1a. (i) To solve for k, consider the boundary conditions.

$$\text{At } x = 0, \quad \Psi(0) = 0$$
$$\text{At } x = L, \quad \Psi(L) = 0$$

for continuity, as $\Psi(x) = 0$ in the barriers.

$$\Psi(0) = A\sin(k.0) = 0$$
$$\Psi(a) = A\sin(kL) = 0$$
$$\therefore kL = n\pi$$
$$\therefore k = \frac{n\pi}{L}$$

To solve for A, recall that the probability of finding particle is 1. We need to normalise Ψ.

$$P = \int_0^L |\Psi|^2 \, dx = 1$$
$$\therefore \int_0^L A^2 \sin^2(kx)\, dx = 1$$
$$\therefore A^2 \int_0^L \left(\frac{1}{2} - \frac{1}{2}\cos(kx)\right) dx = 1$$
$$\therefore A^2 \left[\frac{1}{2}x - \frac{1}{2k}\sin(kx)\right]_0^L = 1$$
$$\therefore \frac{A^2 L}{2} = 1$$
$$\therefore A = \sqrt{\frac{2}{L}}$$

(ii) There are 2 ways.

* Either recall de Broglie

$$p = \frac{h}{\lambda} = \hbar k$$

and use $E = \frac{1}{2} m v^2$

$$= \frac{1}{2} \frac{p^2}{m} \qquad (\text{as } p = mv)$$

$$\therefore \quad E = \frac{1}{2} \frac{\hbar^2 k^2}{m}$$

$$= \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2 m L^2}$$

* Or use the Schrödinger equation.

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + 0 \Psi = E \Psi$$

Substitute in $\Psi = A \sin kx$

$$-\frac{\hbar^2}{2m} \left(-A k^2 \sin kx \right) = E \left(A \sin kx \right)$$

$$\therefore \quad +\frac{\hbar^2}{2m} k^2 = E$$

$$= \frac{\hbar^2 n^2 \pi^2}{2 m L^2}$$

(iii) $\Delta x = L$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{\hbar}{2\Delta x} = \frac{\hbar}{2L}$$

$$E = \frac{1}{2} m v^2$$

$$= \frac{1}{2m} p^2$$

$$\therefore \Delta E = \frac{1}{2m} (\Delta p)^2$$

$$\geq \frac{1}{2m} \frac{\hbar^2}{2^2 L^2}$$

$$= \frac{\hbar^2}{2m} \left(\frac{1}{4L^2}\right)$$

$$= \frac{\hbar^2}{2m} \left(\frac{1}{2L}\right)^2$$

Compare to answer in (ii)

$$E = \frac{\hbar^2}{2m} k^2$$

$$= \frac{\hbar^2}{2m} \left(\frac{n\pi}{L}\right)^2$$

The ground state energy, $E_{n=1}$, must be larger than the uncertainty, ΔE. This is true,

as $$\underbrace{\frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2}_{E_{n=1}} > \underbrace{\frac{\hbar^2}{2m}\left(\frac{1}{2L}\right)^2}_{\Delta E}$$

(b) (i) We have TISE:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V_0 \Psi = E\Psi$$

