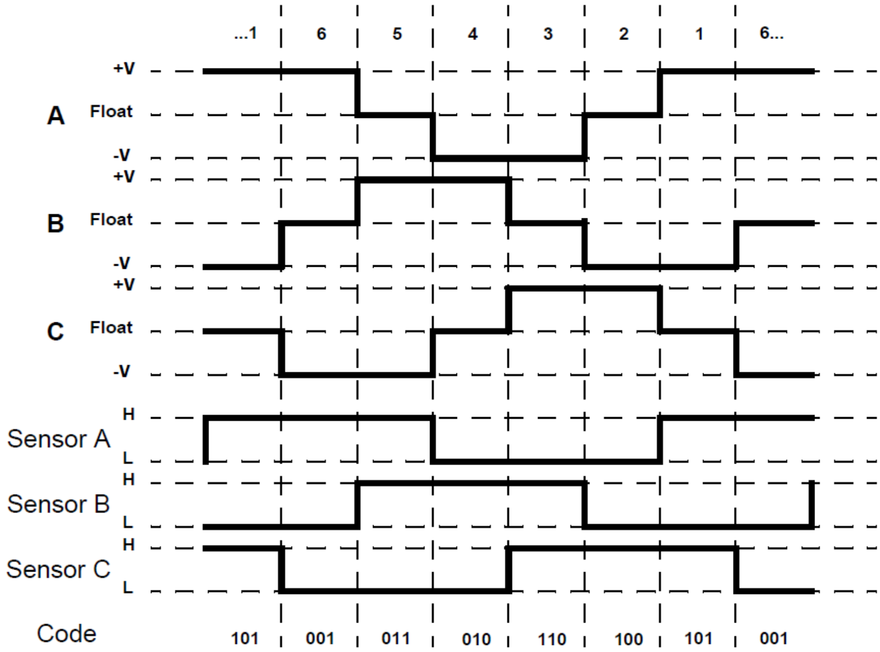
$$\Rightarrow \omega_r = 237.86\tanh(t/13.83)$$

(d)(iii)

0.95*237.86 = 237.86*tanh(t/13.83) → t = 13.83*$\tanh^{-1}$(0.95) = 25.3 s

(e)

Hall sensors provide speed/position feedback; sensor transitions are aligned with applied voltage transitions:



Sensored control is preferred when the drive is required to operate at low speeds and/or has many stop/starts, since back-emf zero-crossing detection is difficult in these cases.

QUESTION 2

(a)(i)

The restoring torque of the motor is:

$T_m = -\hat{T}\sin(N_t\theta)$ where Nt is the number of rotor teeth and $\hat{T}$ is the peak restoring torque.

For a purely inertial load, where the combined moment of inertia of the load and rotor is J, and ignoring damping, this torque can be equated with the inertial torque:

$$T_m = -\hat{T}\sin(N_t\theta) = J\frac{d^2\theta}{dt^2}$$

For small displacements about the equilibrium angular position of zero, sin(Ntθ) can be approximated as Ntθ giving:

$$-\frac{N_t\hat{T}}{J}\theta = \frac{d^2\theta}{dt^2}$$

This is the differential equation for simple harmonic motion and the solution is a pure sinusoid with the natural frequency given by:

$$\omega_0^2 = \frac{N_t \hat{T}}{J} \rightarrow f_0 = \frac{1}{2\pi}\sqrt{\frac{N_t \hat{T}}{J}}$$

(a)(ii)

