ENGINEERING TRIPOS PART IIA

Friday 2 May 2014   9.30 to 11

Module 3B5 - SEMICONDUCTOR ENGINEERING

CRIB

1   (a)   The Time Independent Schrödinger Equation (TISE) is a statement of the conservation of energy using quantum mechanical operators. In a time-invariant situation, the wavefunction must be a valid solution of the TISE.   [10%]

(b)   In spherical polar coordinates, the wavefunction for the electron around the hydrogen atom, Ψ, may be split into $\Psi = R(r)\Phi(\phi)\Theta(\theta)$. The solution to the R function gives the principal quantum number, n, which determines the energy and spatial extent of the electron. The solution to the Θ function gives the orbital quantum number, l, which determines the orbital angular momentum of the electron. The solution to the Φ function gives m_l, which is the orientation quantum number of orbital angular momentum. This determines the component of angular momentum along one axis.   [20%]

(c)   (i)   As there is no θ or ϕ dependence to the Ψ_{1s} wavefunction, the expression for $\nabla^2\Psi$ simplifies to

$$\nabla^2\Psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \Psi}{\partial r}\right)$$

We need to evaluate this for the Ψ_{1s} wavefunction, as follows:

$$\frac{\partial \Psi_{1s}}{\partial r} = \frac{-1}{\sqrt{\pi}}\left(\frac{1}{a}\right)^{5/2}\exp\left(\frac{-r}{a}\right)$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial \Psi_{1s}}{\partial r}\right) = \frac{-1}{\sqrt{\pi}}\left(\frac{1}{a}\right)^{5/2}\left[\frac{-r^2}{a}\exp\left(\frac{-r}{a}\right) + 2r\exp\left(\frac{-r}{a}\right)\right]$$

$$\therefore \nabla^2\Psi_{1s} = \frac{-1}{a}\left(\frac{-1}{a} + \frac{2}{r}\right)\Psi_{1s}$$

This should now be substituted into the TISE:

$$\frac{\hbar^2}{2m}\frac{1}{a}\left(\frac{-1}{a} + \frac{2}{r}\right)\Psi_{1s} - \frac{e^2}{4\pi\varepsilon_0 r}\Psi_{1s} = E\Psi_{1s}$$

$$E = \frac{\hbar^2}{2ma^2} + \frac{\hbar^2}{mar} - \frac{e^2}{4\pi\varepsilon_0 r} \quad \text{Eqn. 1}$$

As E has no r dependence,

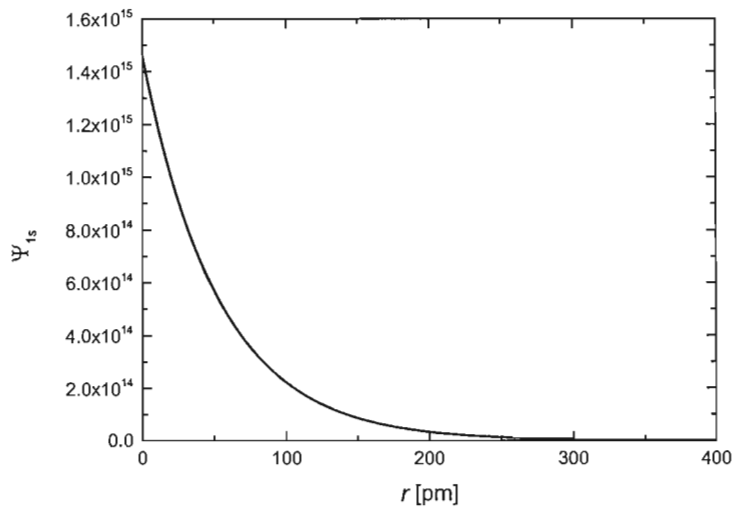$$\frac{\hbar^2}{mar} = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\therefore a = \frac{4\pi\varepsilon_0 \hbar^2}{me^2} = \frac{4\pi \times 8.854 \times 10^{-12} \times (1.055 \times 10^{-34})^2}{9.109 \times 10^{-31} \times (1.602 \times 10^{-19})^2} = \underline{5.29 \times 10^{-11}} \text{ m}$$

We can also evaluate E from Eqn. 1 knowing that the r terms cancel:

$$E = \frac{-\hbar^2}{2ma^2} = \frac{-(1.055 \times 10^{-34})^2}{2 \times 9.109 \times 10^{-31} \times (5.29 \times 10^{-11})^2} = -2.18 \times 10^{-18} \text{ J} = \underline{-13.6 \text{ eV}}$$