Solution Ψ is of form

$$\Psi = C\exp(\alpha x) + D\exp(-\alpha x)$$

For $-\infty < x < 0$, boundary conditions are: $\Psi(-\infty) = 0$

$$\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

$\Downarrow$

$D = 0$

For $L < x < \infty$, boundary conditions are: $\Psi(\infty) = 0$

$$\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

$\Downarrow$

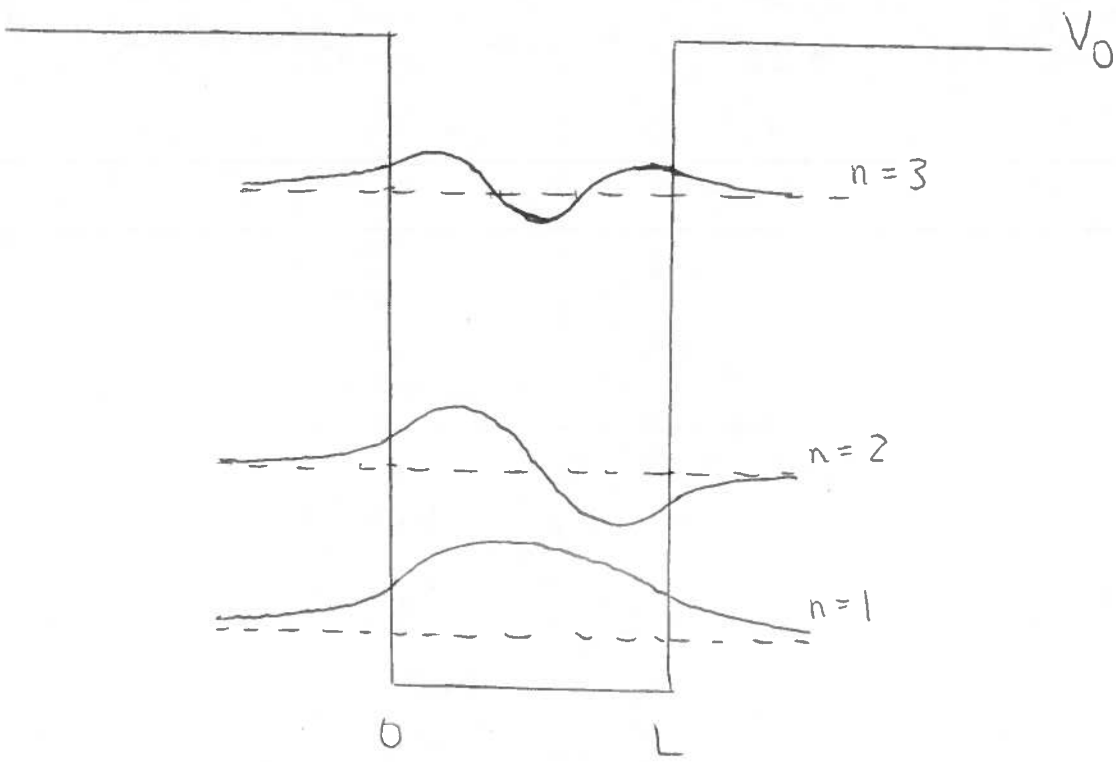
$C = 0$

For $-\infty < x < 0$: $\Psi(x) = C\exp\left(\frac{\sqrt{2m(V_0-E)}}{\hbar}x\right)$

For $L < x < \infty$: $\Psi(x) = D\exp\left(\frac{\sqrt{2m(V_0-E)}}{\hbar}(L-x)\right)$

* Wave function extends beyond barriers, even if $E < V_0$. This is known as tunnelling, a quantum mechanical effect.
* Finite, non-zero probability of finding electron in classically forbidden regions.

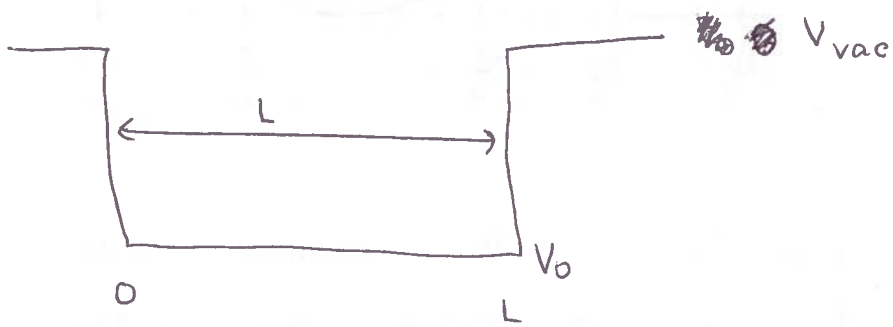
(ii)



(iii) As wells are brought closer together, the wave functions of the highest occupied energy levels spatially extend across the ~~two~~ wells. The Pauli exclusion principle is invoked, such that no 2 electrons can exist in the same state. The wave functions split into N ~~levels~~, levels where N is the number of electrons. These levels form a "band" of states.

2 (a) Assumptions:

- Electrons behave like a gas of free particles which are free to move in the solid
- Removal of a valence electron leaves a positively charged ionic nucleus. The charge due to these ionic cores is assumed to create a uniform potential throughout the solid.
- There is no interaction between free electrons.



