Question 1

(a)(i)

$$\sigma = ne\mu_e + pe\mu_h$$
$$= n_i e(\mu_e + \mu_h)$$
$$= 1.3 \times 10^{13} \times 1.6 \times 10^{-19} \times (0.49 + 0.07)$$
$$= 1.17 \times 10^{-6} \ \Omega^{-1} m^{-1}$$

$$\rho = \frac{1}{\sigma}$$
$$= \frac{1}{1.17 \times 10^{-6}}$$
$$= 8.57 \times 10^{5} \ \Omega m$$

(ii)

$$n = N_C \exp\left(-\frac{E_C - E_F}{kT}\right)$$

$$\therefore \ E_C - E_{F_i} = -kT \ln\left(\frac{n_i}{N_C}\right)$$
$$= \frac{-1.381 \times 10^{-23} \times 300}{1.602 \times 10^{-19}} \ln\left(\frac{1.3 \times 10^{13}}{5.7 \times 10^{23}}\right)$$
$$= 0.634 \ eV$$

$$p = N_v \exp\left(\frac{E_V - E_F}{kT}\right)$$

$$\therefore \ E_{F_i} - E_V = -kT \ln\left(\frac{p}{N_V}\right)$$
$$= \frac{-1.381 \times 10^{-23} \times 300}{1.602 \times 10^{-19}} \ln\left(\frac{1.3 \times 10^{13}}{1.1 \times 10^{25}}\right)$$
$$= 0.710 \ eV$$

→ Position of intrinsic Fermi level above valence band edge.

$$E_g = (E_C - E_{F_i}) + (E_{F_i} - E_V)$$
$$= 0.634 + 0.710$$
$$= 1.344 \ eV$$

The position of the intrinsic Fermi level is slightly above mid-gap because N_V is bigger than N_C.

(iii)

$$\mu = \frac{q\tau}{m^*}$$

$$\therefore \ \tau = \frac{\mu m^*}{q} = \frac{0.49 \times 0.08 \times 9.109 \times 10^{-31}}{1.602 \times 10^{-19}}$$
$$= 2.23 \times 10^{-13} \ s$$
$$= 223 \ fs$$

(iv) $d = v\tau$

$= 4.1 \times 10^{5} \times 2.23 \times 10^{-13}$

$= 9.21 \times 10^{-8}$

$= 92$ nm

(v) This distance is significantly larger than the lattice spacing. Clearly, the electrons are not being scattered every time they pass an atom.

This is consistent with the nearly free electron model. The electron is travelling as a wave and interacting with the lattice as a wave. The electron's wavefunction is given by

$$\Psi(x) = A \exp(jkx)\, u(x) \qquad \text{(one-dimensional form)}$$

where the first term is a plane wave and it is multiplied by $u(x)$ which is a function with the same periodicity as the lattice itself. This is Bloch's theorem.

(b) (i)

$$n_i = N_C \exp\left(-\frac{E_C - E_{Fi}}{kT}\right) = N_V \exp\left(\frac{E_V - E_{Fi}}{kT}\right)$$

$$\therefore \quad kT \ln\left(\frac{N_V}{N_C}\right) = -(E_C - E_{Fi}) - (E_V - E_{Fi})$$

$$\therefore \quad kT \ln\left(\frac{N_V}{N_C}\right) = -(E_C + E_V) + 2E_{Fi}$$

$$\therefore \quad E_{Fi} = \frac{E_C + E_V}{2} + kT \ln\left(\frac{N_V}{N_C}\right)$$

$$n = N_C \exp\left(-\frac{E_C - E_F}{kT}\right) \qquad p = N_V \exp\left(\frac{E_V - E_F}{kT}\right)$$

$$\therefore \frac{n}{p} = \frac{N_C}{N_V} \exp\left(-\frac{E_C - E_F}{kT} - \frac{E_V - E_F}{kT}\right)$$

$$= \frac{N_C}{N_V} \exp\left(\frac{2E_F - (E_C + E_V)}{kT}\right)$$

$$\therefore \quad kT \ln\left(\frac{n}{p}\frac{N_v}{N_c}\right) = 2E_F - (E_c + E_v)$$

$$\therefore \quad E_F = \frac{kT}{2}\ln\left(\frac{n}{p}\frac{N_v}{N_c}\right) + \frac{E_c + E_v}{2}$$

