Version SH/5

EGT2
ENGINEERING TRIPOS PART IIA

Monday 22 April 2024 9.30 to 11.10

Module 3B5

SEMICONDUCTOR ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Attachment: Sheet of Formulae and Constants (2 pages)

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) A Si bar has its n-type doping concentration spatially varied as $N_D = N_0 \times \exp(-ax)$. Si has an intrinsic carrier concentration of $n_i = 1.0 \times 10^{16}$ m^{-3} and a band gap of 1.12 eV at room temperature. Draw a band diagram and indicate the Fermi level across the bar for the range where $N_D >> n_i$. Find an expression for the equilibrium built-in field $E(x)$ over that range. Hint: Use the balance of diffusion and drift currents in equilibrium and Einstein's relation. State all other assumptions. [20%]

(b) A so-called high-low junction is formed between two differently p-doped Si regions, with the doping density at one side being 2×10^{18} m^{-3} and 10^{20} m^{-3} on the other side. Calculate the position of the Fermi level with respect to mid gap on either side of the junction and sketch an equilibrium band diagram of the junction. Assume room temperature conditions (300 K). [15%]

(c) Sketch the I-V characteristics of a Si p-n junction that is illuminated by light with a photon energy of 2 eV. In which quadrant of the I-V curve can the device deliver power. [10%]

(d) A metal is evaporated onto Ge that is doped with an aluminium concentration of 10^{23} m^{-3}.

(i) What metal work function is required to form a Schottky contact? Assume that the Ge has a band gap of 0.66 eV, an electron affinity of 4.0 eV and effective densities of state in the conduction and valence bands of $N_C = 1.04 \times 10^{25}$ m^{-3} and $N_V = 6.0 \times 10^{24}$ m^{-3}, respectively. Assume room temperature and ideal junction conditions. State all other assumptions. [15%]

(ii) Sketch the variation of the electric field across such a Schottky contact with no external bias applied. Use the Poisson equation to derive an expression for the maximum electric field. State all assumptions. [20%]

(iii) It is found that the Schottky barrier height is determined to be 0.5 eV due to Fermi level pinning by interface states. Draw a band diagram of the junction and indicate the barrier. Another Schottky diode junction is formed using Ge with a higher aluminium doping concentration of 10^{25} m^{-3}. Discuss how the junction capacitances compare between the diodes. Which diode will show a more ideal I-V behaviour based on Thermionic Emission Theory? [20%]

2 (a) Sketch the $E - k$ curve for an electron in a 1-dimensional crystal according to the nearly-free electron approximation. Identify the 1st, 2nd and 3rd Brillouin Zones and explain the formation of the energy gaps in the framework of the nearly-free electron approximation. [15%]

(b) Derive the expression for the effective mass m^*:

$$m^* = \hbar^2 \left(\frac{d^2E}{dk^2} \right)^{-1}$$

[25%]

(c) For the 1st Brillouin Zone, in the nearly-free electron approximation, sketch the velocity-wavenumber and effective mass–wavenumber curves. [15%]

(d) In a semiconductor where the bottom of the conduction band is at $k = 0$, an electron moves with a wavenumber $k = 2 \times 10^9\ \mathrm{m}^{-1}$.

(i) If the effective mass of the electrons in the conduction band is $0.5\, m_e$, what is the energy of this electron measured from the bottom of the conduction band? [20%]

(ii) Electrons in the conduction band will occupy some states up to until about k_BT above the bottom of the conduction band. What is the maximum value of k that electrons would have from thermal fluctuations at 21 °C? What consequences does this have for the spectral emission characteristics of a light emitting diode based on such semiconductor? [25%]

3 (a) The time-independent Schrödinger equation in 1-dimension is given by:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$$

Identify each of the terms in this equation and explain its overall physical significance. [10%]

(b) A particle exists in a 1-dimensional, infinitely deep potential well, where $V = \infty$ for $x < 0, x > L$ and $V = 0$ for $0 \le x \le L$.