[40%]

(ii)   The sketch of the Ψ_{1s} wavefunction follows a straightforward exponential decay with a characteristic decay length of a:

The probability of finding the electron at a particular radius is a little more complex. $|\Psi_{1s}|^2$ gives the simple probability per unit volume, but we need to consider the volume of a thin shell of space at a radius r as being $4\pi r^2 dr$, so that the probability as a function of radius becomes

$$P(r)dr = |\Psi|^2 4\pi r^2 \, dr = \frac{4r^2}{a^3} \exp\left(\frac{-2r}{a}\right) dr$$
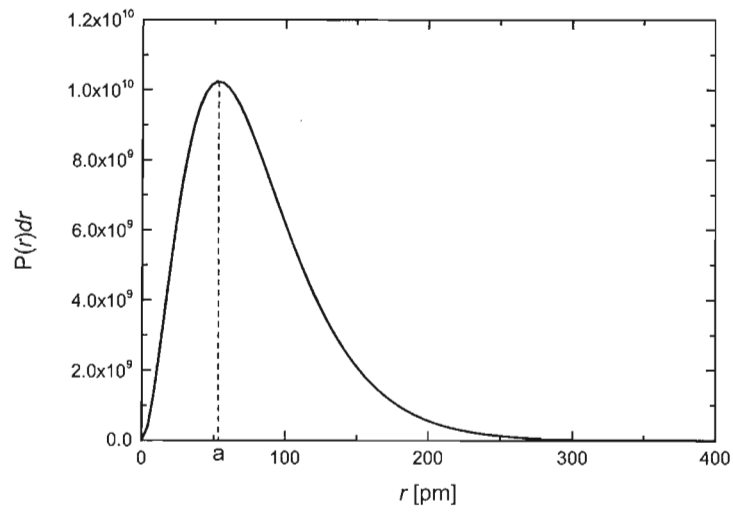
[10%]

(iii) The maximum probability is when $\partial P(r)/\partial r = 0$. Hence

$$\frac{\partial P(r)}{\partial r} = \frac{r}{a^3}\exp\left(\frac{-2r}{a}\right)\left[\frac{-8r}{a}+8\right] = 0 \qquad \text{[20\%]}$$

$$\therefore r = a = \underline{5.29 \times 10^{-11} \text{ m}}$$

2   (a)   (i)   Electrons are Fermions, and therefore obey the Pauli Exclusion Principal meaning that no two electrons can exist in the same state. Hence, Boltzmann statics cannot be used.

[15%]

(ii)   In low occupation situations, the probability of two electrons occupying the same state is so low that Boltzmann statistics can be used again.

[15%]

(b)   (i)   The total energy is given by

$$E_t = \int_0^\infty E n(E) dE$$
$$= \int_0^\infty E g(E) f(E) dE$$
$$= \int_0^{E_F} E g(E) dE$$ [25%]
$$= \int_0^{E_F} \frac{V}{2\pi^2 \hbar^3} (2m)^{3/2} E^{3/2} dE$$
$$E_t = \frac{V}{5\pi^2 \hbar^3} (2m)^{3/2} E_F^{5/2} dE$$

(ii)   We know that the number of electrons per unit volume is given by

$$\frac{N}{V} = \frac{1}{V} \int_0^{E_F} g(E) dE$$
$$= \frac{(2m)^{3/2}}{2\pi^2 \hbar^3} \int_0^{E_F} E^{1/2} dE$$
$$\frac{N}{V} = \frac{(2m)^{3/2}}{3\pi^2 \hbar^3} E_F^{3/2}$$

Now, the average energy per electron is then

$$E_a = \frac{E_t}{N}$$
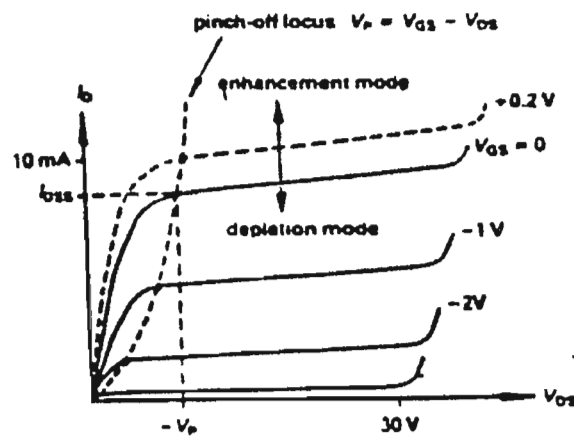$$= \frac{V}{N} \frac{(2m)^{3/2}}{5\pi^2 \hbar^3} E_F^{5/2}$$
$$= \frac{3\pi^2 \hbar^3}{(2m)^{3/2}} \frac{1}{E_F^{3/2}} \frac{(2m)^{3/2}}{5\pi^2 \hbar^3} E_F^{5/2}$$ [25%]
$$E_a = \frac{3E_F}{5}$$

