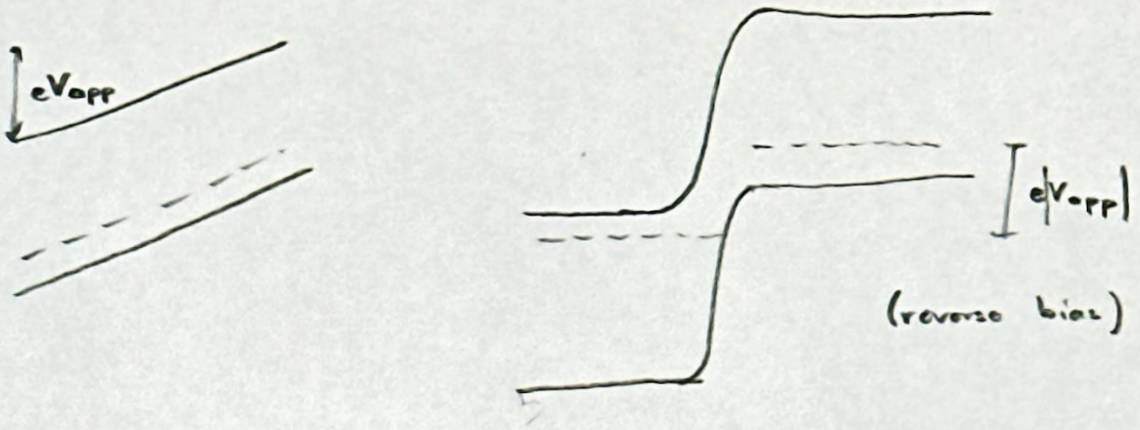
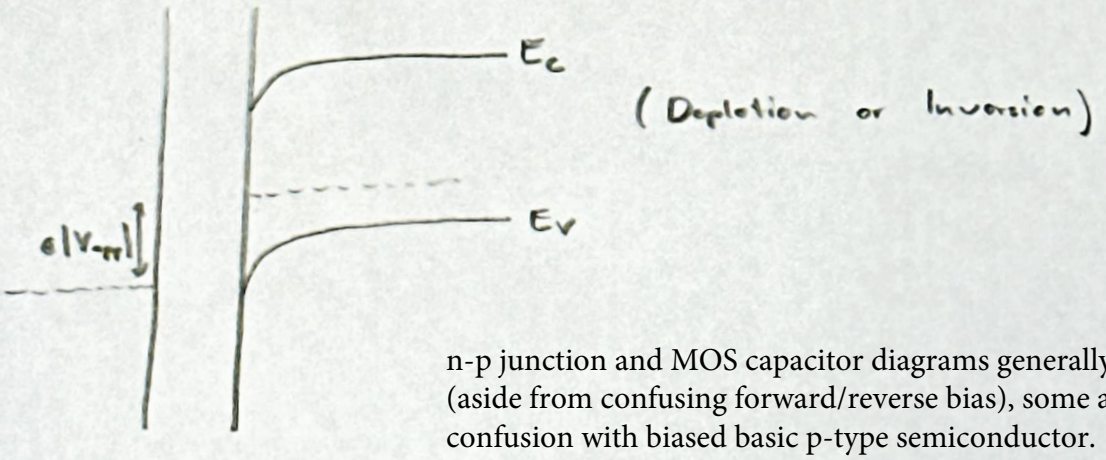
Q1 (a)





n-p junction and MOS capacitor diagrams generally well-drawn (aside from confusing forward/reverse bias), some amount of confusion with biased basic p-type semiconductor.

(b) Hall effect: have Hall bar arrangement and apply magnetic field perpendicular to current flow. This will lead to build-up of electric field and Hall voltage

$$V_H = E_y w \qquad (w = \text{width})$$

with Hall coefficient $R_H = \frac{1}{pe}$

measure conductivity σ to get ρ:

$$\mu_h = \sigma \cdot R_H \qquad (\text{majority p-type})$$

get diffusivity via Einstein relationship

$$D_h = \mu_h \cdot \left(\frac{kT}{e}\right)$$

Many good answers, recognising that its majority carriers so Hall effect can be used. Some confusion with Haynes-Shockley experiment (minority carriers), with some candidates dropping lecture note content without adapting to question.