$$= \frac{E_c + E_v}{2} + \frac{kT}{2}\ln\left(\frac{N_v}{N_c}\right) + \frac{kT}{2}\ln\left(\frac{n}{p}\right)$$

$$= E_{F_i} + \frac{kT}{2}\ln\left(\frac{n}{p}\right)$$

$$\therefore \quad x = \frac{kT}{2}$$

(ii)

$$E_F = E_{F_i} + \frac{kT}{2}\ln\left(\frac{n}{p}\right)$$

$$= 0.710 + \frac{1.381 \times 10^{-23} \times 300}{2 \times 1.602 \times 10^{-19}} \ln\left(\frac{1}{10^{20}}\right)$$

$= 0.115$ eV above the valence band edge.

Could alternatively be answered as follows:

$$\frac{p}{n} = 10^{20} \qquad \therefore \; n = \frac{p}{10^{20}}$$

$$n_i^2 = np$$

$$= \frac{p}{10^{20}} \cdot p$$

$$= \frac{p^2}{10^{20}}$$

$$\therefore \quad p = 10^{10} n_i$$

$$= 1.3 \times 10^{23} \ m^{-1}$$

$$E_F = E_v - kT \ln\left(\frac{p}{N_v}\right)$$

$$= E_v - \frac{1.381 \times 10^{-23} \times 300}{1.602 \times 10^{-19}} \ln\left(\frac{1.3 \times 10^{23}}{1.1 \times 10^{25}}\right)$$

$= E_v + 0.115$ ie. 0.115 eV above V.B. edge.

(iii) Si atoms substitute onto P lattice sites. Si has one fewer valence electron than P. The InP lattice forces it to bond with the 4 neighbouring In atoms, but it is missing a bonding electron. The missing electron acts like a hole.

The most popular question. Calculations of resistivity, band gap, Fermi level, scattering time and distance travelled between scattering events were performed well by most candidates. Some students calculated only the conductivity but not the resistivity in (a)(i). Many students neglected to include appropriate units of measure. A number of students correctly observed that in (a)(ii) the Fermi level lies slightly above mid-gap. (a)(v) was only answered correctly by a handful of students, who explicitly mentioned the nearly free electron model. In their answers to (a)(v), some students incorrectly mentioned tunnelling, others mentioned thermionic emission and others mentioned the lack of ionised impurities as scattering sites. Only a handful of students correctly identified that the Si atoms substitute on phosphorus lattice sites in (b)(iii).

Q2 a) i) For n-side (assume full dopant ionisation)

$$E_F - E_i = kT \ln\left(\frac{N_D}{n_i}\right) = 0.47 \text{ eV}$$

, Law of mass action

$N_D \cdot p_{no} = n_i^2 \rightarrow p_{no} = \frac{n_i^2}{N_D} = 2.2 \cdot 10^8 m^{-3}$

for p-region:

$$E_i - E_F = kT \ln\left(\frac{N_A}{n_i}\right) = 0.28 \text{ eV}, \quad n_{po} = \frac{n_i^2}{N_A} = 2.2 \cdot 10^{11} m^{-3}$$

→ built-in potential

$$eV_0 = 0.75 \text{ eV}$$

ii) $V_{bias} = 0.5V$, electrical injection of minority carriers

$$n(0) = n_{po} \exp\left(\frac{eV}{kT}\right)$$

excess $\quad n - n_{po} = n_{po}\left(\exp\left(\frac{eV}{kT}\right) - 1\right) = 5.5\ 10^{19}\ m^{-3}$

iii) steady state and assume no drift for $x \geq 0$

→ continuity equation reduces to

$$D_e \frac{\partial^2 \Delta n}{\partial x^2} = \frac{\Delta n}{\tau_e}$$

general solution is

$$\Delta n = n - n_{po} = A \exp\left(\frac{-x}{L_e}\right) + B \exp\left(\frac{x}{L_e}\right)$$

Δn can only decay → $B = 0$

$$n - n_{po} = A \exp\left(\frac{-x}{L_e}\right) \quad \text{where} \quad L_e = \sqrt{D_e \tau_e}$$

A found with injection at $x = 0$ (see ii))

$$\rightarrow n - n_{po} = n_{po}\left(\exp\left(\frac{eV}{kT}\right) - 1\right) \exp\left(\frac{-x}{L_e}\right)$$

a) iv) Charge storage capacitance arises from excess minority carriers at either side of junction (see Figure), which causes lagging of voltage as current changes. Switching speed can be improved by adding recombination centres to reduce lifetime of injected carriers. A second approach is to make p-type region here shorter than L_e, so called narrow base diode approach.

b) i) This is so called junction capacitance due to extend of depletion at the junction.