(iii)   Aluminium is in group 3 of the Periodic Table, and so $N/V = 3$. Now

$$E_a = \frac{(2m)^{3/2}}{2\pi^2 \hbar^3} \int_0^{E_F} E^{1/2} dE$$

$$= \frac{3\hbar^2}{10m} \left(3\pi^2 \frac{N}{V}\right)^{2/3}$$

$$= \frac{3\left(1.055 \times 10^{-34}\right)^2}{10 \times 9.109 \times 10^{-31}} \left(3\pi^2 \times 3 \times 5.303 \times 10^{28}\right)^{2/3}$$

$$E_a = 1.03 \times 10^{-18} \text{ J} = \underline{6.43 \text{ eV}}$$

[20%]

3 (a) Output characteristics of the JFET (see lecture notes):

The pinch-off voltage is the potential difference that has to exist between the gate and the drain for the channel in the JFET to be completely depleted at the drain end.
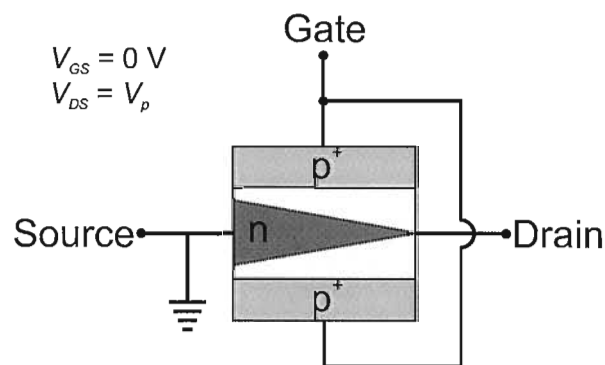
Further increase in V_{DS} has no significant effect upon I_{DS}, which remains at its saturated level, and this is called the saturated region.   [20%]

(b) As the doping density in the n-type channel is much lower than in the p^+ gate, the depletion region in the gate is negligible. Hence, considering the n-side of the junction, the Poisson equation states

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0 \varepsilon_r} = \frac{-eN_D}{\varepsilon_0 \varepsilon_r},$$

where $-eN_D$ is the charge density due to ionised donors. Given that V only varies in the x direction across the junction, this becomes

$$\varepsilon_0 \varepsilon_r \frac{d^2 V}{dx^2} = eN_D.$$

Integrating this and applying the boundary condition that the electric field (and hence dV/dx) is zero at the edge of the depletion region of width w gives

$$\frac{dV}{dX} = \frac{eN_D}{\varepsilon_0 \varepsilon_r}(x - w).$$

Integrating again and applying the boundary condition that $V = 0$ at $x = w$ gives

$$V = \frac{eN_D}{\varepsilon_0 \varepsilon_r}\left(\frac{x^2}{2} - wx + \frac{w^2}{2}\right).$$

Hence, the built-in potential (at $x = 0$) is

$$V_0 = \frac{eN_D w^2}{2\varepsilon_0 \varepsilon_r}.$$

We can rearrange this equation to make w the subject, and allow for the fact that we are also applying a voltage, V, to the gate with respect to the channel,

$$w = \left(\frac{2\varepsilon_0\varepsilon_r (V_0 - V)}{eN_D} \right)^{1/2}.$$

The channel will just be pinched off when the depletion region width at the drain end of the channel ($x = L$) is half the channel height, t. At this point, the externally applied voltage across the junction will be

$$V_p = V_{GS} - V_{DS},$$

and hence

$$\frac{t}{2} = \left(\frac{2\varepsilon_0\varepsilon_r (V_0 - V_p)}{eN_D} \right)^{1/2}.$$

Assuming that $V_0 << V_p$ and rearranging gives

$$\underline{V_p = \frac{-t^2 eN_D}{8\varepsilon_0\varepsilon_r}.}$$

[35%]

(c) For a short channel length, high electric fields lead to an electron velocity saturation. The saturated current follows this velocity saturation. [15%]