Q1 (c) Junction capacitance $C_d = \left| \frac{dQ}{dV} \right|$

take n-side :

$$Q = eN_D x_n = \left(2\varepsilon_0 \varepsilon_r e(V_0 - V_{app})\right)^{1/2} \left(\frac{N_A N_D}{N_A + N_D}\right)^{1/2}$$

$$\rightarrow C_d = \frac{dQ}{dV} = \sqrt{\frac{\varepsilon_0 \varepsilon_r N_A N_D}{2(V_0 - V_{app})(N_A + N_D)}}$$

For a one-sided junction eg p^+n with $N_A \gg N_D$

this simplifies to

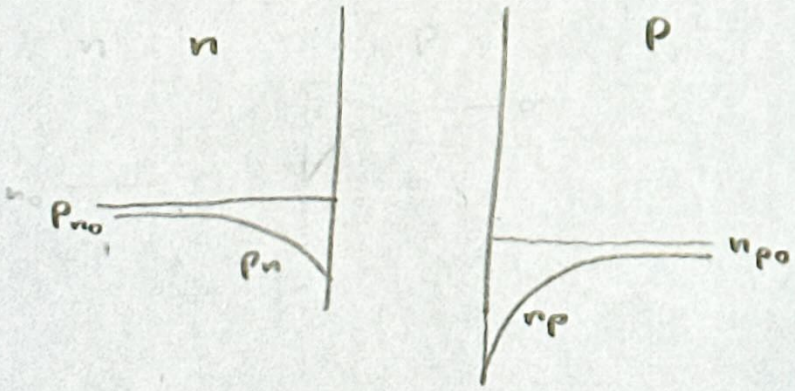
$$C_d = \sqrt{\frac{\varepsilon_0 \varepsilon_r N_D}{2(V_0 - V_{app})}}$$

so N_D (the lower doping density) can be extrapolated from C-V measurement and getting the slope of a C_d vs $(V_0 - V_{app})^{-1/2}$ plot.

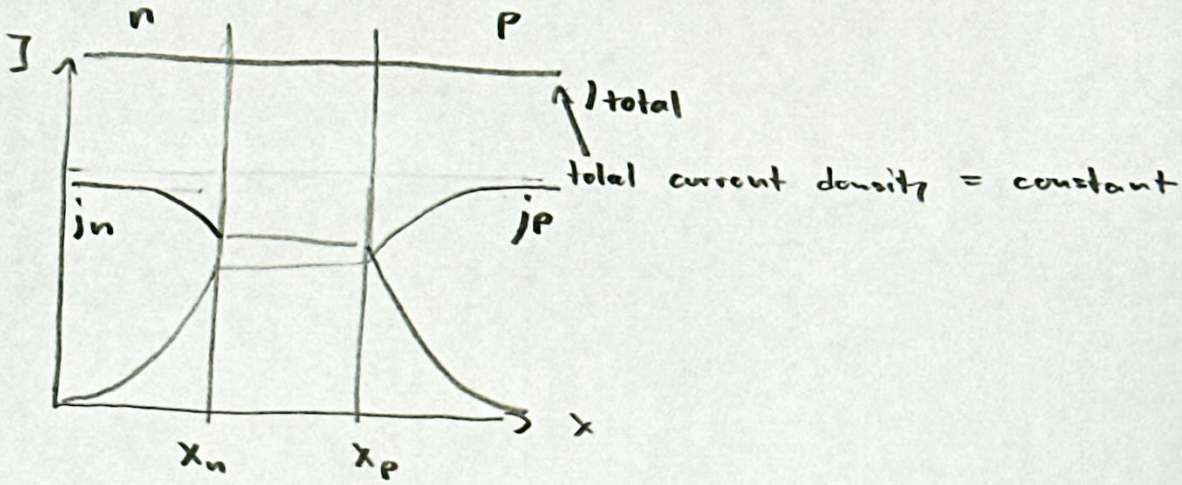
This requires ability to dope semiconductor both n- and p-type.