Junction Capacitance $C_D = \frac{dQ}{dV} = \frac{\varepsilon_0 \varepsilon_r}{W}$

Assume one-sided junction as $N_D \gg N_A$, ie depletion all in p-type region

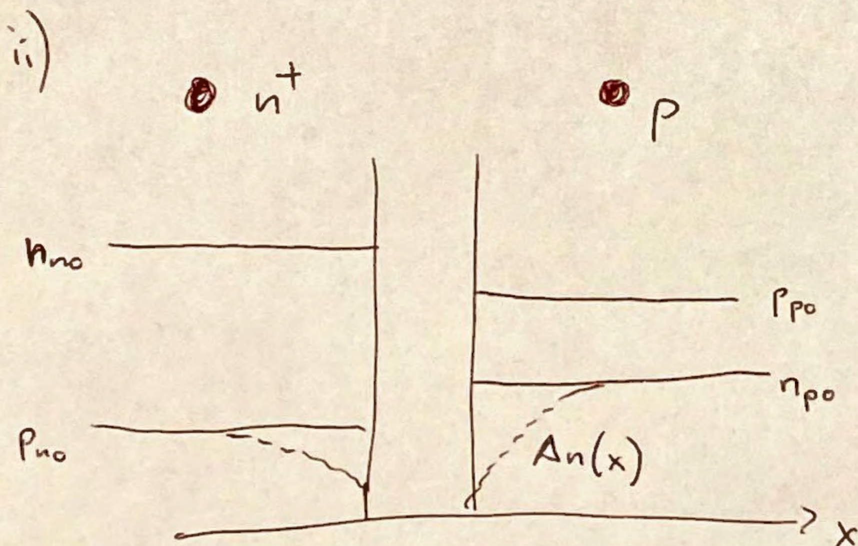
$$\rightarrow \quad W = x_p = \sqrt{\frac{2\varepsilon_0\varepsilon_r (V_0 - V_{app})}{e N_A}}$$

assume $V_{app} \gg V_0$

$W \sim \sqrt{\frac{V_{app}}{N_A}}$ $\rightarrow$ W increases by $\sqrt{2}$

$\rightarrow$ C_D decreases by $\frac{1}{\sqrt{2}}$ $\rightarrow$ $C_D' = 0.21$ pF

ii)



Reverse saturation current dominated by minority carrier extraction and their drift across depletion region.

c) Linearly graded junction $N_D - N_A = Gx$

↑ grade constant

From Poisson equation:

$$\frac{d\mathcal{E}}{dx} = \frac{e}{\varepsilon_r \varepsilon_0} Gx$$

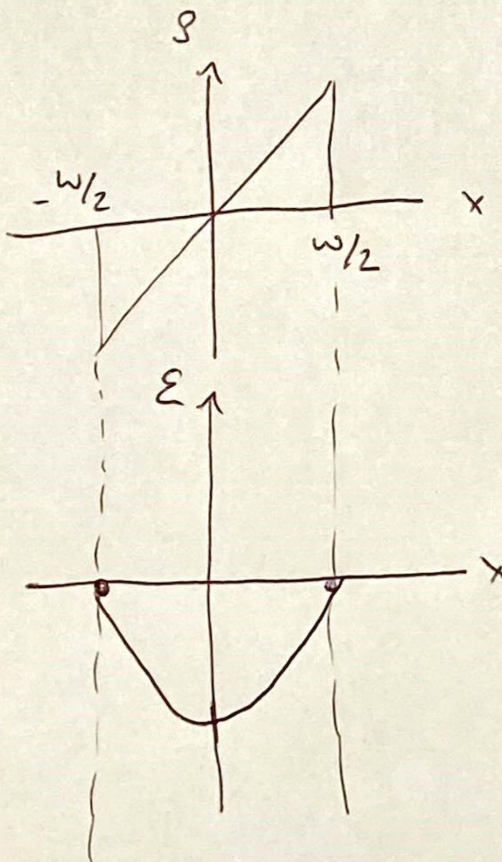
Assume no $\mathcal{E}$-field outside junction, $\mathcal{E} = 0$ at $x = \frac{W}{2}$