Boundary conditions

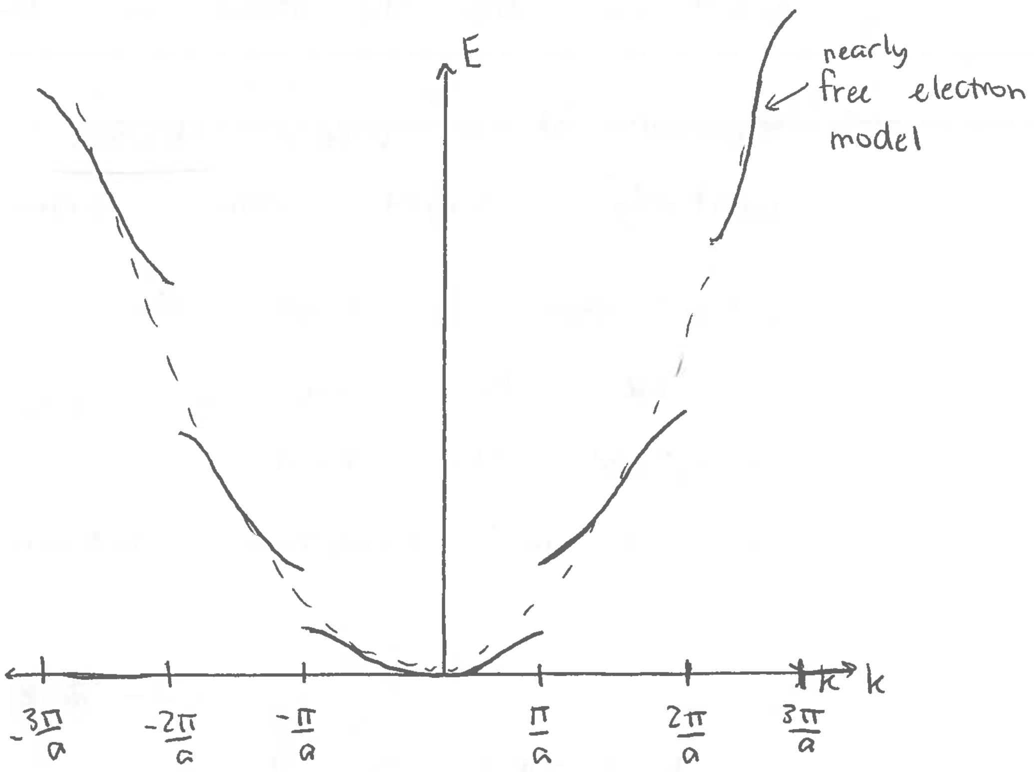
$$\Psi(x, y, z) = \Psi(x+L, y, z)$$
$$= \Psi(x, y+L, z)$$
$$= \Psi(x, y, z+L)$$

These boundary conditions give TRAVELLING WAVE solutions to TISE (and TDSE, of course).

Comment: Many students knew the assumptions but forgot the boundary conditions.

(b) At Brillouin zone boundaries, $k = \pm \frac{\pi}{a} n$

where n is an integer.



dashed line is free electron model
solid line is nearly free electron model.

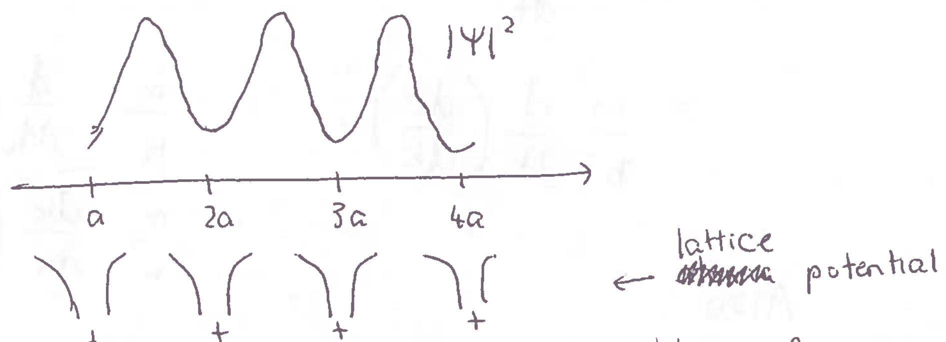
Gaps emerge when $k = \pm \frac{\pi}{a} n$, that is, when $\frac{n\lambda}{2}$ is close to the atomic spacing, a.

The spacing $a = \frac{n\lambda}{2}$ meets Bragg's criteria for diffraction. A wave travelling left to right will be Bragg reflected to travel from right to left, and vice versa. The wave then becomes a standing wave that has "locked on" to the periodicity of the lattice. The electron cannot propagate as a travelling wave.