Rather than using C=dQ/dV, many candidates went to calculate xp and total extend of depletion region. Second part, how active doping density can be extrapolated, often lacked basic details.

(d) Reverse bias, this means extraction of minority carriers

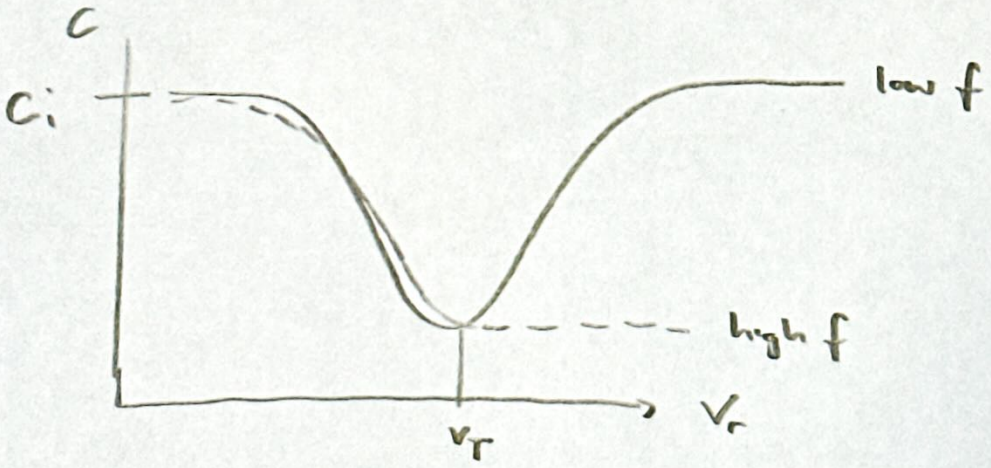


Q1 (d) hole and electron densities across n-p junction



Some confusion on forward/reverse bias, and what plots were asked for.

(e) MOS capacitor – p-type semiconductor



takes time to get inverted carriers to interface. If switching too fast, then no inverted carriers, hence deep depletion.

Very well answered, with many details.

Q1 f) Minimum MOS capacitance

$$C_{min} = \frac{C_i C_{dmin}}{C_i + C_{dmin}}$$ where C_i oxide capacitance

and C_d capacitance due to depletion at MOS interface, which is minimal at maximum depletion w_{max}

$$C_{dmin} = \frac{\varepsilon_0 \varepsilon_r}{w_{max}}$$ where w_{max} depends on doping density.

Thus in principle from measuring C_{min}, the active doping density can be extrapolated.

Well answered, with candidates showing good understanding.

Q2 (a)

(i) Find the value of r for which $P(r)$ is a maximum [25%]

$dP=4\pi r^2|\psi(r)|^2dr$

$P(r)=4\pi|A|^2r^2e^{-2ar}$

$dP/dr=4\pi|A|^2(2re^{-2ar}-2ar^2e^{-2ar})=0$

$r=1/a$

(ii) Find the value of the normalization constant A [25%]

$$\int_0^\infty dP = 4\pi|A|^2\int_0^\infty e^{-2ar}r^2dr = 1$$

$\Rightarrow A=(a^3/\pi)^{1/2}$

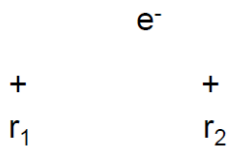
Generally OK. Many mistakes in a(i) and (ii) due to either maths errors or forgetting 4π.

(b)

(i) By using a symmetry argument, derive the expressions for ψ_1 and ψ_2. [25%]

When the two atoms are identical, this gives rise to *covalent bonding.*

Let us consider the case of a ionized hydrogen molecule, H_2^+, i.e. a H_2 molecule that has lost one electron.



What is the wavefunction for an electron subject to the potential generated by two protons?