Using equation in (a)(i), f_0 = 154 Hz

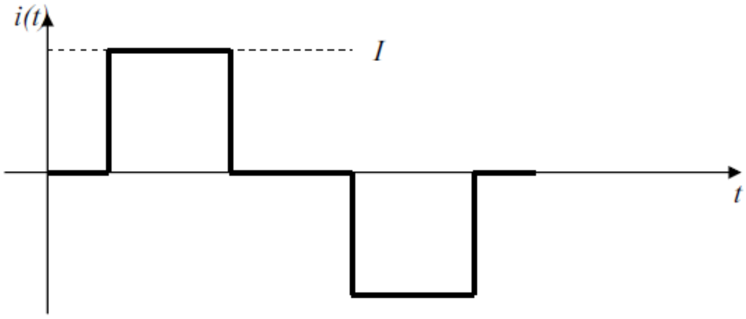
Each excitation pulse rotates the rotor 1/200 revolutions → 154/200 = 0.77 rps = 46.2 rpm

(a)(iii)

At this speed, the frequency of the stepping will excite the natural frequency of the motor/load, possibly resulting in increasingly large oscillations. This could then result in missed steps and loss of stability.

To avoid this problem: 1) accelerate through, minimising the time the motor operates at this speed, so that oscillations do not build up; 2) microstepping, instead of full-stepping, which results in a smoother torque at critical speeds ; 3) employ special couplings able to dissipate the energy, i.e., damping.

(a)(iv)

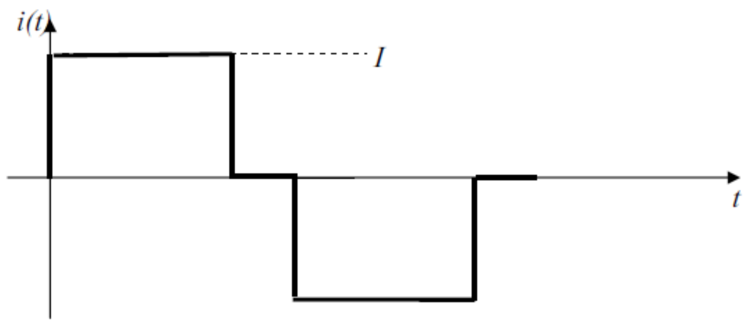


I = √2*I_{rated} = √2 A = 1.414 A

(a)(v)

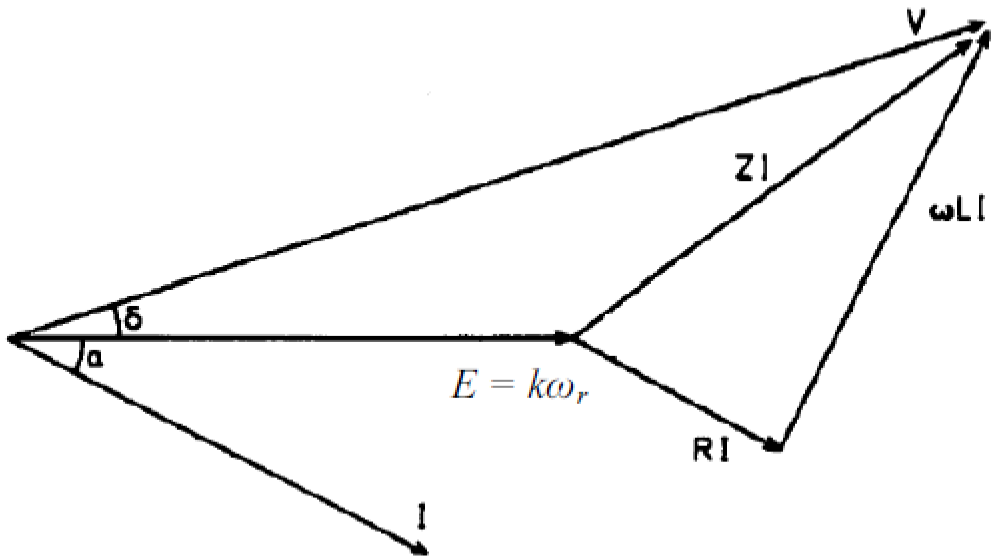
Excitation sequence:

$$A, AB, B, B\overline{A}, \overline{A}, \overline{A}\,\overline{B}, \overline{B}, \overline{B}A, A, AB....$$



I = √(4/3) *I_{rated} = 1.115 A

(b)(i)



δ = load angle, δ+α = power factor angle, E = induced emf, V = voltage; note: resistance cannot be ignored like sinusoidal BLDCM

(b)(ii)

One complete period of phase excitation = 4 rotor steps. 200 rotor steps per revolution, so 150 rpm = 2.5 rps → $f_{electrical}$ = 125 Hz.