There are 2 possibilities for the standing wave:

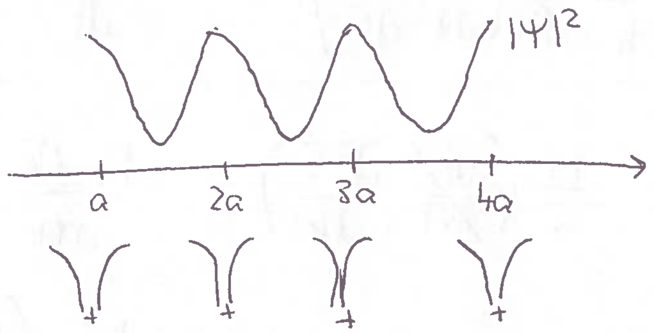
1. There is a low probability of the electron being near the nuclei.

   → This state will have a higher energy than predicted by free electron theory: $E = E_{free} + \Delta E$



2. There is a high probability of the electron being near the nuclei

   → This state will have a lower energy than predicted by free electron theory: $E = E_{free} - \Delta E$.



Comments: This question was answered well.

(c)

$$v = \frac{d\omega}{dk} \qquad (E = \hbar\omega)$$

$$= \frac{1}{\hbar}\frac{dE}{dk} \quad 1$$

$$F = ma \quad 1$$

$$= m\frac{dv}{dt}$$

$$= \frac{m}{\hbar}\frac{d}{dt}\left(\frac{dE}{dk}\right) = \frac{m}{\hbar}\frac{d}{dk}\left(\frac{dk}{dt}\frac{dE}{dk}\right)$$

$$= \frac{m}{\hbar}\frac{dk}{dt}\left(\frac{d^2E}{dk^2}\right)$$

Also

$$F = \frac{dp}{dt} \quad 1$$

$$= \frac{d}{dt}(\hbar k)$$

$$= \hbar\frac{dk}{dt}$$

$$\therefore \frac{m}{\hbar}\frac{d}{dk}\left(\frac{dk}{dt}\frac{dE}{dk}\right) = \hbar\frac{dk}{dt}$$

$$\therefore \frac{m}{\hbar}\left(\cancel{\frac{dk}{dt}}\frac{d^2E}{dk^2}\right) = \hbar\cancel{\frac{dk}{dt}}$$

$$\therefore m = \hbar^2\left(\frac{d^2E}{dk^2}\right)^{-1}$$

Comments: Some students did not equate the forces.

(d) (i)

$$p = N_V \exp\left(\frac{\ldots}{kT}\right)$$

$$= 1.8 \times 10^{19} \times \exp\left(\frac{-150}{25.8}\right)$$

$$= 5.37 \times 10^{16} \ \text{cm}^{-3}$$

Assuming all acceptors are ionised, $N_A = p = 5.4 \times 10^{16} \ \text{cm}^{-3}$ (2 significant figures)

$$n = \frac{n_i^2}{p}$$

$$= \frac{(1.0 \times 10^{16})^2}{5.4 \times 10^{16}}$$

$$= 1.86 \times 10^{15} \ \text{cm}^{-3}$$

$$= 1.9 \times 10^{15} \ \text{cm}^{-3}$$ (2 significant figures)

$\mu_n = 0.16 \ \text{m}^2/\text{Vs}$ (databook)

$\mu_p = 0.05 \ \text{m}^2/\text{Vs}$ (databook)

$$\sigma = ne\mu_n + pe\mu_p$$

$$= e\left[(1.86 \times 10^{15} \times 0.16) + (5.37 \times 10^{16} \times 0.05)\right] \times 10^4$$