(i) Show that the wavefunction of the particle is given by:

$$\psi = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi n x}{L}\right)$$

Explain the meaning of n. [25%]

(ii) Derive an expression for the lowest possible energy that the particle can have. [25%]

(iii) If the particle is an electron and $L = 2$ nm, what is the minimum energy of this electron if the Heisenberg uncertainty principle is to be satisfied? How does this compare to the energy found using the expression derived in part (b)(ii)? Comment on the results. [20%]

(iv) Sketch the probability of finding the particle as a function of position for $0 \le x \le L$ for the two lowest energy states that the particle can exist in. Explain how this would be different if the particle was behaving classically. [20%]

4 (a) Fig. 1 shows the capacitance-voltage (C-V) characteristics of an ideal metal-oxide-semiconductor (MOS) capacitor formed on boron doped Si. Si has a band gap of 1.12 eV, an electron affinity of 4.05 eV, a relative dielectric constant of 11.9, and effective densities of state in the conduction and valence bands of $N_C = 2.8 \times 10^{25}\,\mathrm{m^{-3}}$ and $N_V = 1.04 \times 10^{25}\,\mathrm{m^{-3}}$, respectively. A metal with an effective work function of 5.12 eV is used. The maximum capacitance per unit area is measured as $C_{max} = 7 \times 10^{-4}\ \mathrm{F\,m^{-2}}$, and the minimum capacitance per unit area $C_{min} = 2 \times 10^{-4}\ \mathrm{F\,m^{-2}}$. The band diagram of the system exhibits no band bending at equilibrium, i.e., with no applied bias voltage. Assume room temperature conditions.

(i) Explain the characteristic shape of curve 1 in Fig. 1. Calculate the thickness of the insulating layer separating the doped Si and metal. [15%]

(ii) Calculate the density of active boron dopants. [15%]

(iii) Calculate the maximum width of the depletion area in the doped Si and draw a band diagram of the MOS capacitor at 0.8 V applied bias. [20%]

(iv) Discuss under which conditions curve 2 in Fig. 1 was measured for the same MOS capacitor. [10%]

(b) A MOS field-effect transistor (MOSFET) is fabricated using this MOS capacitor.

(i) Sketch the transfer characteristics of the most energy efficient MOSFET configuration. Discuss what is referred to as sub-threshold conduction. Estimate the fundamentally lowest supply voltage for which an on-off ratio of 4 decades can be achieved. [15%]

(ii) For real MOS systems deleterious interfacial charges have to be considered. Estimate the shift in threshold voltage for an effective interfacial charge density of $1.6 \times 10^{-8}\ \mathrm{C\,cm^{-2}}$. [15%]

(iii) Draw a band diagram of such real MOS capacitor with a bias applied that is opposite equal to the threshold voltage shift calculated in part (b)(ii). [10%]

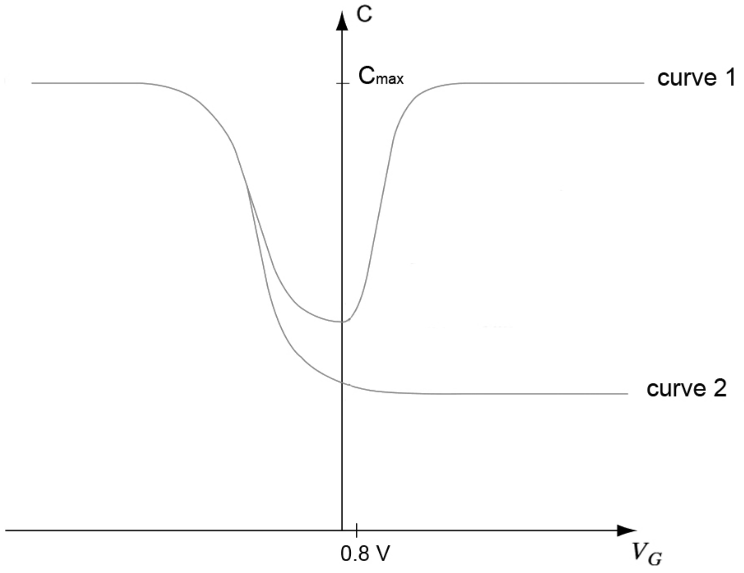

Fig. 1

END OF PAPER

Numerical answers: Q1 (b) 0.24eV/0.14eV, (d) (i) $\phi_{metal}<4.55$eV; Q2 (d)(i) 0.305eV, (d)(ii) 5.77×10^{8} m^{-1}; Q3(b)(iii) 2.38meV/93.95meV ; Q4 (a) (i) 48nm, (a)(ii) 1.5×10^{24} m^{-3}, (a)(iii) 3.8×10^{-7} m, (b)(i) 240mV, (b)(ii) -0.23V