Approximation: Linear combination of atomic wavefunctions
(aka, LCAO, Linear Combination of Atomic Orbitals)

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = A\psi_1(\mathbf{r} - \mathbf{r}_1) + B\psi_2(\mathbf{r} - \mathbf{r}_1)$$

where ψ_1 and ψ_2 are the 1s wavefunctions of the isolated atoms, centred in $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively

What values for A and B?
We use the symmetry of the problem: if we interchange proton 1 and proton 2, the solution must not change.

So, if we write:

$$\psi(\mathbf{r}_2, \mathbf{r}_1) = Ae^{-a|\mathbf{r}-\mathbf{r}_2|} + Be^{-a|\mathbf{r}-\mathbf{r}_1|}$$

We must have:

$$|\psi(\mathbf{r}_1, \mathbf{r}_2)|^2 = |\psi(\mathbf{r}_2, \mathbf{r}_1)|^2$$

This requires

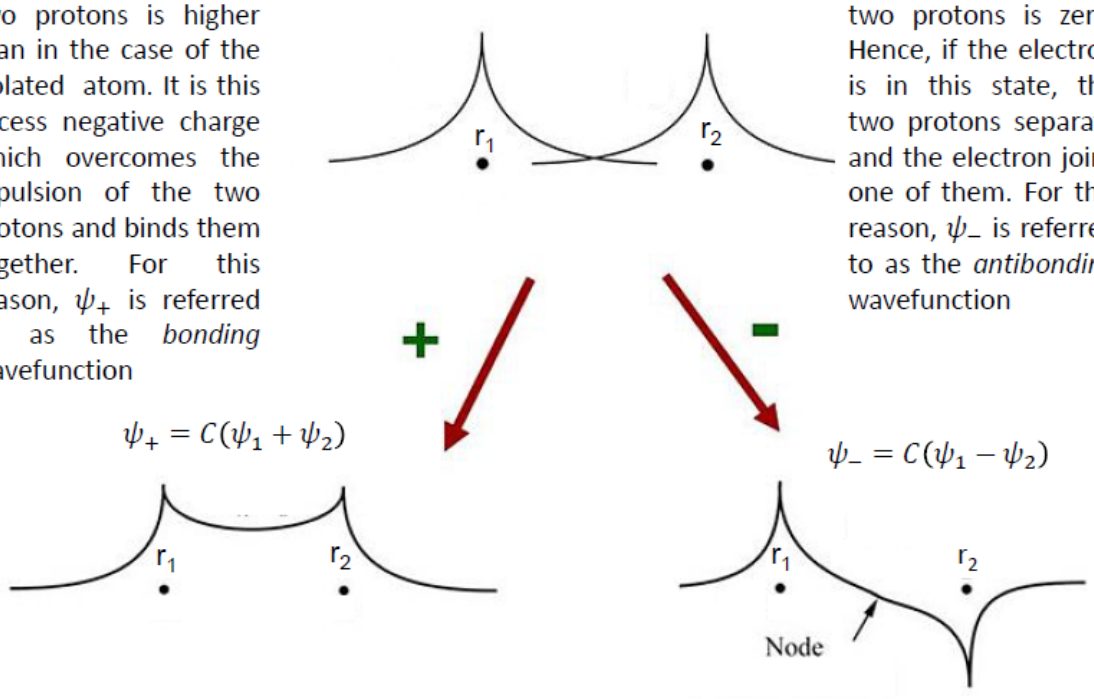
A=± B

Thus, we have 2 wavefunctions as solution:

$$\psi_+ = C(\psi_1 + \psi_2)$$
$$\psi_- = C(\psi_1 - \psi_2)$$

(ii) Sketch the electronic charge, corresponding to ψ_1 and ψ_2, as a function of x, the straight line through the centres of the two protons. With reference to your graph, explain the meaning of *bonding* and *anti-bonding states.* [25%]

The negative charge midway between the two protons is higher than in the case of the isolated atom. It is this excess negative charge which overcomes the repulsion of the two protons and binds them together. For this reason, ψ_+ is referred to as the *bonding* wavefunction



The charge density midway between the two protons is zero. Hence, if the electron is in this state, the two protons separate and the electron joins one of them. For this reason, ψ_- is referred to as the *antibonding* wavefunction

The energy corresponding to the *antibonding* wavefunction is higher than that of the *bonding* wavefunction

b(i), (ii), mostly OK, save for those students who clearly did not remember any of the premises.

