Version SH/2

EGT2
ENGINEERING TRIPOS PART IIA

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Monday 28 April 2025    9.30 to 11.10

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**Module 3B5**

**SEMICONDUCTOR ENGINEERING**

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Engineering Data Book
Attachment: Sheet of Formulae and Constants (2 pages)

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

**You may not remove any stationery from the Examination Room.**

1 (a) Figure 1 shows band diagrams for a p-type semiconductor, a n-p junction and a metal-semiconductor-oxide (MOS) capacitor. Redraw each of the band diagrams with a positive voltage bias applied to the left-hand side of the respective material or junction. Indicate the external voltage drop(s) in each case. [15%]

(b) Outline an experiment that can show that a semiconductor is p-type and allow the extrapolation of its hole diffusivity. [15%]

(c) The extent $x_n$ of the depletion region for the n-side of an unbiased p-n junction is given by

$$x_n = \sqrt{\frac{2\epsilon_0\epsilon_r V_0}{e}}\sqrt{\frac{N_A}{N_A N_D + N_D^2}}$$

where $V_0$ is the built-in potential, and $N_A$ and $N_D$ are the acceptor and donor densities, respectively. Derive an expression for the junction capacitance for the biased n-p junction in part (a). Discuss how the active doping density of a semiconductor can be extrapolated via C-V measurements of a p-n junction formed from it. State all requirements. [20%]

(d) For the biased n-p junction in part (a), sketch the variation of minority carrier concentrations across the junction. Explain and sketch the resulting variation of hole and electron current densities across the junction. [20%]

(e) Sketch the expected C-V characteristics for the MOS capacitor in Fig. 1. Explain why the measurements will be frequency dependent and sketch both low- and high-frequency behaviour. [20%]

(f) Outline how, in principle, the active doping density of the semiconductor can be extrapolated from C-V measurements of a MOS capacitor formed from it. [10%]

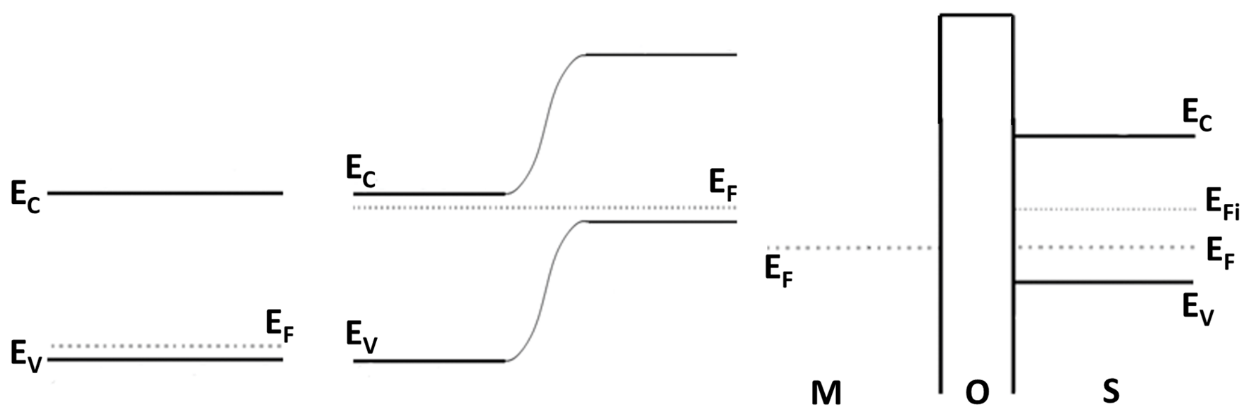


Fig. 1

2 (a) The wavefunction of an electron in the ls state of a hydrogen atom is given by $\psi(r) = A\exp(-ar)$, where $A$ is a normalization constant, $r$ is the distance from the centre of the nucleus and $a$ is a constant. The differential probability of finding the electron at a distance $r$ from the nucleus is given by $dP = P(r)dr$.

(i) Find the value of $r$ for which $P(r)$ has a maximum. [25%]

(ii) Find the value of the normalization constant $A$. [25%]

(b) In an ionized hydrogen molecule $H^{2+}$ a single electron is subject to the field of two protons at positions $r_1$ and $r_2$. The two lowest energy levels for the electrons are $E_1$ and $E_2$, with $E_1 < E_2$, and the corresponding wavefunctions are $\psi_1$ and $\psi_2$.