2.5 rps = 5π rad/s, so E = $k\omega_r$ = 1.4*5π = 22 V

Z = R + jωL = 1.6 + j2π*125*4.8 x 10^{-3} = 1.6 + j3.77 = 4.1∠67°

Power factor, cos φ = cos (δ+α) = cos(10°+α) = 0.9 lagging → 10°+α = 25.84° → α = 15.84°

Assuming **E** at angle of zero, **V** = **E** + **I***Z** = 22 + 1∠-15.84° * 4.1∠67° = 26.1∠51.16°

Input power, P_{in} = 2VIcosφ = 2*26.1*1*0.9 = 47 W

Power loss, P_{loss} = $2I^2R$ = $2*1^2*1.6$ = 3.2 W

Output power, P_{out} = P_{in} – P_{loss} = 47 – 3.2 = 43.8 W

Torque = P_{out}/ω_s = 43.8/5π = 2.79 Nm

Cribs.

Q3:

(a) $m = \dfrac{48}{3\times2\times2} = 4$

If not short pitched:

P_1 N	P_1 S	P_2 N	P_2 S
AAAA $\bar{C}\bar{C}\bar{C}\bar{C}$ BBBB	$\bar{A}\bar{A}\bar{A}\bar{A}$ CCCC $\bar{B}\bar{B}\bar{B}\bar{B}$	AAAA $\bar{C}\bar{C}\bar{C}\bar{C}$ BBBB	$\bar{A}\bar{A}\bar{A}\bar{A}$ CCCC $\bar{B}\bar{B}\bar{B}\bar{B}$
AAAA $\bar{C}\bar{C}\bar{C}\bar{C}$ BBBB	$\bar{A}\bar{A}\bar{A}\bar{A}$ CCCC $\bar{B}\bar{B}\bar{B}\bar{B}$	AAAA $\bar{C}\bar{C}\bar{C}\bar{C}$ BBBB	$\bar{A}\bar{A}\bar{A}\bar{A}$ CCCC $\bar{B}\bar{B}\bar{B}\bar{B}$

If 2 slots short pitched:

P_1 N	P_1 S	P_2 N	P_2 S
AAAA $\bar{C}\bar{C}\bar{C}\bar{C}$ BBBB	$\bar{A}\bar{A}\bar{A}\bar{A}$ CCCC $\bar{B}\bar{B}\bar{B}\bar{B}$	AAAA $\bar{C}\bar{C}\bar{C}\bar{C}$ BBBB	$\bar{A}\bar{A}\bar{A}\bar{A}$ CCCC $\bar{B}\bar{B}\bar{B}\bar{B}$
AA $\bar{C}\bar{C}$ $\bar{C}\bar{C}$ BB BB $\bar{A}\bar{A}$	$\bar{A}\bar{A}$ CC CC $\bar{B}\bar{B}$ $\bar{B}\bar{B}$ AA	AA $\bar{C}\bar{C}$ $\bar{C}\bar{C}$ BB BB $\bar{A}\bar{A}$	$\bar{A}\bar{A}$ CC CC $\bar{B}\bar{B}$ $\bar{B}\bar{B}$ AA

$$B_{rms} = \frac{\pi}{2\sqrt{2}}\bar{B} = \frac{3.14}{2\times1.414}\times1.1 = 1.22\ \text{T}$$

