EGT2
ENGINEERING TRIPOS PART IIA

Monday 25 April 2022   9.30 to 11.10

Module 3B5

SEMICONDUCTOR ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM CUED approved calculator allowed.
Engineering Data Book.

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 InP is a III–V semiconductor and has the following properties at 300 K:

Effective density of states in conduction band:	$5.7 \times 10^{23}\,\mathrm{m^{-3}}$,
Effective density of states in valence band:	$1.1 \times 10^{25}\,\mathrm{m^{-3}}$,
Electron effective mass:	$0.08\,\mathrm{m}_e$,
Hole effective mass:	$0.6\,\mathrm{m}_e$,
Atomic lattice spacing:	0.6 nm,
Intrinsic carrier density:	$1.3 \times 10^{13}\,\mathrm{m^{-3}}$.

(a) A sample of intrinsic InP has electron and hole mobilities of $0.49\,\mathrm{m^2V^{-1}s^{-1}}$ and $0.07\,\mathrm{m^2V^{-1}s^{-1}}$ respectively.

(i) Calculate the resistivity of the intrinsic InP sample. [10%]

(ii) Calculate the position of the intrinsic Fermi level, E_{Fi}, and the band gap energy. Comment on the position of the Fermi level in the intrinsic InP. [20%]

(iii) Calculate the average time between scattering events for electrons in InP. [10%]

(iv) Using your answer to (a)(iii), and given that the average thermal electron velocity is $4.1 \times 10^5\,\mathrm{ms^{-1}}$, calculate the average distance travelled by electrons between scattering events. [10%]

(v) Compare your answer in (a)(iv) with the lattice spacing. Explain your results with reference to the appropriate models for electron transport. [10%]

(b) (i) Show that in a doped semiconductor, the Fermi level E_F is related to the Fermi level E_{Fi} of the intrinsic material according to the function:

$$E_F = E_{Fi} + x \ln \frac{n}{p}$$

where n and p are the electron and hole densities, respectively, and find an expression for x. [20%]

(ii) An InP sample is deliberately doped with Si acceptors so that there are 10^{20} times more holes than there are electrons. Calculate the Fermi level. [10%]

(iii) The electronic configuration of Si is $1s^2 2s^2 2p^6 3s^2 3p^2$. Explain qualitatively how Si acceptor atoms substitute into the InP lattice to introduce free holes. [10%]

Note:

$$\sigma = ne\mu_e + pe\mu_h$$

$$\mu = \frac{q\tau}{m^*}$$

$$n = N_C \exp\left(-\frac{E_C - E_F}{kT}\right)$$

$$p = N_V \exp\left(\frac{E_V - E_F}{kT}\right)$$

$$n_i^2 = np$$

2 (a) A n-p junction is formed from Si with a concentration of donors of $N_D = 10^{24}\,\mathrm{m}^{-3}$ and a concentration of acceptors of $N_A = 10^{21}\,\mathrm{m}^{-3}$. Unless otherwise stated, assume the junction to be abrupt and ideal. The intrinsic carrier concentration of Si is $1.5\times10^{16}\,\mathrm{m}^{-3}$. Assume room temperature (300 K) operation. State any other assumptions made.

(i) Calculate the built-in potential and equilibrium minority carrier concentrations p_{n0} and n_{p0} on either side of the junction, as labelled in Figure 1. [10%]

(ii) A forward bias of 0.5 V is applied. Calculate the excess electron concentration Δn at $x = 0$, as schematically plotted in Figure 1. [10%]

(iii) Using the continuity equation, derive an expression of how the excess electron concentration Δn varies with x for $x \geq 0$. [20%]

(iv) Explain what is referred to as charge storage capacitance of the n-p junction. Give two approaches how to minimise such capacitance. [10%]

(b) The n-p junction of above is now reverse biased at 10 V and a capacitance of 0.3 pF is measured.

