3B5 - 2023 - Crib v2

Question 1

(a)(i) Substitute solution into Schrödinger equation:

$$\frac{-\hbar^2}{2m} \nabla^2 \psi(\underline{r}) = E \, \psi(\underline{r})$$

$$\therefore \frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(A e^{j(k_x x + k_y y)}\right) = E \left(A e^{j(k_x x + k_y y)}\right)$$

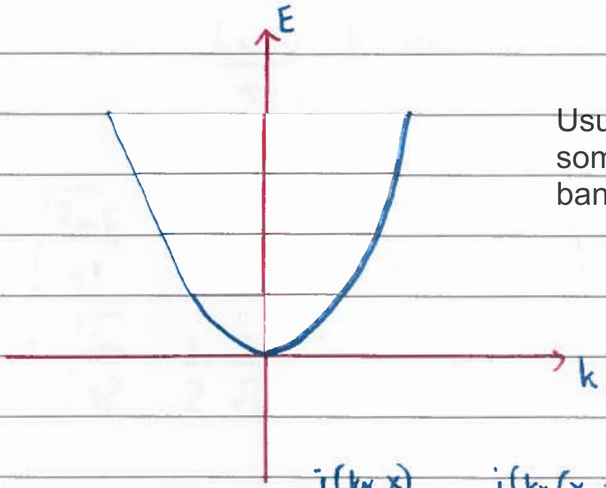
$$\therefore -\frac{\hbar^2}{2m} A \left(-k_x^2 - k_y^2\right)\left(A e^{j(k_x x + k_y y)}\right) = E\left(A e^{j(k_x x + k_y y)}\right)$$

LHS = RHS when $\frac{\hbar^2}{2m}(k_x^2 + k_y^2) = E$

$$\therefore \frac{\hbar^2 k^2}{2m} = E$$

$$\therefore k = \sqrt{\frac{2mE}{\hbar^2}}$$

Band diagram



Usually answered well except some students forgot to plot the band diagram

(ii) Boundary conditions require $e^{j(k_x x)} = e^{j(k_x(x + L_x))}$ and $e^{j(k_y y)} = e^{j(k_y(y + L_y))}$

$$\therefore k_x L_x = 2\pi n_x \quad \text{and} \quad k_y L_y = 2\pi n_y$$

$$\therefore k_x = \frac{2\pi n_x}{L_x} \quad \text{and} \quad k_y = \frac{2\pi n_y}{L_y}$$

where n_x and n_y are integers

These boundary conditions allow travelling plane wave solutions to the

Schroedinger equation, which, being travelling waves, are a suitable model for free electron transport / motion.

Many students did not recognise that the boundary conditions allow a travelling wave solution. Many students gave wavevectors that were out by a factor of 2, because they chose standing wave solutions

(iii) The "area" of k-space per point $= \left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)$

i.e. every time we add an extra state to n_x or n_y, we increase k-space by $\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)$

The density of points is the reciprocal, times the area of the annulus:

$$\frac{L_x L_y}{4\pi^2} \cdot 2\pi k \, dk$$

And remember spin gives twice the states so

$$g_k(k)\, dk = 2 \frac{L_x L_y}{4\pi^2} \cdot 2\pi \, k \, dk$$

$$= \frac{L_x L_y}{\pi} k \, dk$$

Some students, incorrectly, attempted to derive the formula from the 3D density of states formula. Some students forgot that there are 2 spins which doubles the density of states.

