EGT2
ENGINEERING TRIPOS PART IIA

Monday 24 April 2023 9.30 to 11.10

Module 3B5

SEMICONDUCTOR ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Attachment: Sheet of Formulae and Constants (2 pages)

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Consider a two-dimensional potential well with sides of length L_x and L_y. The potential inside the well is given by $V(x, y) = 0$. Electrons of mass m exist within the well.

(i) Show that $\psi(x, y) = A\mathrm{e}^{(j(k_x x + k_y y))}$ is a valid solution to the time independent Schrödinger equation and derive an expression relating the wavenumber $k = \sqrt{k_x^2 + k_y^2}$ to the total energy of the electron, E. Plot the E–k energy band diagram for the electrons. [15%]

(ii) The boundary conditions are chosen to be $\psi(x, y) = \psi(x+L_x, y) = \psi(x, y+L_y)$. Derive expressions for the allowed values of k_x and k_y in terms of L_x and L_y. Explain why boundary conditions such as these are appropriate for a free-electron model. [15%]

(iii) Within two-dimensional **k**-space, an infinitesimally thin annulus of thickness dk positioned at a wavevector k contains an area $V_k = 2\pi k \mathrm{d}k$, as shown in Fig. 1. Show that the density of states $g_k(k)\mathrm{d}k$ within an annulus of thickness dk is given by

$$g_k(k)\mathrm{d}k = \frac{L_x L_y k}{\pi}\mathrm{d}k.$$

[15%]

(iv) Find an expression for the density of states $g(E)\mathrm{d}E$ as a function of energy. [15%]

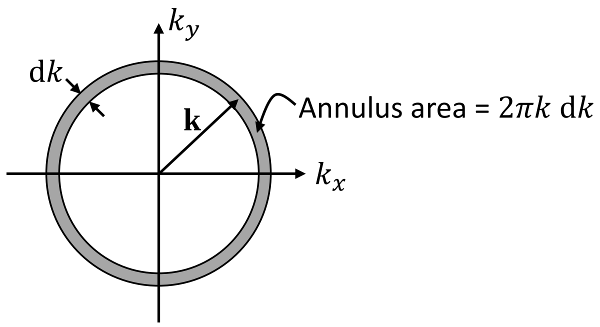

Fig. 1

(b) Consider a two-dimensional crystal with an internal periodic potential with a period given by the atomic lattice spacing a in both x and y directions. The crystal has sides of length L_x and L_y, where $L_x, L_y \gg a$.

(i) Letting $k_y = 0$, sketch the E–k_x energy band diagram according to the nearly-free electron approximation for the first three Brillouin zones. [15%]

 (cont.

(ii) Explain why it is conventional for a band diagram to show only the first Brillouin zone. [10%]

(iii) For this crystal, the density of states in the conduction band is given by

$$g(E)\mathrm{d}E = \begin{cases} 0 & E < E_C \\ BL_xL_y\mathrm{d}E & E \geq E_C \end{cases},$$

where E_C is the energy at the conduction band minimum and B is a constant. Show that if $E_C - E_F \gg k_BT$, the free electron concentration, n, is given by

$$n = N_{C,2D} \exp\left(\frac{E_F - E_C}{k_BT}\right),$$

where $N_{C,2D}$ is the effective density of states in the conduction band and E_F is the Fermi level. Derive an expression for $N_{C,2D}$. What are the units of measurement for $N_{C,2D}$? [15%]

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2 (a) InP is used as channel material to fabricate a Metal-Semiconductor Field-Effect Transistor (MESFET). InP has a band gap of 1.35 eV, an electron affinity of 4.38 eV, and an effective density of states in the valence band of 10^{25} m^{-3}. Assume room temperature (300 K) operation.

(i) A p-type doping density of $N_A = 10^{21}$ m^{-3} is desired. Give a suitable element that can act as acceptor and indicate the substitutional lattice site it should take. Calculate the shift in Fermi level this doping causes with respect to intrinsic InP. State all assumptions made. [15%]

(ii) Assume ideal work functions Φ for the following metals: Φ(Au) = 5 eV, Φ(Ti) = 4.3 eV, Φ(Pd) = 5.5 eV. Discuss which of these metals would be suitable to form the MESFET gate with the p-doped InP. Sketch a band diagram of the resulting unbiased Gate-Channel cross-section. [15%]

(iii) Explain how the same metal could be used to form Source and Drain contacts. Sketch the MESFET device and graphically explain the term pinch-off voltage. [15%]

(iv) The drain-source current I_{DS} is found to saturate without pinch-off of the channel. Explain this behaviour. Sketch the output and transfer characteristics of this MESFET. [20%]

(b) InGaAs can support very high electron mobilities and can be grown epitaxially on InP support. For such a High-Electron-Mobility Transistor (HEMT) design an InAlAs buffer layer is used, as shown in Fig. 2. The bandgap of InAlAs is 1.5 eV and that of InGaAs is 0.8 eV. InAlAs/InGaAs form a type II heterojunction, with the conduction band offset being approximately $\frac{2}{3}$ of the band gap difference.