(i) By using a symmetry argument, derive the expressions for $\psi_1$ and $\psi_2$. [25%]

(ii) Sketch the distribution of electronic charge, corresponding to $\psi_1$ and $\psi_2$, as a function of $x$, whereby the x-axis is the straight line through the centres of the two protons. With reference to your graph, explain the meaning of bonding and anti-bonding states. [25%]

3 Figure 2 shows a schematic diagram of an experiment that demonstrates the Hall Effect in a sample of silicon, where a current, I, is flowing in the positive $x$ direction. The intrinsic carrier concentration of silicon is $10^{16}$ m$^{-3}$ and the mobility of holes and electrons is 0.048 m$^2$ V$^{-1}$s$^{-1}$ and 0.14 m$^2$ V$^{-1}$s$^{-1}$, respectively.

(a) Explain how the Hall Effect can be used to determine both the sign and density of the majority carriers. [20%]

(b) Assuming that the current flows uniformly through the cross-section of the sample in the $y - z$ plane, show that the measured voltage, $V_H$, identified in Fig. 2, is given by $V_H = \frac{I \times B}{q \times t}$, where $B$ is the magnetic flux density and $q$ is the charge density of majority carriers in the semiconductor. [25%]

(c) The sample is doped with a number density of 5x$10^{22}$ m$^{-3}$ of Boron atoms. It has dimensions of $w$ = 2 mm, $t$ = 2 $\mu$m, and $l$ = 2 mm. What $V_H$ is measured when $I$ = 150 nA and $B$ = 0.2 T? State any assumptions made. [25%]

(d) Calculate the total conductivity of the silicon sample and the ratio of the conductivity due to electrons with respect to holes. How will the answers change if the number density of Boron atoms is increased to 5x$10^{24}$ m$^{-3}$? [30%]

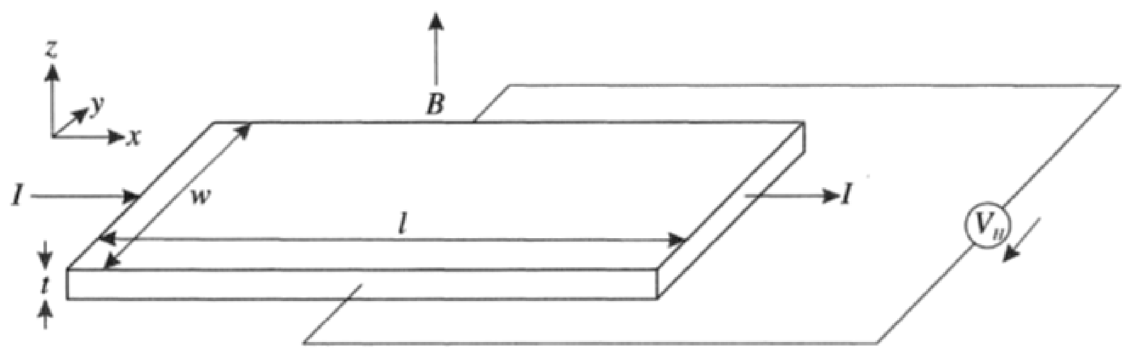


Fig. 2

4 (a) A semiconductor with a direct bandgap of 2 eV is illuminated with 3 eV photons.

(i) Describe the fundamental processes that will occur. Calculate the wavelength of light that will be emitted by the semiconductor. [10%]

(ii) Emission from the semiconductor will stop within $10^{-8}$ s after the optical excitation is turned off. Discuss how the semiconductor material can be modified to maintain emission for minutes after removing the excitation. Illustrate the proposed mechanism in a band diagram. [15%]

(b) A metal with a work function of 5 eV is evaporated onto GaAs with a band gap of 1.43 eV, which is doped with a density of $10^{22}$ $m^{-3}$ Te (group VI) atoms. Assume GaAs has an electron affinity of 4.07 eV, a dielectric constant of 13.1, and effective densities of states in the conduction band and valence bands of $N_C = 4.7\times10^{23}$ $m^{-3}$ and $N_V = 7\times10^{24}$ $m^{-3}$, respectively. Assume no interface traps and room temperature (300 K) conditions. State all other assumptions.