(i) Explain the origin of this capacitance. Calculate the change of this capacitance if the p-doping is doubled and the reverse bias increased to 40 V. State any assumptions made. Note that the extend of the depletion region in the p-type region can be expressed as

$$x_p = \sqrt{\frac{2\epsilon_0\epsilon_r(V_0 - V_{App})}{e}} \times \sqrt{\frac{N_D}{N_A N_D + N_A^2}}$$

where V_0 is the built-in bias, and V_{App} the externally applied bias. [15%]

(ii) Redraw Figure 1 for the case of reverse bias, and explain what process dominates the reverse saturation current. [10%]

(iii) For an implanted p-n junction the abrupt junction model is found inadequate. At the junction, the doping profiles are found to compensate each other, leading to a linearly graded junction profile as shown in Figure 2. N_A and N_D represent the dopant concentrations, and w the width of the depletion region. Derive an expression for the maximum electric field in such a linearly graded p-n junction at equilibrium, i.e. with no external bias applied. State any assumptions made. Sketch the electric field across the unbiased junction. [25%]

Note: The continuity equation for electrons is given by

$$\frac{\partial(\Delta n)}{\partial t} = -\frac{\Delta n}{\tau_e} + \mu_e \varepsilon \frac{\partial(\Delta n)}{\partial x} + D_e \frac{\partial^2(\Delta n)}{\partial x^2}$$

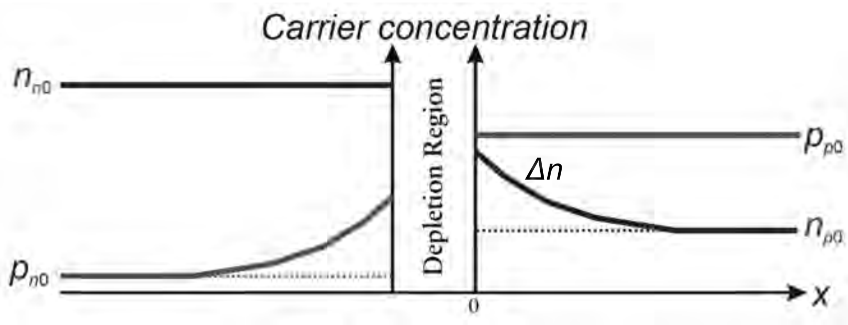

Fig. 1

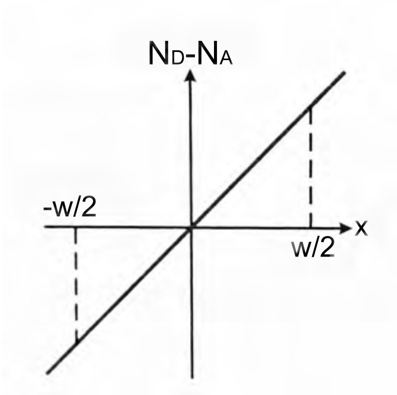

Fig. 2

3 (a) Sketch an equilibrium band diagram for each of the following device structures. State all your assumptions.

(i) A junction between p-type cuprous oxide (Cu_2O) and Cu metal. Such so called metal rectifiers found early applications to replace valves in receivers. [10%]

(ii) A junction between heavily n-doped AlGaAs and lightly n-doped GaAs. The AlGaAs has a band gap of 1.85 eV and the GaAs band gap is 1.4 eV. In the type I heterostructure the band gap difference is accommodated approximately 2/3 in the conduction band and 1/3 in the valence band. Indicate where a two-dimensional electron gas can form. [15%]

(iii) A n-p-n bipolar junction transistor formed by an emitter junction of heavily n-doped AlGaAs and p-doped GaAs and a collector junction made of GaAs. [10%]

(b) The doping density across a 1 μm wide piece of p-type Si is exponentially varied from $10^{21}\,\text{m}^{-3}$ to $10^{24}\,\text{m}^{-3}$. Sketch a band diagram and calculate the resulting electric field at room temperature. State all your assumptions. [15%]