(d) (i) For MESFET, same expression for V_p as in (b)

$$V_p = \frac{-t^2 eN_D}{8\varepsilon_0\varepsilon_r} = -1.7V$$

(ii) The carrier density is given by
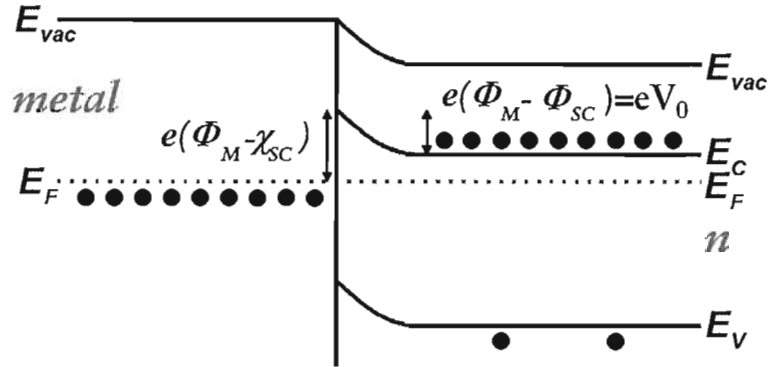
$$n = N_C \exp\left(\frac{E_F - E_C}{kT} \right).$$

Assuming that all donors are ionized, i.e. $n \approx N_D$, this becomes

$$E_F - E_C = kT \ln\left(\frac{N_D}{N_C} \right) = 0.862 \times 10^{-4}.298 \ln\left(\frac{10^{22}}{4.7 \times 10^{23}} \right) = -0.1\,eV\ .$$

The work function ϕ is related to the electron affinity χ by

$$\phi = \chi - (E_F - E_C) = 4.07 + 0.1 = 4.17\ eV$$

(iii) For an n-type semiconductor-metal junction, a rectifying Schottky barrier will be formed if the work function of the metal is larger than that of the semiconductor.

[30%]

4 (a) Assuming equilibrium:

$$\frac{\partial(\Delta p)}{\partial t} = 0$$

As there is no electric field, the master equation simplifies to

$$0 = -\frac{\Delta p}{\tau_h} + D_h \frac{\partial^2(\Delta p)}{\partial x^2}$$

General solution is given by
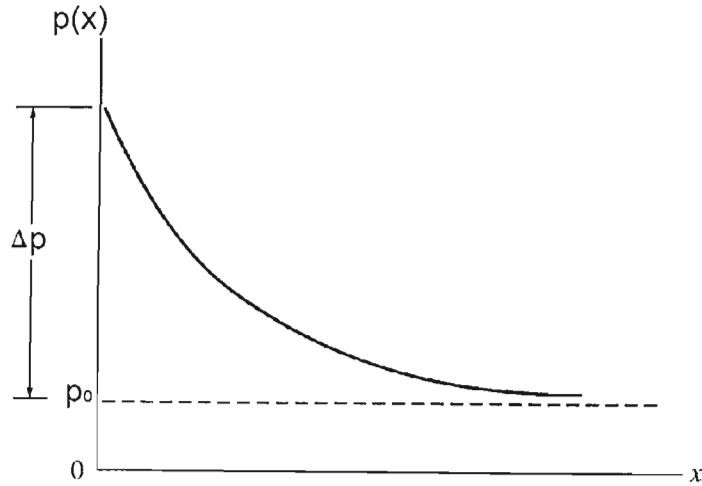
$$\Delta p = \Delta p_0 \exp\left(\frac{-x}{L_h}\right)$$

with $L_h^2 = D_h \tau_h$

Assuming all photons generate an e-h pair, then at the surface (x = 0) holes must be diffusing away at the rate at which they are created in steady state, so

$$\phi = -D_h \frac{d\Delta p}{dx}\bigg|_{x=0} = \frac{D_h \Delta p_0}{L_h}$$
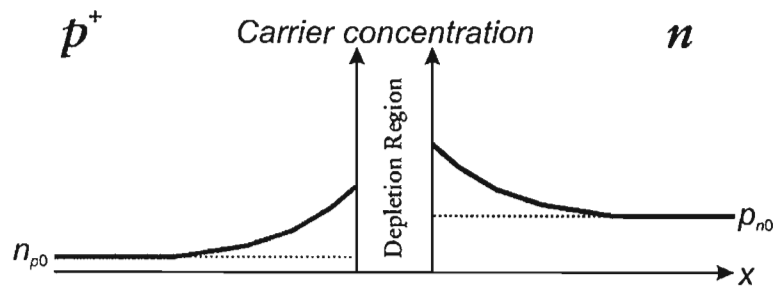
and hence

$$\Delta p = \phi \left(\frac{\tau_h}{D_h} \right)^{1/2} \exp\left(\frac{-x}{L_h} \right)$$