Q3

(a) Explain how the Hall Effect can be used to determine both the sign and density of the majority carriers. [20%]

The Hall Effect can be used to determine both the sign and number density of majority carriers in a semiconductor by applying a magnetic field, B, to a semiconductor sample in a perpendicular direction to the flow of a current that is driven through the material. The movement of carriers due to the current will result in the carriers experiencing a force due to the magnetic field. This force will be perpendicular to each of the current flow direction and the magnetic field. As a result, majority carriers will be deflected towards one side of the semiconductor, and an electric field will result. An equilibrium will be reached when the force on the carriers due to the magnetic field is balanced by the force due to the electric field. This will result in a constant Hall voltage, V, being developed. The sign of the Hall voltage indicates the sign of the charge of the majority carriers, and hence whether they are electrons or holes. Its magnitude allows the carrier concentration to be determined.

Generally well answered.

(b) Assuming that the current flows uniformly through the cross-section of the sample in the y-z plane, show that the measured voltage, V_H, identified in Fig. 1 is given by: $V_H=IB/qt$, where B is the magnetic flux density and q is the charge density of majority carriers in the semiconductor.

[25%]

The force, F_B, acting on a carrier moving with velocity, v, due to the magnetic field will be:

$F_B=evB$

The force, F_E, due to the electric field is

$F_E=eE=eV_H/w$

At equilibrium, these two forces are equal:

$evB=eV_H/w$

$\Rightarrow V_H=vBw$

The current, I, is related to v by: $I=qvwt$

$\Rightarrow V_H=IB/qt$

Generally well answered.

(c) The sample is doped with a number density of $5\text{x}10^{22}\text{m}^{-3}$ of boron atoms. It has dimensions of w = 2 mm, t = 2 μm and l = 2 mm. What V_H is measured when I= 150 nA and B = 0.2 T? State any assumptions made. [25%]

We assume all acceptors are ionised. Boron ia a p-type dopant in Si, so holes will be the majority carriers. Thus p is equal to the doping density of boron atoms. Then:

$q=pe=5\text{x}10^{22}\text{m}^{-3}\ 1.602\ \text{x}\ 10^{-19}\ \text{C} = 8.01\ \text{x}\ 10^{3}\ \text{C m}^{-3}$

$V_H = IB/qt$= 150 10^{-9}C/s x 0.2 T/ (8.01 x 10^3 C m^{-3} x 2 x 10^{-6} m)= 1.87 μ V

Generally well answered

(d) Calculate the total conductivity of the silicon sample and the ratio of the conductivity due to electrons with respect to holes. [15%]

The number density of electrons can be determined as the number density of holes is known from the law of mass action.

$n = n^2_i/p = (10^{16})^2/5 \times 10^{22} = 2 \times 10^9\ m^{-3}$

Therefore the conductivity due to electrons is:

$\sigma_e = ne\mu_e = 2 \times 10^9 \times 1.602 \times 10^{-19} \times 0.14 = 4.49 \times 10^{-11}\ \Omega^{-1}m^{-1}$

while that due to holes is:

$\sigma_h = pe\mu_h = 5 \times 10^{22} \times 1.602 \times 10^{-19} \times 0.048 = 4.49 \times 10^{-11}\ \Omega^{-1}m^{-1} = 385\ \Omega^{-1}m^{-1}$

Therefore the total conductivity is essentially the same as that of holes alone and the ratio of electron conductivity to that of the holes is: 1.17 x 10^{-13}

How do the answers to change if the number density of boron atoms is increased to 5x 10^{24} m^{-3}? [15%]

If the number density of Boron atoms was increased by a factor 100, then the ratio of electrons to holes would change by $100^2 = 10^4$. One might expect the conductivity to increase by a factor 100. However, doping densities over $10^{23}\ m^{-3}$ are very high, and impurity scattering will reduce mobility. Hence the increase in conductivity will be less than 100, and may be as little as 10.