Integrate

$$\mathcal{E} = \frac{eG}{2\varepsilon_r \varepsilon_0} x^2 + C$$

$$C = -\frac{eGW^2}{8\varepsilon_r \varepsilon_0} \quad \text{from boundary condition}$$

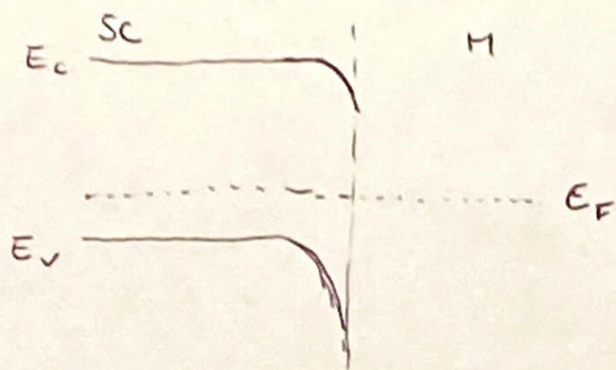
$$\rightarrow \quad \mathcal{E}_{max}\Big|_{x=0} = -\frac{eGW^2}{8\varepsilon_0 \varepsilon_r}$$



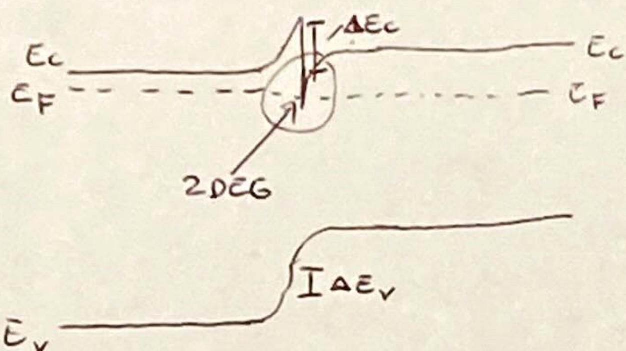
Basic junction details were generally well answered. A number of candidates got confused with (a)(ii), which was just asking for forward injection condition, and with (a)(iv), where a few candidates confused charge storage capacitance with junction capacitance. A surprisingly large number of candidates lost points on part (b)(i), not simplifying to the case of a one-sided junction. Most challenging proved part (b)(iii) which asked for field integration for a linearly graded junction. It was nice to see a number of very good answers to this as this had not been covered in lecture notes.