(iv) Remember $k = \sqrt{\frac{2mE}{\hbar^2}}$

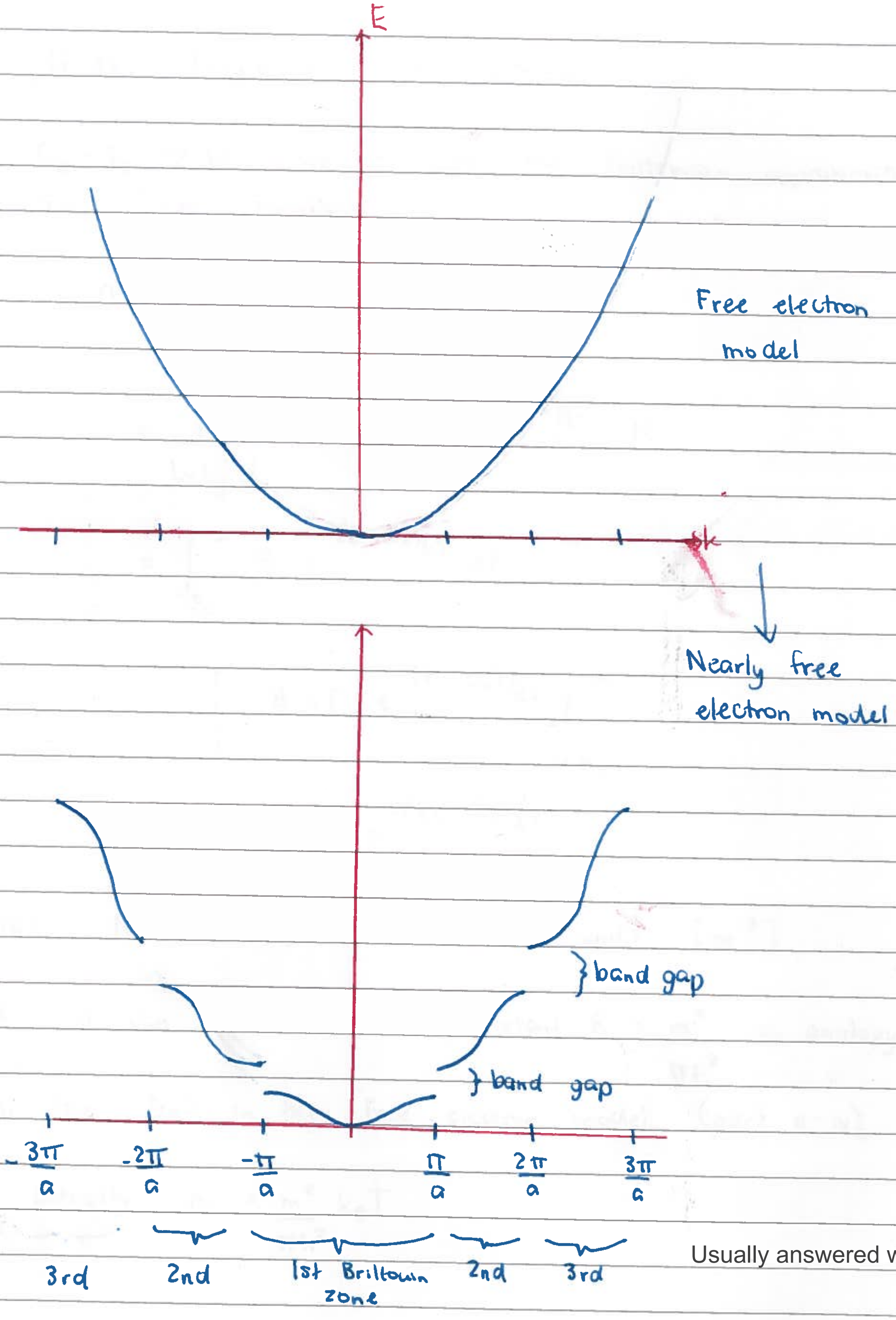
$$\therefore \frac{dk}{dE} = \sqrt{\frac{2m}{\hbar^2}} \cdot \frac{1}{2} \frac{1}{\sqrt{E}}$$

Substitute the above into $g_k(k)\, dk$ to get:

$$g(E)\, dE = \frac{L_x L_y}{\pi} \sqrt{\frac{2mE}{\hbar^2}} \cdot \sqrt{\frac{2m}{\hbar^2}} \cdot \frac{1}{2} \frac{1}{\sqrt{E}}$$

$$= \frac{L_x L_y m}{\pi \hbar^2}$$

Usually answered well though some students didn't cancel out the *E* terms (b)(i)



Usually answered well

(b)(ii) All the information can be contained in the first Brillouin zone because Bloch's theorem actually shows that the E-k curve can be translated by $n\frac{2\pi}{a}$.