$$= e\left[2.98 \times 10^{18} + 2.69 \times 10^{19}\right]$$

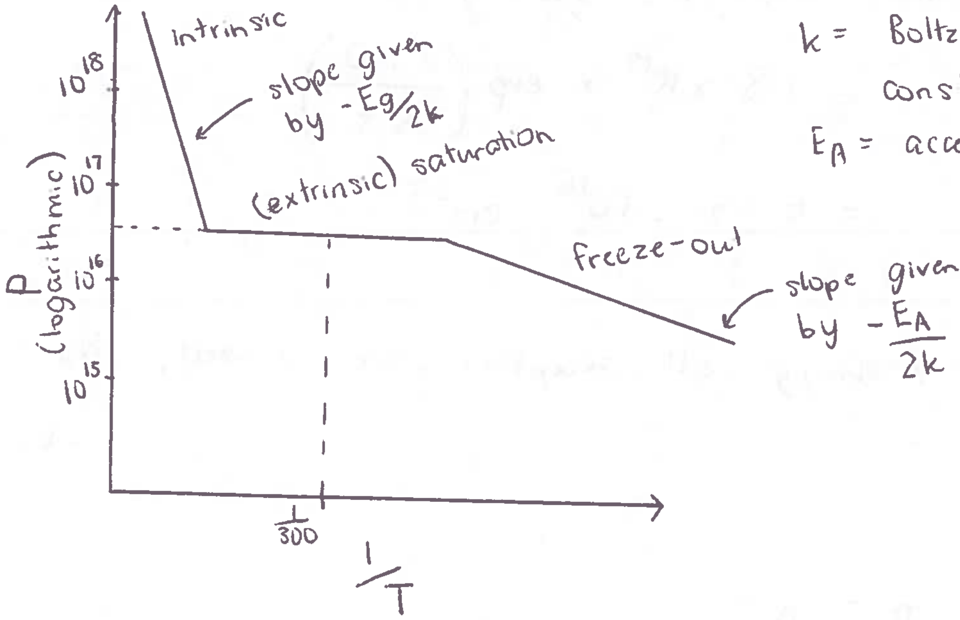
$$= 1.602 \times 10^{-19} \times 2.98 \times 10^{19}$$

$$= 4.78 \ \Omega^{-1} \text{cm}^{-1}$$

$= 500 \ \Omega^{-1} \text{m}^{-1}$ (appropriate number of significant figures)

Comments: Some students struggled to remember the units for conductivity. Most converted between cm^{-3} and m^{-3} with ease.

(iii)



k = Boltzmann's constant.

E_A = acceptor level.

Comments: Only 1 student stated the slopes $-E_g/2k$ and $-E_A/2k$.

3 a) - base transport factor $\beta = \frac{I_c}{I_{Ep}}$

where I_{Ep} is hole component of emitter current and I_c is hole current at collector.

β should be close to unity, so that a small base current can control a large I_c (difference between I_{Ep} and I_c is due to hole recombination).

- emitter injection efficiency $\gamma = \frac{I_{Ep}}{I_{Ep} + I_{En}}$

where I_{En} is electron component of emitter current, i.e. electron injected from base into emitter.

γ should also be close to unity, as I_{En} should be $\ll I_{Ep}$

2

b) (i) $D_h = \frac{kT}{e}\mu_h = 25\,\text{meV} \cdot 400 \frac{cm^2}{Vs} = 10 \frac{cm^2}{s}$, assume RT

$L_h = \sqrt{D_h \cdot \tau_h} = \sqrt{10 \frac{cm^2}{s} \cdot 10^{-6} s} = 3.2 \cdot 10^{-3}\,cm = 32\,\mu m$ 2

(ii) Excess hole concentration

$$\Delta p_n(0) = p_{no}\left[\exp\left(\frac{eV_{EB}}{kT}\right) - 1\right]$$, assume RT and low level injection

where p_{no} is equilibrium number density of holes in base when BJT is unbiased

$$n_i^2 = p_{no} N_D \rightarrow p_{no} = \frac{n_i^2}{N_D} = \frac{3.24 \cdot 10^{24}}{10^{22}}\ m^{-3}$$

$$= 324\ m^{-3}$$

3

$$\rightarrow \Delta p_n(0) = 324\ m^{-3}\left(e^{24}\right) = 8.6 \cdot 10^{12}\ m^{-3}$$

c) Assume no electric fields outside depletion regions

→ continuity equation for holes becomes

$$\frac{\Delta p_n}{\tau_h} = D_h \frac{\partial (\Delta p_n)}{\partial x^2}$$

which has general solution

$$\Delta p_n(x) = C \exp\left(\frac{-x}{L_n}\right) + D \exp\left(\frac{x}{L_n}\right)$$

boundary conditions

$$\Delta p_n(0) = C + D$$

$$\Delta p_n(W_b) = 0 = C \exp\left(\frac{-W_b}{L_n}\right) + D \exp\left(\frac{W_b}{L_n}\right)$$

hence

$$C = \frac{\Delta p_n(0) \exp\left(\frac{W_n}{L_n}\right)}{\exp\left(\frac{W_b}{L_n}\right) - \exp\left(\frac{-W_n}{L_n}\right)}$$