$(\alpha = 15^\circ,\ \beta = 7.5^\circ)$

$$k_w = \frac{\sin(4\times2\times7.5/2)}{4\cdot\sin(2\cdot\frac{7.5}{2})}\cdot\cos\left(\frac{2\times15}{2}\right)$$

$$= \frac{0.5}{0.522}\times0.966 = 0.925$$

$$N_{ph} = \frac{E_{rms}}{k\,k_w B_{rms}} = \frac{E_{rms}\cdot p}{l\omega d\cdot k_w\cdot B_{rms}} = \frac{690/\sqrt{3}\times2}{0.5\times50\times2\pi\times0.3\times0.925\times1.22}$$

$$= 15.05\ \text{(turns)}$$

$$N_{coil} = \frac{N_{ph}}{m\cdot n\cdot p} = \frac{15.05}{4\times2\times2} = 0.94 \approx 1\ \text{(turn)}$$

(b)

$$\bar{J} = \left(\frac{2\times3}{d\pi} N_{ph} k_w I_{ph}\right)$$

$$I_{ph} = \frac{S}{3\cdot V_{ph}} = \frac{13.6\times10^3}{3\times 690/\sqrt{3}} = 11.38A$$

$$\bar{J} = \left(\frac{2\times3}{0.3\times3.14} \times 15.05\times0.925\times11.38\right) = 1009 A/m$$

The specific electric loading $\bar{J}$ is the axial average current per meter of circumference of the air gap of the machine.

The specific magnetic loading $\bar{B}$ is the radial average flux density over the cylindrical surface of the air gap of the machine.

(C)

$$\hat{B}_t = \left(1+\frac{W_s}{W_t}\right)\frac{\pi}{2}\bar{B}$$

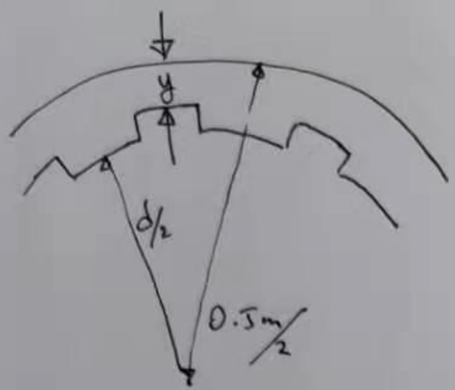
$$\frac{W_s}{W_t} = \frac{2}{\pi}\left(\frac{\hat{B}_t}{\bar{B}}\right) - 1$$

The maximum $\hat{B}_t$ should be 2.2T.

$$\frac{W_s}{W_t} = \frac{2}{\pi}\cdot 2 - 1 = 0.274.$$

It is not sensible to have $W_s = W_t$ otherwise the machine will have saturation at the tooth. To avoid saturation, the width of slot (w_s) should be 0.274 or less than the width of ~~the~~ tooth (w_t).

(d)



$$\hat{B}_c = \frac{1}{2y}\,\frac{\pi d}{2p}\,\bar{B}$$

$$y = \frac{0.5-0.3}{2} = 0.1\text{m}$$

$$\hat{B}_c = \frac{1}{2\times 0.1} \times \frac{3.14\times 0.3}{2\times 2} \times 1.1$$

$$= 1.3\,\text{T}$$

The $\hat{B}_c$ is considerably less than the saturation point. Therefore, the depth of the slot, i.e. the width of the yoke y can be reduced without saturating the stator. The $\hat{B}_c$ is independent to the $\hat{B}_t$ so the increase of $\hat{B}_c$ will not affect ~~s~~ the $\hat{B}_t$. The increase of slot area will give more space for conductors so the $\bar{J}$ can be increased so does the power rating.