[25%]

(b) positive bias applied to the p-type region (forward bias):

In forward biased p+n junction minority carriers are electrically injected, similar to the optical injection in (a). No electrical fields are assumed outside depletion region, hence master equation and boundary conditions (and hence general solution) are the same for injected holes in n-region and for (a).   [15%]

(c) If $W_b \ll L_h$, then there will be almost no recombination in the base, and the excess hole concentration will drop linearly across the base. If $W_b > L_h$, then there will be significant recombination of carriers in the base, and the current will fall between the emitter and the collector. The excess carrier concentration then follows a curve below the straight line. As for (b), there is no electric field assumed in the BJT base region, ie master equation to solve is the same. However, the

emitter and collector end present two boundary conditions, hence general solution for BJT is given by

$$\Delta p_n(x) = C \exp\left(\frac{-x}{L_h}\right) + D \exp\left(\frac{x}{L_h}\right)$$

For high base to collector current amplification factor, ie improved transistor operation, recombination should be minimised, so $W_b \ll L_h$ is the preferable situation.
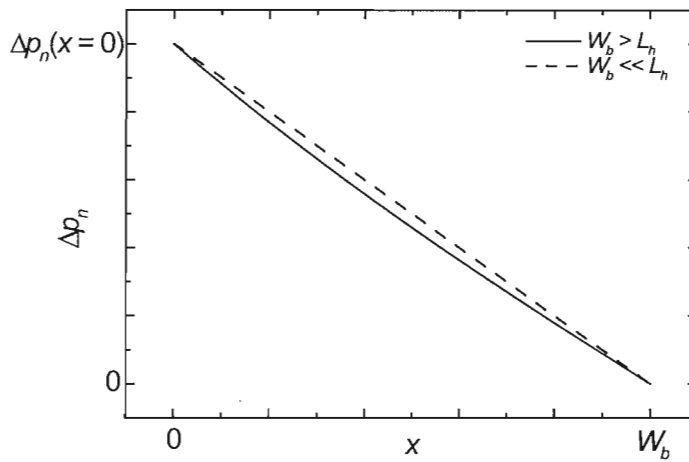
(d) (i) Diffusion time across base $\tau_{diff} = \dfrac{W_b^2}{D_e}$

using the Einstein relation

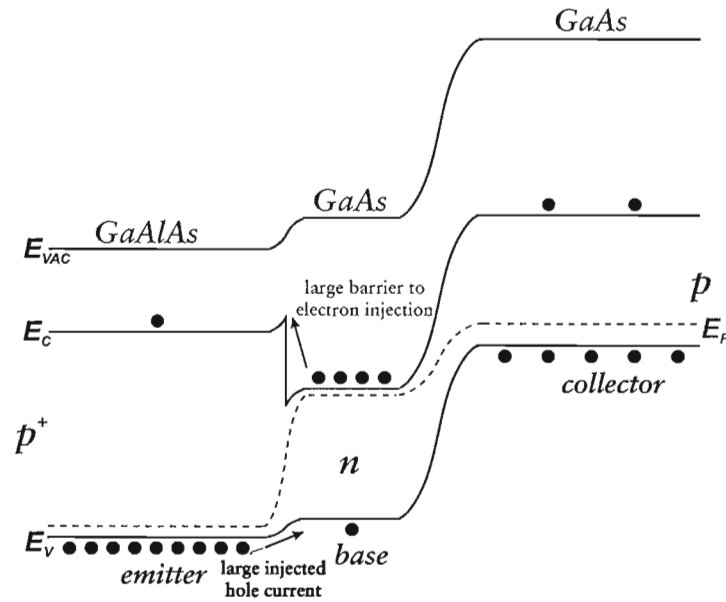$$\tau_{diff} = \frac{W_b^2}{(\dfrac{kT\mu_h}{e})} = \frac{10^{-12}\,m^2}{(8.62\times10^{-5}\times298\times0.05\dfrac{m^2VK}{VKs})} = 8\times10^{-10}\,s$$

(ii) Drift time across base is $\tau_{drift} = \dfrac{W_b}{\mu_h\varepsilon} = \dfrac{10^{-6}\,mVsm}{0.05m^2100000V} = 2\times10^{-10}\,s$

This is four times faster compared to (i).

[20%]

(e) In a heterojunction bipolar transistor a different semiconductor with a large band gap for the emitter compared with the base and collector is used. This creates a large barrier to electron injection from the base into the emitter whilst retaining a high doping density in the base.

[15%]