(i) Explain the advantage of transfer doping, i.e. why the InAlAs layer is doped but not the InGaAs channel in this HEMT design. [10%]

(ii) Draw a band diagram of the heterojunction between the n-doped InAlAs and InGaAs layer. Indicate where a so called two-dimensional electron gas (2-DEG) is formed. Explain the meaning of this term and outline how this can lead to quantised electron levels. Estimate the ground state energy of such a 2-DEG assuming its width is 0.5 nm. [25%]

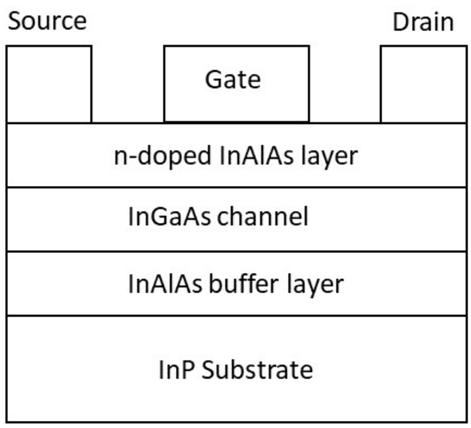

Fig. 2

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3 (a) An n-type Si sample is measured at 300 K in a Hall effect set-up, as illustrated in Fig. 3. The sample under test has thickness t, length l and width w. An electric field E_x establishes a current of density J_x flowing in the positive x-direction. The magnetic flux density B_z is applied in the positive z-direction. The Hall voltage V_H is defined as illustrated in Fig. 3. The sample has a donor density of $N_D = 4.5 \times 10^{21}$ m^{-3}. In Si at 300 K, the effective mass of electrons is 0.36 m$_e$, the bandgap is 1.12 eV and the intrinsic carrier density is $n_i = 1.0 \times 10^{16}$ m^{-3}.

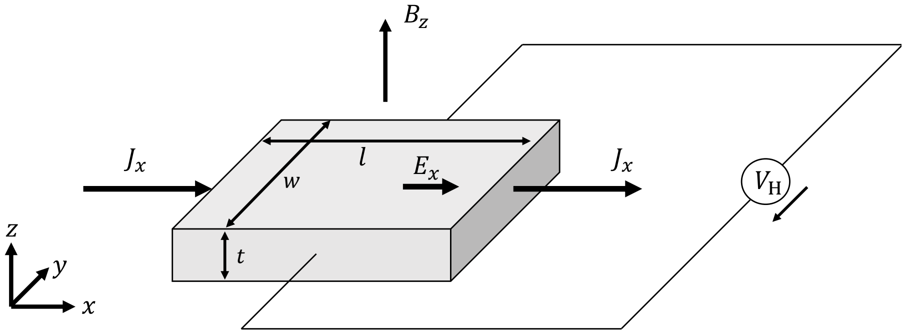

Fig. 3

(i) Calculate the energy of the Fermi level, E_F, with respect to the valence band minimum, E_V, in the n-type Si. State all assumptions made. [15%]

(ii) Calculate the mobility of electrons in the sample given that the magnitude of the applied electric field is $E_x = 0.64$ Vm^{-1} and the current density is $J_x = 40$ Am^{-2}. Calculate the average time between electronic scattering events. State all assumptions made. [20%]

(iii) Derive an expression for the magnitude of the Lorentz force experienced by the electrons in terms of J_x and B_z. What is the polarity of the resulting Hall voltage V_H, considering its defined direction in Fig. 3 ? [15%]

(iv) The Hall coefficient is defined as

$$R_H = \frac{V_H}{w J_x B_z}.$$

Find the relationship between the Hall coefficient R_H and the density of free electrons, n. Calculate the Hall voltage if $w = 1.0$ mm, the current density $J_x = 40$ Am^{-2} and the magnetic flux density $B_z = 0.2$ T. [15%]