Q3

a) i) Schottky junction - p-type SC - metal



ii) n^+ AlGaAs - n GaAs

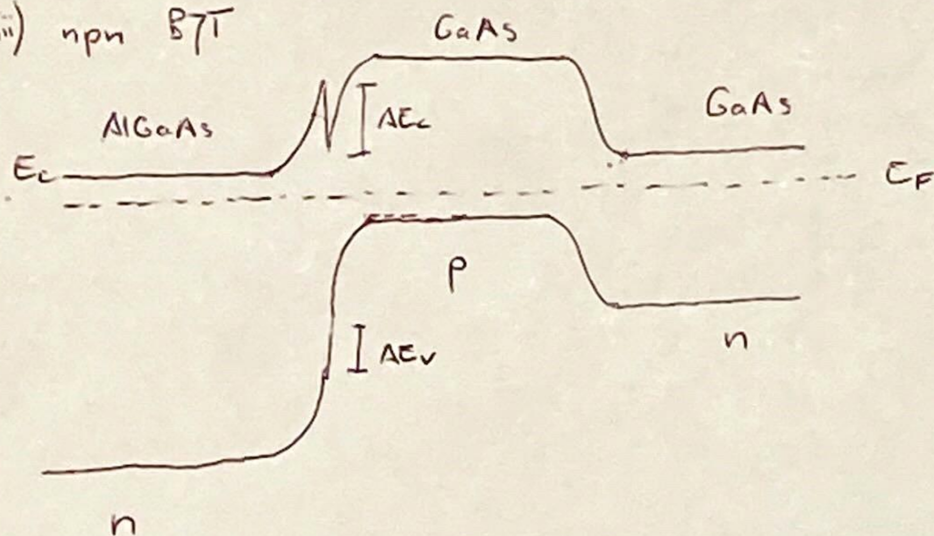


$\Delta E_{gap} = 0.45eV$

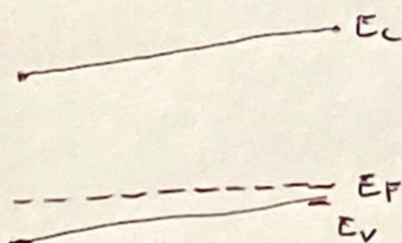
$\rightarrow$ $\Delta E_c = 0.3eV$

$\Delta E_v = 0.15eV$

iii) npn BJT



b) p-type Si - N_A exponentially varied from $10^{21} - 10^{24} m^{-3}$



Assume full dopant ionisation :

$$E_F = kT \ln \frac{N_V}{N_A}$$

$$\Delta E_F = kT \ln \frac{N_{A2}}{N_{A1}}$$

resulting electric field uniform as profile is exponential over $W = 1\mu m$ - Assume RT

$$|\mathcal{E}| = \frac{kT}{W} \ln \frac{N_{A2}}{N_{A1}} = 177 \frac{kV}{m}$$

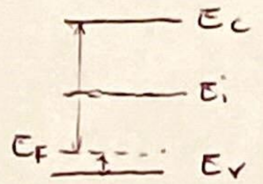
c) i) Calculate WF of p-Si:

Assume full dopant ionisation

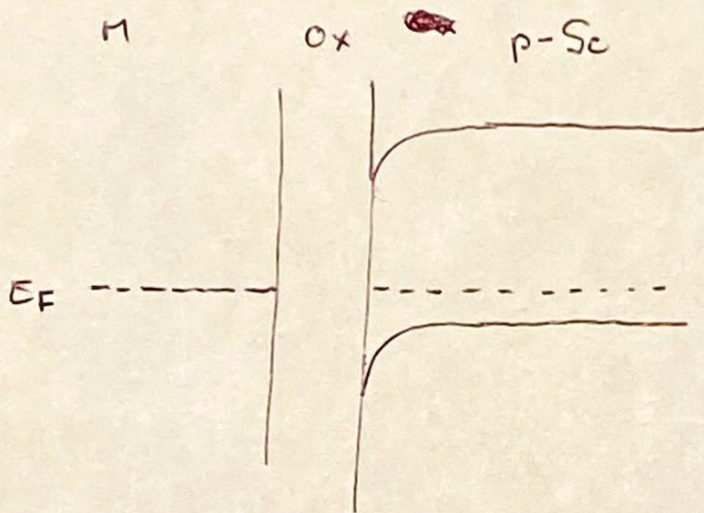
$$p = N_A = N_V \exp\left(\frac{-E_F}{kT}\right)$$