Not all students explicitly mentioned Bloch's theorem. Some students, erroneously, described the validity as being related to the occupancy of the bands (b)(iii) $g(E)\,dE = L_x L_y B \; dE$ if $E > E_c$

If $E_c - E_F \gg kT$, we can use the Boltzmann approximation to the Fermi function

$$n = \frac{1}{L_x L_y} \int_{E_c}^{\infty} g(E)\, f(E)\, dE$$

$$= \frac{1}{L_x L_y} \int_{E_c}^{\infty} L_x L_y B\, e^{-(E-E_F)/kT}\, dE$$

$$= \int_{E_c}^{\infty} B\, e^{-(E-E_F)/kT}\, dE$$

$$= \Big[-B\, kT\, e^{-(E-E_F)/kT} \Big]_{E_c}^{\infty}$$

$$= B\, kT\, e^{-(E_c-E_F)/kT}$$

where $N_c = B k_B T$ and has units $[m^{-2}]$.

N.B. It can be shown that constant $B = \frac{m^*}{\pi \hbar^2}$ in analogy with the DoS in the free electron model (part a-iv),

so actually $N_c = \frac{m^*}{\pi \hbar^2} k_B T$

Usually answered well. Many students had incorrect units though (especially stating m-3 and units containing J)

Q2 (a)(i) InP p-doping

suitable acceptors

group II (Zn, Cd) element on III (In) site, or

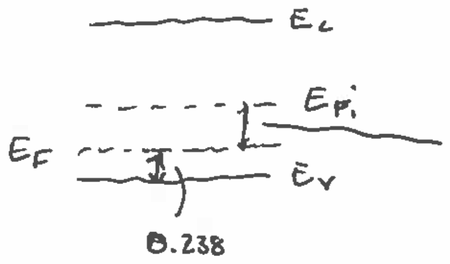
group IV element (C, Ge, Si) on V (P) site

$$p = N_V \exp\left(\frac{-(E_F - E_V)}{kT}\right)$$ (see formula sheet)

Assuming $p = N_A$

$$E_F - E_V = kT \ln\left(\frac{N_V}{N_A}\right) = 0.238 \text{ eV}$$

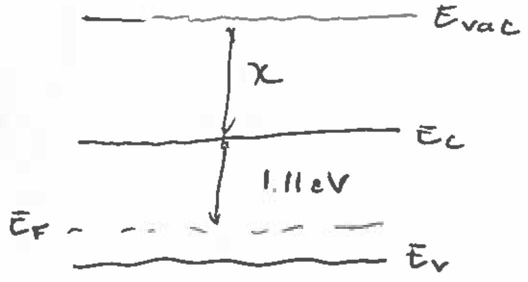
Assume E_{Fi} at mid gap (0.675eV) for intrinsic InP



E_F shifted by 0.437eV towards E_V

Mostly answered well. Range of candidates got confused with suitable acceptor element and site, and with carrier statistics that were meant to be simple here (from formula sheet).

(ii) Calculate work function of p-doped InP



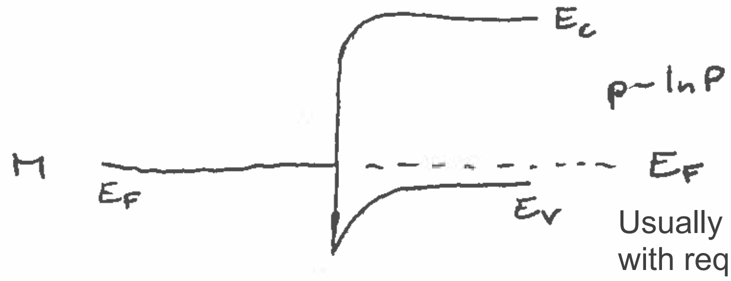
$\phi(\text{InP}) = 4.38 \text{ eV} + 1.11 \text{ eV} = 5.49 \text{ eV}$