(b) Fig. 4 shows an Arrhenius plot of the conductivity $\sigma(T)$ of three different samples, A, B and C, as a function of temperature, T. The conductivity is expressed as a ratio relative to the conductivity at 300 K. Identify each material as insulator, semiconductor or metal and sketch the corresponding band diagram for each material. Compare the sizes of the bandgaps. Comment on the presence of dopants and the position of any dopant energy levels relative to the bands. [20%]

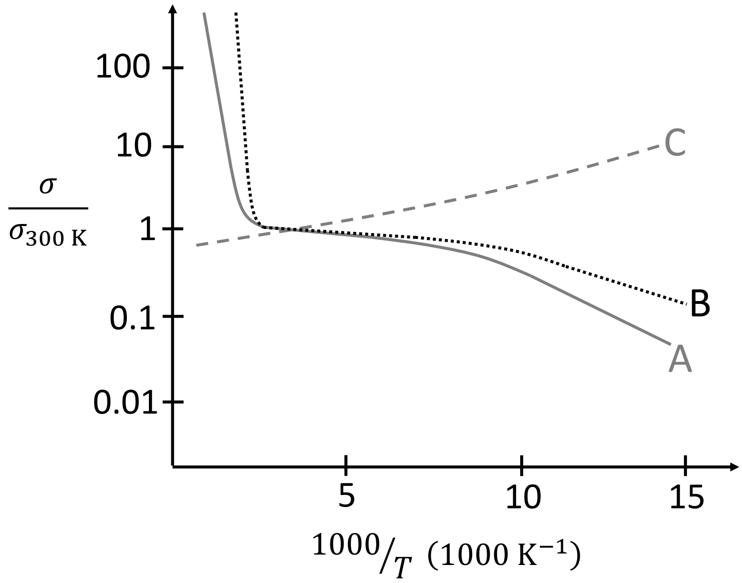

Fig. 4

(c) An electron and a photon have the same energy. At what value of this energy will their respective wavelengths be equal? [15%]

4 (a) Outline an experiment that allows the measurement of the diffusion coefficient and lifetime of holes in a n-type semiconductor. [10%]

(b) A bar made of Si with a doping density of $N_D = 10^{22}\ \mathrm{m}^{-3}$ has a length of 0.03 m and cross-sectional area of $5 \times 10^{-6}\ \mathrm{m}^2$. Fig. 5 shows the carrier mobilities in Si with respect to total impurity concentration.

(i) Calculate the current for a voltage of 10 V applied across the length of the bar. [10%]

(ii) The Si bar is now continuously illuminated, which uniformly creates 10^{26} electron hole pairs per second per m^{-3}. What is the new current at 10 V bias, assuming a life time of excess electrons and holes of 10^{-5} s ? State all assumptions made. [15%]

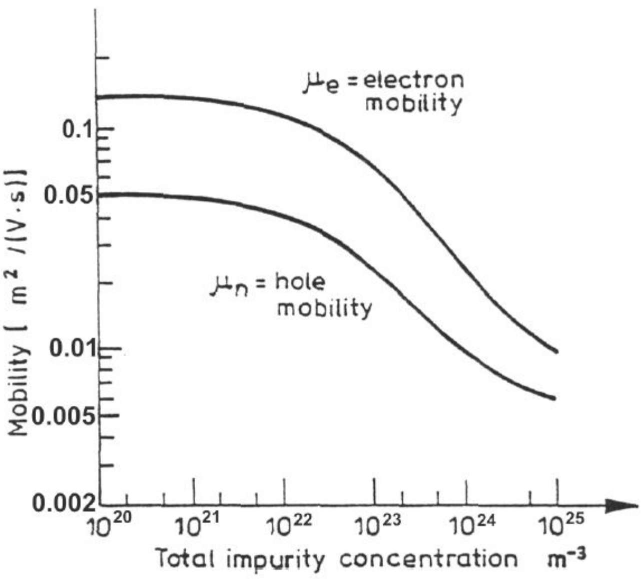

Fig. 5

(c) The same n-doped Si is used to form the base region of a pnp Bipolar Junction Transistor (BJT). The undepleted base width is 2 μm. Assume room temperature (300 K) operation and an intrinsic carrier concentration of $1.5 \times 10^{16}\ \mathrm{m}^{-3}$.