(c) Al is evaporated on an oxidised p-type Si substrate with a doping density of $N_A = 10^{21}\,\text{m}^{-3}$. The SiO_2 thickness is 10 nm and its relative permittivity is 3.9. Assume that the Al has an effective work function of 4 eV and that the resistivity of the oxide is infinite, i.e. that no carrier transport is possible through that layer. Assume the Si has a band gap of 1.12 eV, a relative permittivity of 11.8, an intrinsic carrier concentration of $1.5 \times 10^{16}\,\text{m}^{-3}$, an electron affinity of 4.05 eV, and effective densities of states in the conduction and valence bands of $N_C = 2.8 \times 10^{25}\,\text{m}^{-3}$ and $N_V = 1.04 \times 10^{25}\,\text{m}^{-3}$, respectively. Further assume no interface traps and room temperature (300 K) conditions.

(i) Sketch a band diagram of the resulting MOS capacitor with no external bias applied. Calculate the external voltage V_{FB} that needs to be applied to the Al with respect to the Si to achieve the so called flat band condition. [15%]

(ii) Calculate the threshold voltage V_T which has to be applied to the metal to induce strong inversion. Assume that the width of the depletion region in the p-type Si is 100 nm at strong inversion. State any other assumptions made. [20%]

(iii) Sketch the C-V characteristics of the MOS capacitor and indicate V_T and V_{FB}. Assume that there is no deep depletion. Calculate the minimum capacitance per unit area. [15%]

4 (a) An electron beam is accelerated through an electric potential and is incident on a semiconductor crystal at an angle θ to the crystal planes. The distance between the crystal planes is d.

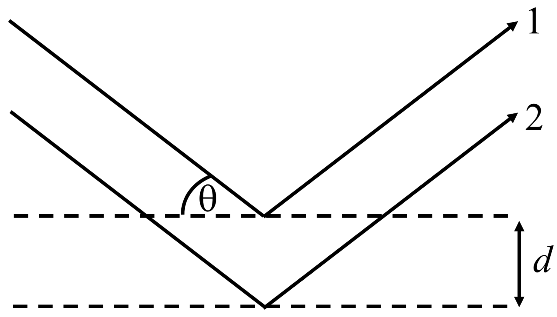

Fig. 3

(i) By considering the phase difference between ray 1 and ray 2 of Figure 3, derive the formula describing the electron wavenumbers at which Bragg diffraction occurs. [10%]

(ii) A set of parallel lattice planes of a crystal are spaced at distances of 2.1×10^{-10} m. Find the lowest value of electron accelerating potential that will produce Bragg reflection for a beam incident on the lattice planes at an angle of 45°. [20%]

(iii) Discuss the significance of Bragg diffraction on electron propagation and electron energy in the nearly-free electron model. [30%]

(b) Consider a one-dimensional wire in which the density of states is given by

$$g(E)\,dE = L\frac{\sqrt{2}}{\pi\hbar}m^{\frac{1}{2}}E^{-\frac{1}{2}}\,dE$$

where L is the length of the wire and m is the electron mass. The electron density per unit length, n, is given by $n = \frac{N}{L}$ where N is the total number of electrons. Derive an expression for the Fermi energy, as a function of n, at absolute zero. [40%]

Note:

$$f(E) = \frac{1}{\exp\left(\frac{E-E_F}{kT}\right) + 1}$$

END OF PAPER

Numerical answers

Q1

(a)(i) $8.57 \times 10^5\ \Omega m$

(ii) E_{Fi}-E_V = 0.710 eV; E_g = 1.344 eV

(iii) 223 fs

(iv) 92 nm

(b)(ii) E_F-E_V = 0.115 eV

Q2

(a)(i) 0.75 eV; $p_{n0} = 2.2 \times 10^8\ m^{-3}$; $n_{p0} = 2.2 \times 10^{11}\ m^{-3}$

(ii) $5.5 \times 10^{19}\ m^{-3}$

(b)(i) decrease by $1/\sqrt{2}$

Q3

(b) 177 kV/m

(c)(i) 0.93 V

(ii) -0.35 V

(iii) $8.4 \times 10^{-4}\ F/m^2$

Q4

(a)(ii) 17.1 V