Need Schottky contact for MESFET gate, hence in WF model

$$\phi(\text{InP}) > \phi(M)$$

→ either Au or Ti are suitable

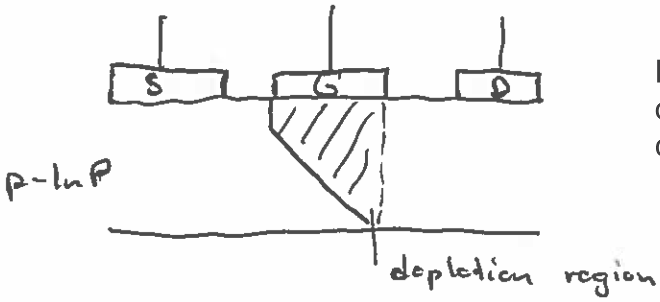
Band diagram



Usually answered well. Some candidates got confused with requirement for Schottky contact for p-type semiconductor.

Q2 a) (iii) Need Ω-contact at Source and Drain. This can be achieved by heavy (p-type) doping of InP close to contacts.

At pinch-off voltage the gate depletion area extends across the channel at drain end.



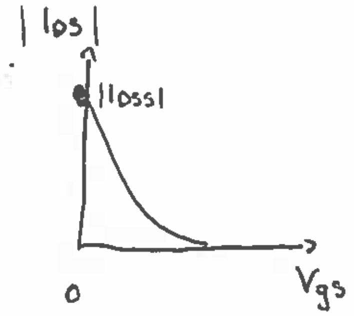
Pinch-off generally well explained. Mixed range of answers re the formation of required Ohmic contacts

(iv) High electric fields can lead to velocity saturation in InP, which leads to I_{DS} saturation without pinch-off

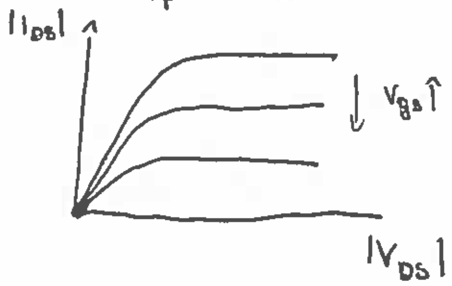
Transfer characteristics:

MESFET in depletion mode,

positive V_{gs} to extend depletion region



Output characteristics ($V_{DS} < 0$)