(i) Base transport is dominated by diffusion. Calculate the hole diffusion length L_h in the base region and the transit time of holes across the base. [15%]

(ii) Outline how amplification is achieved in such a BJT. Estimate the base-to-collector current amplification factor β from the time characteristics of hole transport. [15%]

(iii) A 0.5 V forward bias is applied to the emitter-base junction. Calculate the excess hole concentration injected into the base at the edge of the emitter depletion region. State all assumptions made. [15%]

(iv) Sketch the distribution of excess holes across the base assuming that both the emitter-base and base-collector junctions are forward biased. [10%]

(v) Outline what dictates the upper frequency limit of such BJT. How can this be improved? [10%]

END OF PAPER

Numerical Answers: 2(a)(i) 0.437 eV, (b)(ii) 0.3 eV; 3(a)(i) 0.71 eV, (a)(ii) 0.087 $m^2V^{-1}s^{-1}$, 177 fs, (a)(iv) 11 μV, (c) 1.022×10^6 eV; 4(b)(i) 0.32 A, (b)(ii) 0.37 A, (c)(i) 1.3×10^{-3} m^2s^{-1}, 3 ns, (c)(ii) 3333, (c)(iii) 5.6×10^{18} m^{-3}

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3B5 Semiconductor Engineering: Sheet of Formulae and Constants

Planck's constant	$h = 6.626 \times 10^{-34}$ Js
Reduced Planck's constant	$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}$ Js
Speed of light in a vacuum	$c = 2.998 \times 10^{8}$ m/s
Mass of electron	$m_\mathrm{e} = 9.109 \times 10^{-31}$ kg
Charge of electron	$-e = -1.602 \times 10^{-19}$ C
Permittivity of free space	$\epsilon_0 = 8.854 \times 10^{-12}$ C^2/(Jm)
Boltzmann constant	$k_\mathrm{B} = 1.381 \times 10^{-23}$ J/K
de Broglie relation	$p = \dfrac{h}{\lambda}$
Planck's equation	$E = \hbar\omega$
Bragg's law	$n\lambda = 2d\sin\theta$
Heisenberg's uncertainty principle	$\Delta x \Delta p \geq \dfrac{\hbar}{2}$
Time-dependent Schrödinger equation	$-\dfrac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + V\psi(\mathbf{r},t) = j\hbar\dfrac{\partial}{\partial t}\psi(\mathbf{r},t)$
Time-independent Schrödinger equation	$-\dfrac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r}) + V\Psi(\mathbf{r}) = E\Psi(\mathbf{r})$
Laplacian in Cartesian coordinates	$\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$
Laplacian in spherical polar coordinates	$\nabla^2 = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial}{\partial r}\right) + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial}{\partial\theta}\right) + \dfrac{1}{r^2\sin^2\theta}\dfrac{\partial^2}{\partial\phi^2}$
Density of states in three-dimensional free electron model	$g(E)\mathrm{d}E = \dfrac{V}{2\pi^2\hbar^3}(2m^*)^{\frac{3}{2}}E^{\frac{1}{2}}\mathrm{d}E$
Fermi–Dirac function	$f(E) = \dfrac{1}{\exp\left(\frac{E-E_F}{k_\mathrm{B}T}\right) + 1}$
Law of mass action	$np = n_i^2$

Density of electrons in conduction band	$n = N_C \exp\left(-\frac{E_C - E_F}{k_\mathrm{B}T}\right)$
Density of holes in valence band	$p = N_V \exp\left(\frac{E_V - E_F}{k_\mathrm{B}T}\right)$
Current density	$J = \sigma\varepsilon$
Conductivity	$\sigma = ne\mu_\mathrm{e} + pe\mu_\mathrm{h}$
Mobility	$\mu = \frac{q\tau_\mathrm{scatt}}{m^*}$
Continuity equation for electrons	$\frac{\partial(\Delta n)}{\partial t} = -\frac{\Delta n}{\tau_\mathrm{e}} + \mu_\mathrm{e}\varepsilon\frac{\partial(\Delta n)}{\partial x} + D_\mathrm{e}\frac{\partial^2(\Delta n)}{\partial x^2}$
Continuity equation for holes	$\frac{\partial(\Delta p)}{\partial t} = -\frac{\Delta p}{\tau_\mathrm{h}} - \mu_\mathrm{h}\varepsilon\frac{\partial(\Delta p)}{\partial x} + D_\mathrm{h}\frac{\partial^2(\Delta p)}{\partial x^2}$
Poisson equation	$\nabla^2 V = -\frac{\rho}{\epsilon_0\epsilon_r}$
Einstein's relation	$D = \mu\frac{k_\mathrm{B}T}{e}$
Diffusion length	$L = \sqrt{D\tau}$
Debye screening length	$L_\mathrm{D} = \sqrt{\frac{\epsilon_0\epsilon_r k_\mathrm{B}T}{e^2 n_0}}$