(i) Calculate the built-in potential, $V_0$, and indicate $V_0$ in an equilibrium band diagram of the Schottky junction formed. [15%]

(ii) Calculate the unbiased depletion width $w$ at the junction starting from the Poisson equation. [20%]

(iii) Another metal with work function 4.8 eV evaporated onto this GaAs is found to give the same Schottky barrier height. Give a possible explanation for this finding. [10%]

(c) For n-type InAs it is found that the Fermi level at the interface sits above its conduction band edge for virtually any metal deposited onto it. Draw a band diagram of such an n-type InAs-metal junction and explain what type of contact is formed. [10%]

(d) Sketch a Metal Semiconductor Field Effect Transistor (MESFET) design using the materials described in parts (b) and (c). Note that graded alloys of $Ga_{1-x}In_xAs$ with $x$ varied from 0 to 1 can be grown for interface engineering. Sketch the expected MESFET output characteristics including gate-source voltage variation. [20%]

**END OF PAPER**

Numerical answers: Q2(a)(i) r=1/a, (a)(ii) $A=(a^3/\pi)^{1/2}$; Q3(c) 1.87 μV, (d) 385 $\Omega^{-1}m^{-1}$, 1.17 x $10^{-13}$; Q4(a)(i) 620 nm, (b)(i) 0.83 V, (b)(ii) 347 nm.

# 3B5 Semiconductor Engineering: Sheet of Formulae and Constants

| | |
|---|---|
| Planck's constant | $h = 6.626 \times 10^{-34}$ Js |
| Reduced Planck's constant | $\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}$ Js |
| Speed of light in a vacuum | $c = 2.998 \times 10^{8}$ m/s |
| Mass of electron | $m_{\mathrm{e}} = 9.109 \times 10^{-31}$ kg |
| Charge of electron | $-e = -1.602 \times 10^{-19}$ C |
| Permittivity of free space | $\epsilon_0 = 8.854 \times 10^{-12}$ C$^2$/(Jm) |
| Boltzmann constant | $k_{\mathrm{B}} = 1.381 \times 10^{-23}$ J/K |

de Broglie relation

$$p = \frac{h}{\lambda}$$

Planck's equation

$$E = \hbar\omega$$

Bragg's law

$$n\lambda = 2d\sin\theta$$

Heisenberg's uncertainty principle

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Time-dependent Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t)+V\psi(\mathbf{r},t) = j\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t)$$

Time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r})+V\Psi(\mathbf{r}) = E\Psi(\mathbf{r})$$

Laplacian in Cartesian coordinates

$$\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

Laplacian in spherical polar coordinates

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

Density of states in three-dimensional free electron model

$$g(E)\mathrm{d}E = \frac{V}{2\pi^2\hbar^3}(2m^*)^{\frac{3}{2}}E^{\frac{1}{2}}\mathrm{d}E$$

Fermi–Dirac function

$$f(E) = \frac{1}{\exp\left(\frac{E-E_F}{k_{\mathrm{B}}T}\right)+1}$$

Law of mass action

$$np = n_i^2$$

| | |
|---|---|
| Density of electrons in conduction band | $n = N_C \exp\left(-\frac{E_C - E_F}{k_\mathrm{B}T}\right)$ |
| Density of holes in valence band | $p = N_V \exp\left(\frac{E_V - E_F}{k_\mathrm{B}T}\right)$ |
| Current density | $J = \sigma\varepsilon$ |
| Conductivity | $\sigma = ne\mu_\mathrm{e} + pe\mu_\mathrm{h}$ |
| Mobility | $\mu = \frac{q\tau_\mathrm{scatt}}{m^*}$ |
| Continuity equation for electrons | $\frac{\partial(\Delta n)}{\partial t} = -\frac{\Delta n}{\tau_\mathrm{e}} + \mu_\mathrm{e}\varepsilon\frac{\partial(\Delta n)}{\partial x} + D_\mathrm{e}\frac{\partial^2(\Delta n)}{\partial x^2}$ |
| Continuity equation for holes | $\frac{\partial(\Delta p)}{\partial t} = -\frac{\Delta p}{\tau_\mathrm{h}} - \mu_\mathrm{h}\varepsilon\frac{\partial(\Delta p)}{\partial x} + D_\mathrm{h}\frac{\partial^2(\Delta p)}{\partial x^2}$ |
| Poisson equation | $\nabla^2 V = -\frac{\rho}{\epsilon_0\epsilon_r}$ |
| Einstein's relation | $D = \mu\frac{k_\mathrm{B}T}{e}$ |
| Diffusion length | $L = \sqrt{D\tau}$ |
| Debye screening length | $L_\mathrm{D} = \sqrt{\frac{\epsilon_0\epsilon_r k_\mathrm{B}T}{e^2 n_0}}$ |