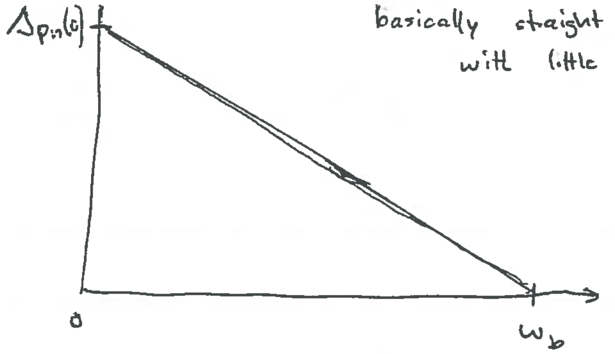
$$D = \frac{-\Delta p_n(0) \exp\left(\frac{-W_n}{L_n}\right)}{\exp\left(\frac{W_b}{L_n}\right) - \exp\left(\frac{-W_b}{L_n}\right)}$$

and

$$\Delta p_n(x) = \Delta p_n(0) \frac{\exp\left(\frac{W_b - x}{L_n}\right) - \exp\left(\frac{x - W_b}{L_n}\right)}{\exp\left(\frac{W_b}{L_n}\right) - \exp\left(\frac{-W_b}{L_n}\right)}$$

43 d) $L_n = 32\ \mu m$

$\rightarrow \dfrac{W_b}{L_n} = \dfrac{1}{16}$



plot basically straight line with little droop

(2)

e) Current is diffusion dominated, ie given by Fick's first law

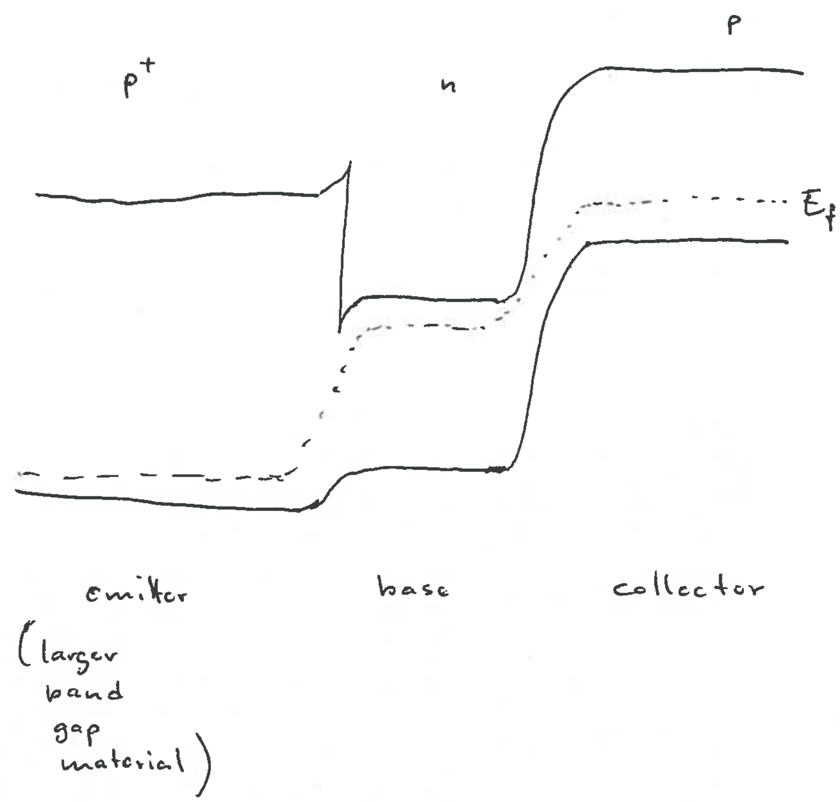
$$I = -eAD_n \frac{d(\Delta p_n)}{dx}$$

(assuming that hole current dominates at these terminals)

emitter and collector currents can be derived by evaluating above at $x = 0$ and $x = W_b$, respectively.

From current conservation, then $I_B = I_E - I_C$ (2)

f)



(2)

4 a (i) Poisson equation:

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\varepsilon\varepsilon_0}$$

here $\frac{d\mathcal{E}}{dx} > 0$ in the semiconductor

$$\left(\frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon\varepsilon_0} \right)$$

hence depleted charge is positive, which means semiconductor is doped n-type.