$\rightarrow$ $E_F = -kT \ln\left(\frac{N_A}{N_V}\right) = 0.24 \text{ eV}$

$\rightarrow$ $e\phi_{Si(p)} = \chi + E_{gap} - E_F = 4.93 \text{ eV}$



$\rightarrow$ $e\phi_{Si(p)} > e\phi_{Al}$ $\rightarrow$ band bending in SC



$$|V_{FB}| = \left|\frac{\phi_M - \phi_{SC}}{e}\right| = 0.93 \text{ V}$$

ii) $$V_T = -\frac{Q_D}{C_{ox}} + V_S + V_{FB}$$

charge per unit area in SC depletion

$$Q_D = -eN_A w_{max} = -1.6 \cdot 10^{-5} \frac{C}{m^2}$$

oxide capacitance per unit area

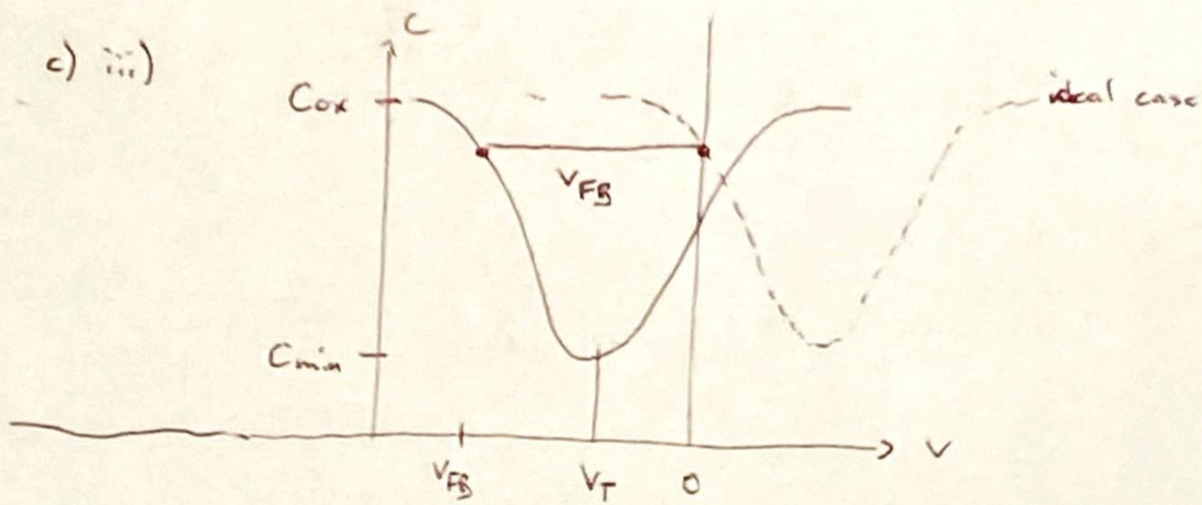
$$C_{ox} = \frac{\varepsilon_0 3.9}{d_{ox}} = 3.5 \cdot 10^{-3} \frac{F}{m^2}$$

$$\phi_F = \frac{kT}{e} \ln\left(\frac{N_A}{n_i}\right) = 0.29 \text{ V}$$ (or via $E_i - E_F$, see i))

strong inversion $V_S = 2\phi_F = 0.58 \text{ V}$

$\rightarrow$ $V_T = \underset{(V_{FB})}{-0.93 \text{ V}} + \underset{(V_S)}{0.58 \text{ V}} + 4.57 \cdot 10^{-3} \text{ V} = -0.35 \text{ V}$

c) iii)



C_{min} is C_{ox} in series with C from SC depletion, C_D:

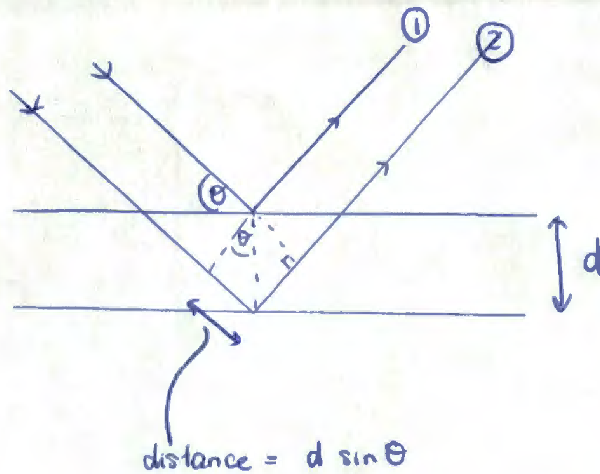
$$C_D = \frac{\varepsilon_0 \, 11.8}{100\,nm} = 1.1 \cdot 10^{-3} \frac{F}{m^2}$$

$$C_{min} = \frac{C_{ox} \cdot C_D}{C_{ox} + C_D} = 8.4 \cdot 10^{-4} \frac{F}{m^2}$$

Drawing of band diagrams in part (a) was generally well done. A surprisingly large number of students struggled with (b), even though this mirrored a question from supervised examples paper 4. The band diagram in part (c)(i) was well answered, whereas many candidates lost points in (ii) and (iii), getting confused with or not calculating the various contributions to the threshold voltage and minimum capacitance. The drawing of C-V characteristics also caused some problems.