Q4

(a) $P_{loss} = P_{out} \cdot \frac{1}{\eta} \cdot (1-\eta)$

$= 2\times10^3 \times \frac{1}{0.9} \times (1-0.9)$

$= 222\,W$

$\tau = \frac{C}{k} = \frac{1000}{2.5} = 400\,s$

$\theta = \frac{P_{loss}}{k}\left(1-e^{-\frac{t}{\tau}}\right) = \frac{222}{2.5}\left(1-e^{-\frac{100}{400}}\right) = 19.64°C$

The temperature rise is 19.64°C

(b) The ambient temperature is 40°C. The motor is cooled to 50°C. therefore, the motor is cooled to 10°C above the ambient. $\theta_0 = 10°C$

$\theta = \theta_0 + \left(\frac{P_{loss}}{k} - \theta_0\right)\left(1-e^{-\frac{t}{\tau}}\right)$

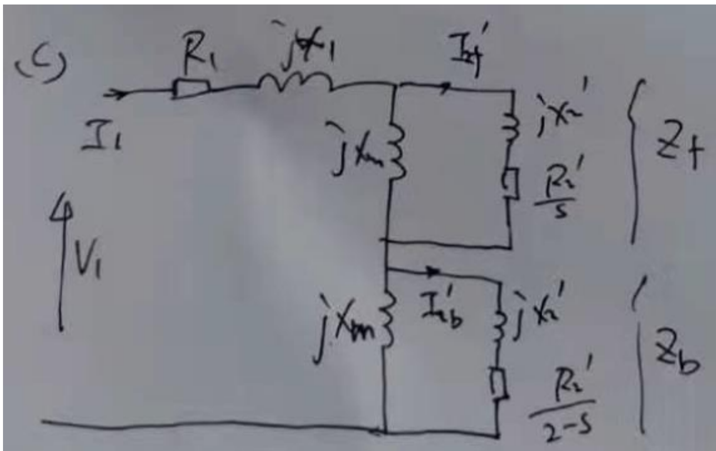
$= 10 + \left(\frac{222}{2.5} - 10\right)\left(1-e^{-\frac{100}{400}}\right)$

$= 27.3°C$

Therefore, the peak temperature of the motor is

$\theta_{pk} = 40 + 27.3 = 67.3°C$

(c)



V_1: single phase stator voltage

I_1: stator current

X_m: magnetising reactance

X_2': referred rotor reactance

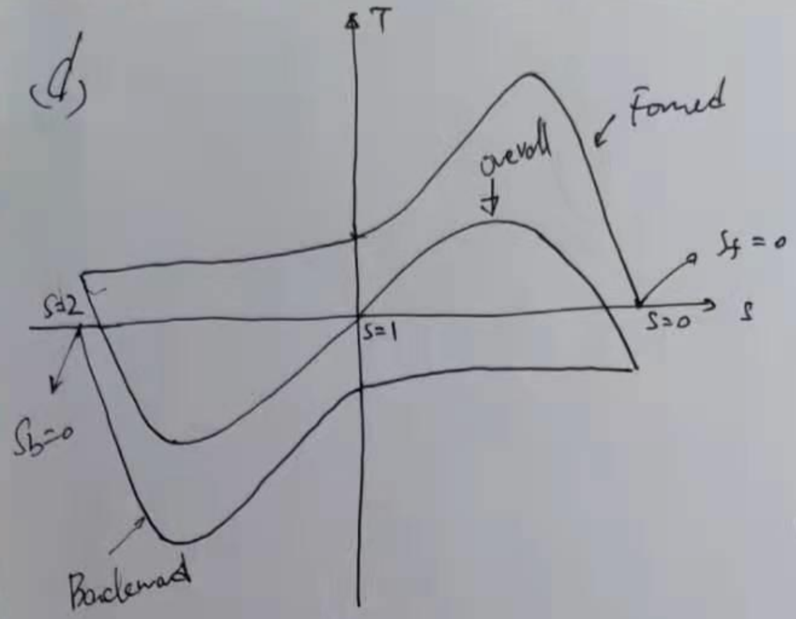
R_2': referred rotor resistance

s: slip

I_{2f}': referred rotor forward current

I_{2b}': referred rotor backward current.