2

follow up issues when wrongly assuming p (no point penalities)

(ii) $$-\mathcal{E} = \frac{dV}{dx}$$

$$V_{bi} = -\frac{1}{2}\,\mathcal{E}(0)\,w = +\frac{1}{2} \times 3.8 \cdot 10^4\ \mathrm{V/cm} \times 0.3 \times 10^{-4}\ \mathrm{cm}$$

$$= 0.57\ \mathrm{V}$$

some calculate via $\mathcal{E}_r$, not noting how $\mathcal{E}_{max}$ and V_0 relate

Work function of semiconductor (ϕ_{sc}):

$$n = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$\rightarrow\ E_F - E_c = kT \ln\left(\frac{n}{N_c}\right) \underset{\substack{\text{assume} \\ n = N_D}}{=} kT \ln\left(\frac{N_D}{N_c}\right)$$

$$= kT \ln\left(\frac{10^{22}}{3 \cdot 10^{25}}\right) = -0.2\ \mathrm{eV}$$

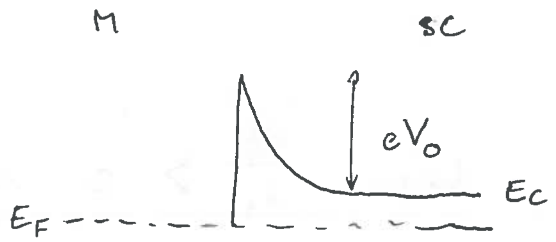
$$e\phi_{sc} = e\chi + 0.2\ \mathrm{eV} = 4.2\ \mathrm{eV}$$

Work function of metal:

$$eV_0 = e(\phi_M - \phi_{sc}) \Rightarrow e\phi_M = 4.77\ \mathrm{eV}$$

5

+ 3 a (iii)



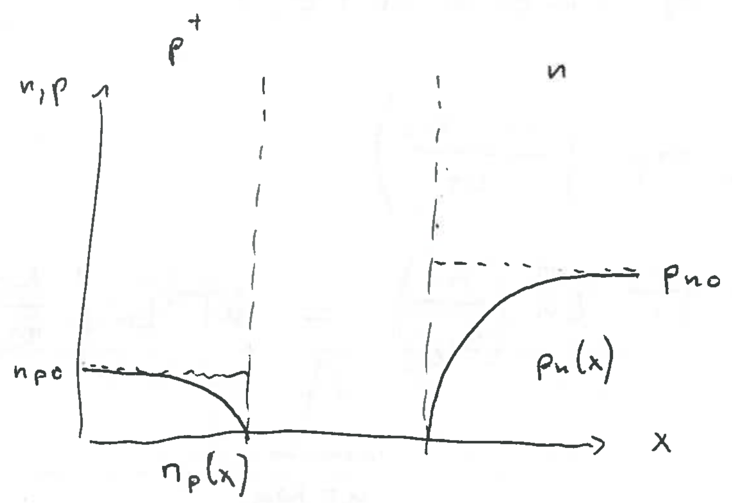
2

b) Ideality factor reflects how closely device follow behaviour derived just considering thermoionic emission current.

The ideality factor will increase for increased doping (i) [2] as well as upon cooling (ii) [1]. In both cases this is due to the relative increase in tunneling current, which is not considered in thermoionic emission theory.

3

c) i)



minority carrier extraction at reverse bias

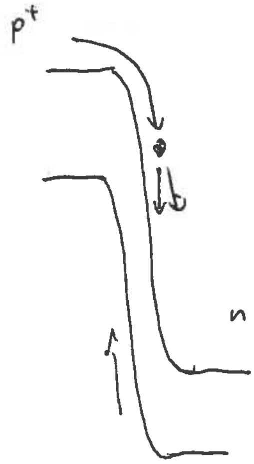
2

ii) For a p^+n diode, I_s is dominated by minority carrier extraction (bipolar device)

For a SBD, I_s is dominated by barrier for majority carrier flow from metal into semiconductor (unipolar device)

3

4 c) iii) Avalanche breakdown: kinetic energy of electrons or holes crossing the junction is sufficient to generate electron hole pairs when the carriers undergo scattering event.



V_{br} increases with band gap of the material, since more energy is required for an ionising collision.

V_{br} (GaAs) > V_{br} (Ge)

3