Question 4

(a)(i)



Extra path length travelled by ray ② is $2d\sin\theta$

Bragg diffraction occurs when the difference in path length is equal to an integer number of wavelengths.

$$n\lambda = 2d\sin\theta \qquad \lambda = \frac{2\pi}{k}$$

$$\therefore \quad k = \frac{\pi n}{d\sin\theta} \qquad n = \text{integer}$$

(ii) $n\lambda = 2d\sin\theta$

$\frac{1}{2}mv^2 = qV$

$\lambda = \frac{h}{p}$

$p = mv$

$$\therefore qV = \frac{p^2}{2m}$$

$$\therefore V = \frac{p^2}{2qm}$$

$$= \frac{h^2}{2qm\lambda^2}$$

$$\approx \frac{h^2}{2qm(2d\sin\theta)^2}$$

$$= \frac{h^2}{8mqd^2\sin^2\theta}$$

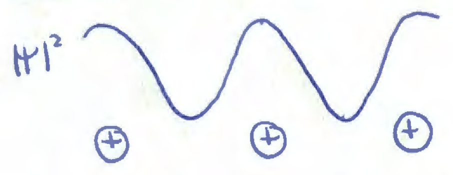
$$= 17.1 \text{ V}$$

(iii) In the nearly free electron model, the electron exists in a periodic potential. When the electron's wavenumber approaches the Bragg diffraction criterion:

$$k = \frac{\pi n}{d} \qquad (\text{N.B. } \theta = 90^\circ)$$

it is Bragg scattered and forms a standing wave, rather than a travelling wave. There are two possibilities for this standing wave:

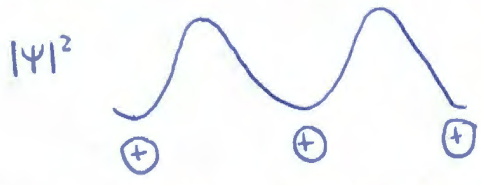
① Wavefunction is such that probability of finding electron is highest nearer the nuclei



$$E = E_{free} - \Delta E$$

This leads to an energy state that is lower than predicted by the free electron model.

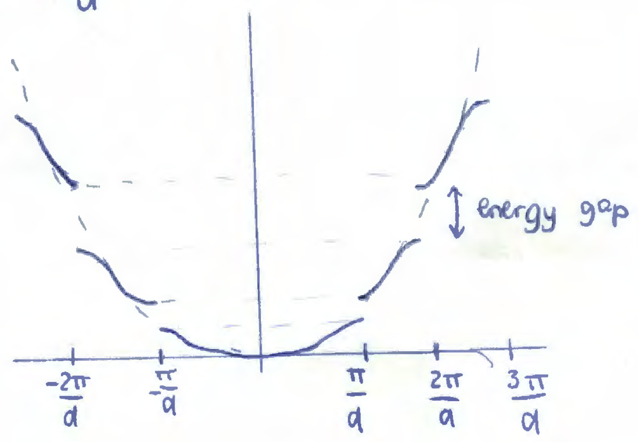
② Wave function is such that probability of finding electron is lowest near the nuclei



$$E = E_{free} + \Delta E$$

This leads to an energy state that is higher than predicted by the free electron model.

An energy gap results between the two states, whenever $k = \pm \frac{\pi n}{d}$. No states are available within the gap.



(b)

$$n = \frac{N}{L} = \int_0^{E_F} \frac{\sqrt{2}}{\pi\hbar} m^{\frac{1}{2}} E^{-\frac{1}{2}} \, dE$$

$$= \frac{\sqrt{2}\sqrt{m}}{\pi\hbar} \int_0^{E_F} E^{-\frac{1}{2}} \, dE$$

$$= \frac{\sqrt{2m}}{\pi\hbar} \left[2E^{\frac{1}{2}} \right]_0^{E_F}$$

$$= \frac{2\sqrt{2}\sqrt{m}}{\pi\hbar} E_F^{\frac{1}{2}}$$

$$\therefore \quad E_F = \left(\frac{\pi\hbar n}{2\sqrt{2}\sqrt{m}} \right)^2$$

$$= \frac{\pi^2 \hbar^2 n^2}{8m}$$

A popular question. The derivation of the Bragg scattering equation (a)(i) was performed well by most candidates but many left their solution in terms of wavelength rather than wavenumber as required. (a)(ii) was answered correctly by less than half of the candidates. A common error was an erroneous assumption that the electrons travel at the speed of light. (a)(iii) was answered well by most candidates. (b) was answered well by the majority of candidates. A common error was that candidates did not note that the calculation was to be performed at absolute zero, which simplified the Fermi function and the corresponding maths considerably.