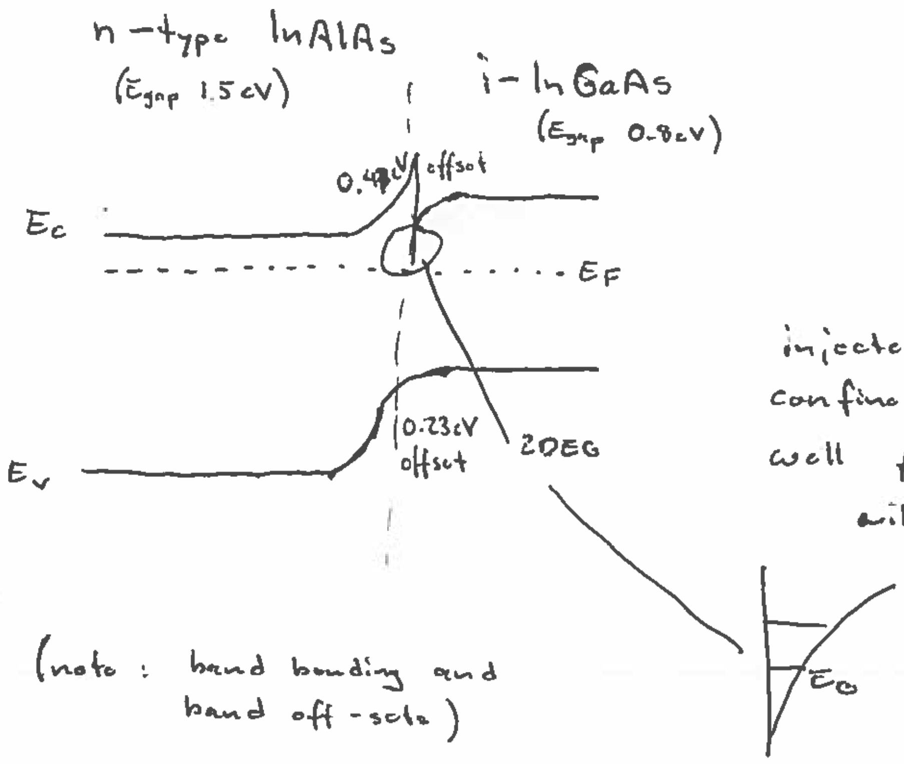


Velocity saturation answered well, but some candidates drew output characteristics of MOSFET here, and few added transfer characteristics

b) (i) Higher doping lowers carrier mobility, hence for high mobility undoped channel is desired. Yet carriers are required, which is achieved by injection from higher band gap material.

Usually answered well

Q2 b) ii)



injected electrons confined in potential well form 2DEG, with states corresponding to 2D quantum well

(note: band bending and band off-sets)

For simple estimate of ground state energy of 2DEG the Heisenberg uncertainty relationship can be used (assume box confinement)

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

$$E_0 = \frac{\Delta p^2}{2m} = \frac{1}{2m}\left(\frac{\hbar}{2\Delta x}\right)^2 = \frac{\hbar^2}{2mw^2} \qquad \left(\text{assume } \Delta x \sim \frac{w}{2}\right)$$

for $w = 0.5$ nm

$E_0 = 0.152$ eV

so $E_{2D} = 2E_0 = 0.3$ eV

(in 2D overall E is sum of individual energies for each degree of freedom)

(any reasonable estimate accepted)

Bandstructure of heterostructure generally well sketched. Not many candidates offered answer to second part which needed oversight and making connection with first part of lectures.

Question 3

(a) (i) Assume all donors are ionised, $n = N_D$

$$n = n_i \exp\left(\frac{E_F - E_{Fi}}{k_B T}\right)$$

$$\therefore k_B T \ln\left(\frac{n}{n_i}\right) + E_{Fi} = E_F$$

$$\therefore E_F = \frac{1.381 \times 10^{-23}}{1.602 \times 10^{-19}} \times 300 \ln\left(\frac{4.5 \times 10^{21}}{1.0 \times 10^{16}}\right) + \frac{1.12}{2}$$

$$= 0.337 + 0.56$$

$$= 0.89 \text{ eV}$$ above the valence band edge

Usually answered well. Some students had surprisingly committed the formula for Nc in terms of m* to memory, giving a more circuitous way to the answer

(ii)

$$J_x = \sigma E_x$$

$$\therefore \sigma = \frac{J_x}{E_x}$$

$$= \frac{40}{0.64}$$

$$= 62.5 \ \Omega^{-1} m^{-1}$$

$$\sigma = ne\mu$$

$$\therefore \mu = \frac{\sigma}{ne}$$

$$= \frac{62.5}{4.5 \times 10^{21} \times 1.602 \times 10^{-19}}$$

$$= 0.087 \ m^2 V^{-1} s^{-1}$$

$$\mu = \frac{q\tau}{m^*}$$

$$\therefore \tau = \frac{\mu m^*}{q} = \frac{0.087 \times 0.36 \times 9.109 \times 10^{-31}}{1.602 \times 10^{-19}}$$

$$= 177 \text{ fs}$$

Usually answered well, although some students forgot the units for mobility

(iii) For electrons $F = q v_x B_z$ $J_x = nqv_x \Rightarrow v_x = \frac{J_x}{nq}$

$$= q \frac{J_x}{nq} B_z = \frac{J_x B_z}{n}$$

The electrons experience the Lorentz force in the negative y-direction. A negative Hall voltage will result.