3B5 Semiconductor Engineering: Sheet of Formulae and Constants

Planck's constant	$h = 6.626 \times 10^{-34}$ Js
Reduced Planck's constant	$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}$ Js
Speed of light in a vacuum	$c = 2.998 \times 10^{8}$ m/s
Mass of electron	$m_{\mathrm{e}} = 9.109 \times 10^{-31}$ kg
Charge of electron	$-e = -1.602 \times 10^{-19}$ C
Permittivity of free space	$\epsilon_0 = 8.854 \times 10^{-12}$ C^2/(Jm)
Boltzmann constant	$k_{\mathrm{B}} = 1.381 \times 10^{-23}$ J/K
de Broglie relation	$p = \dfrac{h}{\lambda}$
Planck's equation	$E = \hbar\omega$
Bragg's law	$n\lambda = 2d\sin\theta$
Heisenberg's uncertainty principle	$\Delta x \Delta p \geq \dfrac{\hbar}{2}$
Time-dependent Schrödinger equation	$-\dfrac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + V\psi(\mathbf{r},t) = j\hbar\dfrac{\partial}{\partial t}\psi(\mathbf{r},t)$
Time-independent Schrödinger equation	$-\dfrac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r}) + V\Psi(\mathbf{r}) = E\Psi(\mathbf{r})$
Laplacian in Cartesian coordinates	$\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$
Laplacian in spherical polar coordinates	$\nabla^2 = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial}{\partial r}\right) + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial}{\partial\theta}\right) + \dfrac{1}{r^2\sin^2\theta}\dfrac{\partial^2}{\partial\phi^2}$
Density of states in three-dimensional free electron model	$g(E)\mathrm{d}E = \dfrac{V}{2\pi^2\hbar^3}(2m^*)^{\frac{3}{2}}E^{\frac{1}{2}}\mathrm{d}E$
Fermi–Dirac function	$f(E) = \dfrac{1}{\exp\left(\frac{E-E_F}{k_{\mathrm{B}}T}\right) + 1}$
Law of mass action	$np = n_i^2$

Density of electrons in conduction band	$n = N_C \exp\left(-\frac{E_C - E_F}{k_\mathrm{B}T}\right)$
Density of holes in valence band	$p = N_V \exp\left(\frac{E_V - E_F}{k_\mathrm{B}T}\right)$
Current density	$J = \sigma\varepsilon$
Conductivity	$\sigma = ne\mu_\mathrm{e} + pe\mu_\mathrm{h}$
Mobility	$\mu = \frac{q\tau_\mathrm{scatt}}{m^*}$
Continuity equation for electrons	$\frac{\partial(\Delta n)}{\partial t} = -\frac{\Delta n}{\tau_\mathrm{e}} + \mu_\mathrm{e}\varepsilon\frac{\partial(\Delta n)}{\partial x} + D_\mathrm{e}\frac{\partial^2(\Delta n)}{\partial x^2}$
Continuity equation for holes	$\frac{\partial(\Delta p)}{\partial t} = -\frac{\Delta p}{\tau_\mathrm{h}} - \mu_\mathrm{h}\varepsilon\frac{\partial(\Delta p)}{\partial x} + D_\mathrm{h}\frac{\partial^2(\Delta p)}{\partial x^2}$
Poisson equation	$\nabla^2 V = -\frac{\rho}{\epsilon_0\epsilon_r}$
Einstein's relation	$D = \mu\frac{k_\mathrm{B}T}{e}$
Diffusion length	$L = \sqrt{D\tau}$
Debye screening length	$L_\mathrm{D} = \sqrt{\frac{\epsilon_0\epsilon_r k_\mathrm{B}T}{e^2 n_0}}$