Generally OK, but very few people understood that an increase of a factor 100 in B atoms would also bring about a significant decrease in mobility, hence the simplistic approach of simply scaling the results with B atoms was not the correct one.

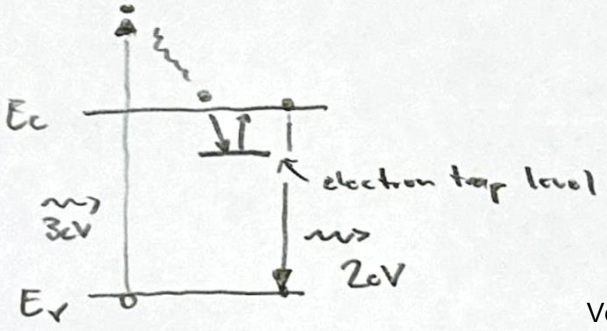
Some evidence of candidates running out of time, reflected by incomplete parts.

Q4(a)(i) 3eV photon absorption will create electron – hole pair. Excited electron will give up energy to lattice until it reaches bottom of conduction band. Direct recombination occurs with the hole, emitting a photon with energy of the band gap.

For photons $E = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{E} = 620\ nm$

Many candidates mistook photon energy as 1eV, or got confused with photoelectric effect.

(ii) So called phosphorescence can occur by the introduction of defects, such as an electron trap. The delay time between excitation and recombination then depends on the trapping probability and thermal re-excitation. If trapping probability is larger than probability for recombination, an electron might get trapped several times before recombining.



Very scattered responses, many speculative, and bringing up heterostructures and confusing with some concepts from 3B6.

(b) (i) To doping makes GaAs n-type.

Built-in potential $V_o = \frac{\phi_M - \phi_{SC}}{e}$

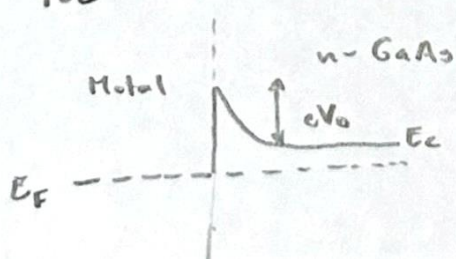
Need to calculate position of Fermi level for n-doped GaAs.

Assume all donors are ionised, $n = N_D$

$$n = N_D = N_C \exp\left(\frac{E_F - E_C}{kT}\right)$$

$$\rightarrow E_F - E_C = kT \ln\left(\frac{N_D}{N_C}\right) = 0.026eV \cdot \ln\left(\frac{10^{22}}{4.7\,10^{23}}\right) = -0.1eV$$

$$\rightarrow \phi_{SC} = 4.07eV + 0.1eV = 4.17eV$$



$\rightarrow V_o = 0.83\ V$

Generally well answered, some answers assumed p-type doping.

Q4 (b) (ii) Poisson equation (10)

$$\frac{d^2V}{dx^2} = -\frac{eN_D}{\varepsilon_0 \varepsilon_r}$$

Assume no fields outside depletion region, ie $\mathcal{E} = 0$ at $x = w$

$$-\mathcal{E} = \frac{dV}{dx} = \frac{eN_D(w-x)}{\varepsilon_0 \varepsilon_r}$$

Integrate again with $V = 0$ at $x = 0$

$$V = \frac{eN_D}{2\varepsilon_0 \varepsilon_r}(2wx - x^2)$$

V_0 is V at $x = w$:

$$V_0 = \frac{eN_D w^2}{2\varepsilon_0 \varepsilon_r}$$

$$\rightarrow w = \sqrt{\frac{2\varepsilon_0 \varepsilon_r V_0}{eN_D}} \quad \text{use} \quad V_0 = \frac{\phi_M - \phi_{SC}}{e}$$