Usually answered well if students remembered F = q v x B

(iv)

Once the Hall voltage is established:

$$q E_y = \frac{J_x B_z}{n}$$

$$R_H = \frac{E_y}{J_x B_z} = \frac{1}{qn} = -\frac{1}{en}$$

$$V_H = E_y w$$

$$R_H = \frac{V_H}{w J_x B_z} = -\frac{1}{en}$$

$$\therefore V_H = -\frac{w J_x B_z}{en}$$

$$= -\frac{1.0 \times 10^{-3} \times 40 \times 0.2}{1.602 \times 10^{-19} \times 4.5 \times 10^{21}}$$

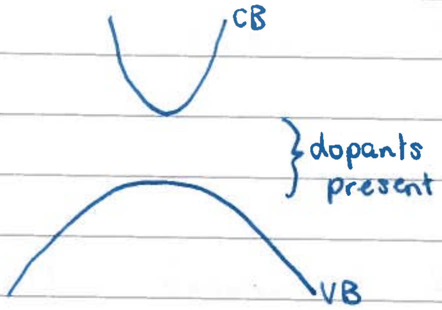
$$= 11 \mu V$$

Usually answered well

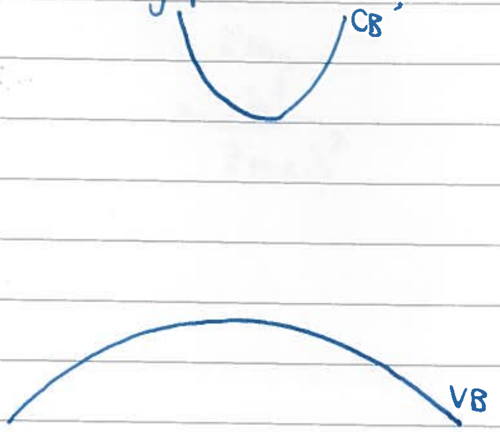
(b)

Examine the slope in the intrinsic region. Slope B > Slope A

Sample A is a semiconductor



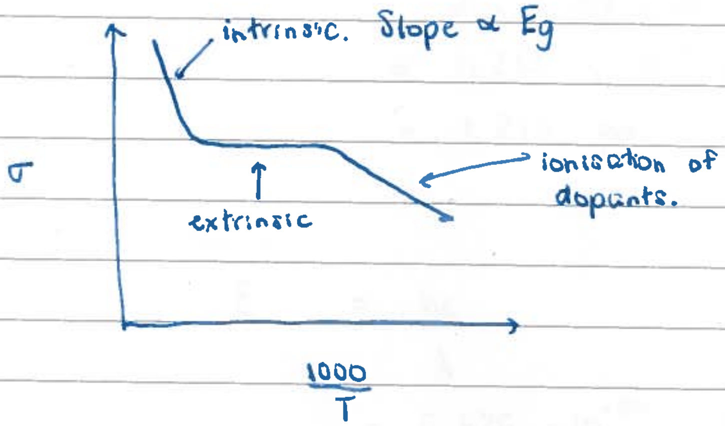
Sample B has a bigger bandgap than A;



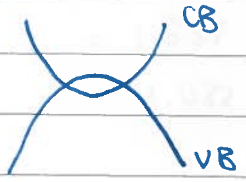
Sample C is metallic as its conductivity drops with increasing temperature.

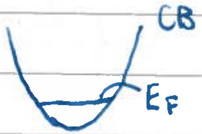
B is also a semiconductor At 300K, the presence of an "extrinsic region" suggests dopants are present and effective.

NB:



Sample C :





Not all students identified C as the metal. Many students didn't identify the slope in the low and high temperature regions as the key indicators of the dopant ionisation energy and bandgap energy, respectively

(c)

Photons

$$E = \hbar\omega = \frac{hc}{\lambda}$$

Electrons

$$E = \frac{p^2}{2m_e} = \frac{h^2}{2m_e\lambda^2}$$

If

$$E_{photon} = E_{electron}$$

$$\frac{hc}{\lambda} = \frac{h^2}{2m_e\lambda^2}$$

$$\therefore \quad c = \frac{h}{2m_e\lambda}$$

$$\therefore \quad \lambda = \frac{h}{2m_e c} = \frac{6.626 \times 10^{-34}}{2 \times 9.109 \times 10^{-31} \times 2.998 \times 10^{8}} = 1.213 \times 10^{-12}\ m = 1.213\ pm$$