(d)



Forward slip

$$S_f = \frac{w_s - w_r}{w_s} = S$$

Backward slip

$$S_b = \frac{-w_s - w_r}{-w_s}$$

$$= 2 - S$$

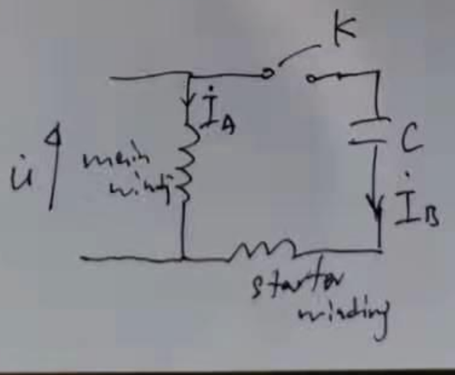
w_s: synchronous speed

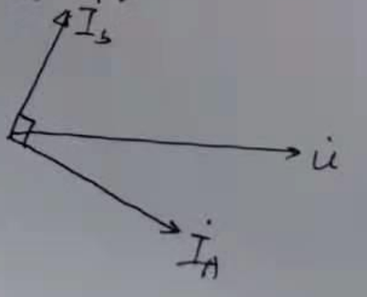
w_r: rotor speed.

S: overall slip

When starting the single phase induction machine, $w_r = 0$, S=1. The overall torque is zero. Therefore, the single phase machine is not able to start itself because the starting torque is zero.

(e) An additional winding, called starter winding is required. This starter winding needs to have a spatial 90° leading displacement with respect to the main winding and the current in this starter winding needs to be 90° leading phase angle with respect to the current at the main winding, in order to have the maximum starting torque.





Assessors' comments

Q1 Trapezoidal brushless DC motor: 37 attempts, mean 12.6/20

A very popular question, with lots of good attempts. The main issue in the early parts of the question was treating the measured open-circuit voltage as its phase rather than line value. This doubles the emf and torque constant, but the resulting errors were treated as consequential and were not penalised. Part (d) caused the greatest difficulty, and whilst most candidates were able to determine the final values of the drive torque and speed, no one successfully integrated the governing differential equation (although many were able to write it down, and get somewhere with solving it).

Q2 Stepper motor: 30 attempts, mean 14.2/20

Another popular question, attracting lots of very good answers. In part (a) the main problem was relating the rated current of the stepper motor to its peak current, and many candidates lost a few marks there. Part (b)(ii) caused the greatest difficulties, with very few candidates getting correct values for all quantities. The main cause was not relating speed to excitation frequency correctly.

Q3 Machine design: 16 attempts, mean 10.9/20

An unpopular question. The machine winding design has normally been unpopular in this module as it's relatively decoupling to the rest of the module. Candidates were able to sketch the coil arrangement and apply the winding factors. However, most of candidates struggled the number of turns per phase of Part (a). Candidates also struggled to fully apply the specific electric loading although most of candidate were able to write down the definition of the specific electric and magnetic loading. For Part (c), most f candidates understood the peak flux density occurs at the tooth. For Part (d), most of candidate have left this part empty, which could due to the time pressure.

Q4 Thermal modelling, duty cycles and single-phase induction motors: 34 attempts, mean 13.7/20

A popular question. Most of candidates have shown good understanding of the temperature of the motor when starting from the same as the ambient for Part (a). The loss calculation has some common errors on using the input power. For Part (b), most of candidates were able to call the right equation for the temperature when initial temperature is not same to the ambient. However, the absolute temperature of the motor needs to be added with the ambient temperature. Nearly all candidates have well answered the Part (c) of single phase induction machine equivalent circuit and terms and about two third candidates were able to sketch the torque speed characteristics correctly for Part (d). Most of candidates understood the capacitor needs to be used for increasing the torque of the single phase induction motor but only a few were able to sketch the equivalent circuit and phasor diagram of this Part (e).