$$= \sqrt{\frac{2\varepsilon_0 \varepsilon_r (\phi_M - \phi_{SC})}{e^2 N_D}}$$

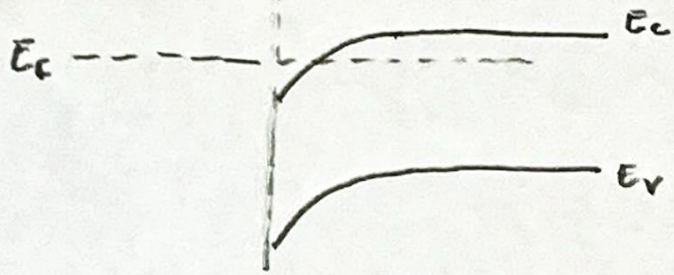
$$= \sqrt{\frac{2 \cdot \varepsilon_0 \, 13.1 \, (0.83_{01})}{e^{0} \, 10^{22} \, m^{-3}}} = 347 \text{ nm}$$

Basic integration generally well done, but many answers getting constants wrong or not calculating width.

(iii) Rather than following the so-called Anderson model, the Schottky barrier is dictated by Fermi level by interface states at the metal-sc interface. The barrier height is thus determined by this pinning, independent of the metal WF.

Many speculative and confused answers.

Q4 (c) Metal | n-InAs

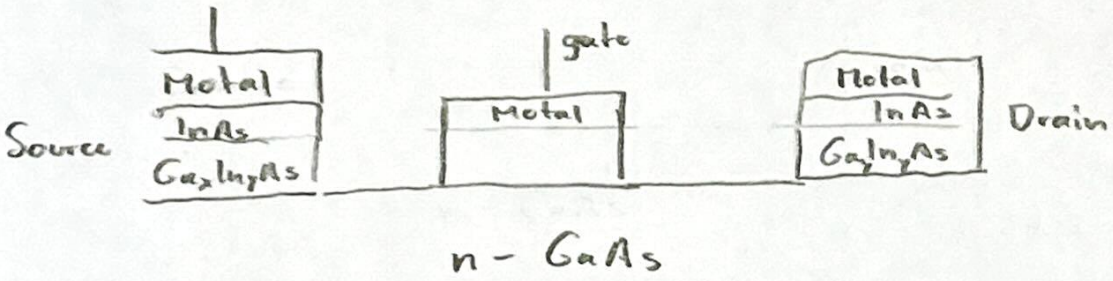


This results in an SZ-contact for virtually any metal.

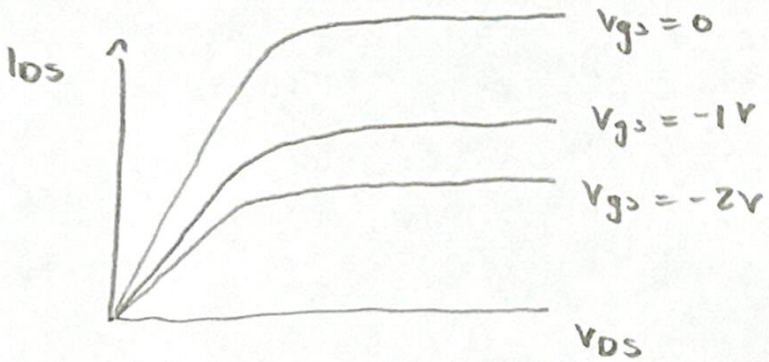
Well answered; some drawings had Fermi level not constant or InAs not as n-type.

(d) MESFET needs SZ contacts at source and drain, and Schottky contact at gate.

Use InAs surface to get SZ-contacts, and (any) metal directly on n-GaAs to get Schottky contact



Output characteristics for n-channel MESFET



Most answers lacked details (some due to running out of time), particularly regarding actual proposed MESFET structure and bias details of output characteristics. Many rough sketches without proper labelling.