(gamma rays if a photon)

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 2.998 \times 10^{8}}{1.213 \times 10^{-12}} = 1.637 \times 10^{-13}\ J = 1.022 \times 10^{6}\ eV$$

Answered well but many students didn't perform the calculation and left the energy in terms of fundamental constants

Q4 (a) Haynes – Shockley experiment:

pulse of minority carriers is created by laser pulse.
(here holes)

An applied drift field makes this pulse move across the semiconductor. Drift mobility of holes can be extracted from run time, diffusion coefficient from pulse broadening (links to mobility via Einstein relationship) and hole life time from total decay of carriers.

Usually answered well. Some answers lacked detail on how lifetime can be extrapolated

(b) (i) electron mobility (from figure): $\mu_e = 0.12 \frac{m^2}{Vs}$

$\rightarrow$ current $I = A n e v = A e N_D \mu_e \mathcal{E}$

$$= A e N_D \mu_e \frac{V}{d} = 0.32 \text{ A}$$

Usually answered well.

(ii) continous illumination:

generated excess electons δn = excess holes δp

$$= g \cdot \tau$$

$$= 10^{21} m^{-3}$$

Both now contribute to current

$$I = Ae \left[(n + \delta n) \mu_e + \delta p \mu_p \right] \mathcal{E}$$

$\mu_p = 0.05 \frac{m^2}{Vs}$ (from figure)

$$\rightarrow I = A e \mathcal{E} \left[(N_D + \delta n) \mu_e + \delta p \mu_p \right]$$

$$= 0.366 \text{ A}$$

Some confusion here, with some candidates trying to start with Master equation and not recognising simple continuous uniform illumination

Q4 (c) (i) From Einstein relation

$$D_h = \frac{kT}{e}\mu_h$$

from (b)(figure) $\mu_h = 0.05 \frac{m^2}{Vs}$

$$\rightarrow D_h = 1.3\ 10^{-3} \frac{m^2}{s}$$

$$L_h = \sqrt{D_h \tau_h} = 114\ \mu m \qquad (\tau_h = 10^{-5} s \text{ from (b)})$$

$$\tau_{transit} = \frac{W_b^2}{D_h} = 3\ 10^{-9} s$$

First part usually answered well, many candidates did not calculate transit time

(ii) Baso current given by small amount of holes that recombine, whereas collector current given by holos swept across. Small base current is maintaining charge neutrality and thus contolling larger collector current.

$\beta = \frac{I_C}{I_B}$ can be reflected by ratio of hole recombination and hole transit time

$$\rightarrow \beta = \frac{\tau_h}{\tau_T} = 3333$$

Explanation of amplification usually answered well, but not many offered quantitative answer.

(iii) Equilibrium hole concentration from law of mass action

$N_D = 10^{22} m^{-3}$, $n_i = 1.5\ 10^{16} m^{-3}$

$$p_{no} = \frac{n_i^2}{N_D} = 2.25\ 10^{10} m^{-3}$$

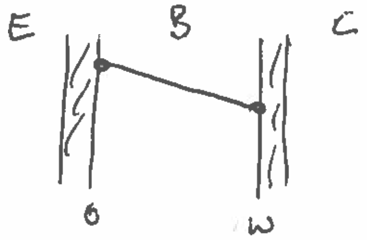
Injection dominated by Boltzmann term

$$\Delta p_n(0) = p_{no} \exp\left(\frac{eV_{EB}}{kT}\right) = 5.6\ 10^{18} m^{-3}$$

position: edge of emitter depletion region

Candidates who attempted this part usually answered well.

Q4(c)(iv)



Note $L_n \gg W_b$

Injection from both sides,

minimal recombination.

Some candidates confused with this part, reflected by large range of different sketches, but not many recognising that both junctions are in forward bias

(v) Upper frequency limit dictated by hole transit time across base. Can be improved by introducing doping gradient and thus additional drift across base. W_b given, but reduction also lowers transit time.

Many candidates confused this with heterostructured emitter junction improvement, and did not connect to transit time argument here