Question

Q1

a) Work function of n-type GaAs :

$$n = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

assuming full donor ionisation

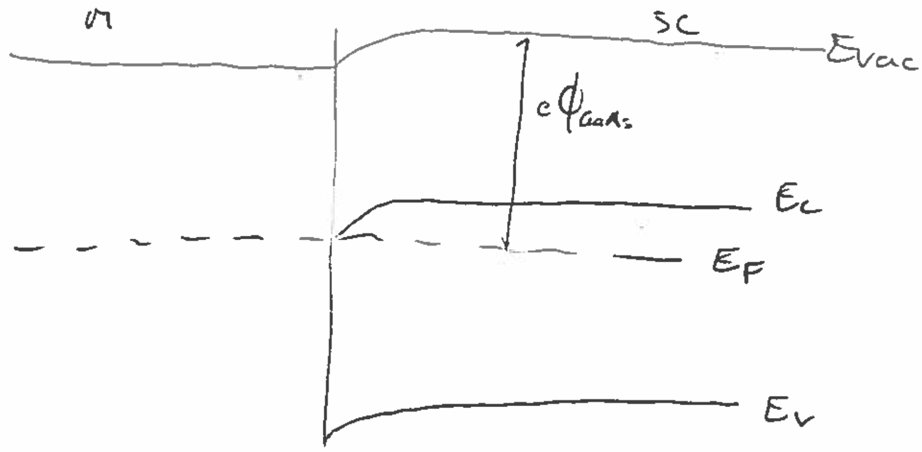
$$n = N_D$$

$$E_F - E_c = kT \ln\left(\frac{N_D}{N_C}\right) = kT \ln\left(\frac{10^{21}}{4.7\,10^{23}}\right) = -0.16\ eV$$

Hence work function

$$e\phi_{GaAs} = e\chi + (E_c - E_F) = (4.07 + 0.16)\ eV = 4.23\ eV$$

For Ω - contact condition is $\phi_M < \phi_{GaAs}$



b) $$\nabla^2 V \overset{(1D)}{=} \frac{d^2V}{dx^2} = \frac{-eN_D}{\varepsilon_0 \varepsilon_r}$$

Integrate with respect to x , using $\mathcal{E} = 0$ at $x = w$ with assumption of no fields outside depletion region

1 b)

$$-\mathcal{E} = \frac{dV}{dx} = \frac{eN_D(w-x)}{\varepsilon_0 \varepsilon_r}$$

Integrating again, using $V=0$ at $x=0$

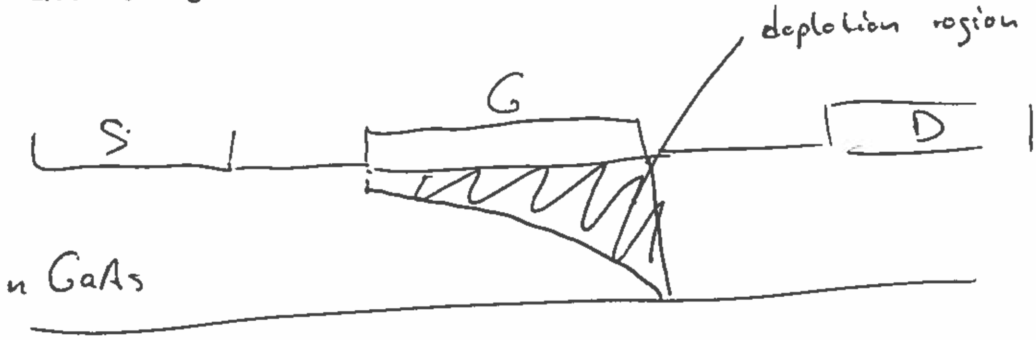
$$V = \frac{eN_D}{2\varepsilon\varepsilon_0}(2wx - x^2)$$

V_0 is potential at $x=w$

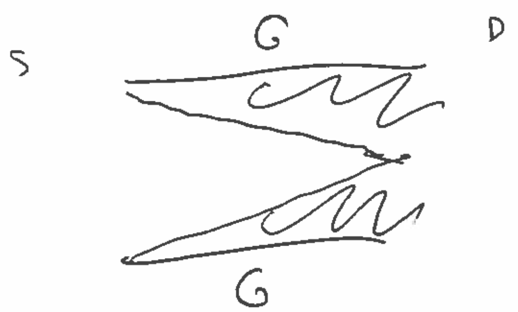
$$V_0 = \frac{eN_D w^2}{2\varepsilon\varepsilon_0}$$

$$\rightarrow \quad w = \left(\frac{2\varepsilon\varepsilon_0 V_0}{eN_D}\right)^{1/2}$$

c) (i) At the pinch-off voltage V_p applied to the MESFET gate the depletion stretches across the whole channel at the drain end



or for double side gate

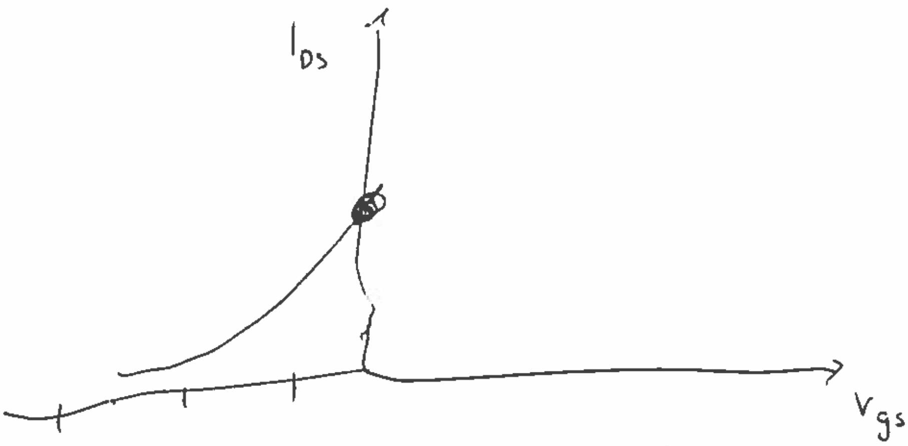


1 c) equation valid for $V_{DS} \leq V_{GS} - V_P$

(ii) saturated I_{DS} for $V_{DS} = V_{GS} - V_P$

$$I_{DS}(sat) = \frac{d h N_D e \mu_e V_P}{L} \left\{ \frac{V_{GS}}{V_P} - \frac{2}{3}\left(\frac{V_{GS}}{V_P}\right)^{3/2} - \frac{1}{3} \right\}$$

$$g_m = \frac{\partial I_{DS}(sat)}{\partial V_{GS}} = \frac{d h N_D e \mu}{L} \left\{ 1 - \left(\frac{V_{GS}}{V_P}\right)^{1/2} \right\}$$



(neg V_{gate} to deplete channel)

1 d) In Figure 1 $I_{DS}(sat)$ variation for the given V_{GS} is basically equidistant, ie not as expected from formulae above. This means that current saturation occurred due to velocity saturation and not due to pinch-off.
For velocity saturation case, g_m is essentially constant.
For choice of material this means not mobility but saturation velocity matters most for short channel devices.

Examiner's comments:
(d) Some confused it with pinch-off.

Question 2

(a) If $E_c - E_F >> kT$ then the Fermi-Dirac distribution is well-approximated by a Boltzmann distribution.

$$\therefore \quad f(E) \approx e^{\left(\frac{-(E-E_F)}{kT}\right)}$$

So,

$$n = \frac{1}{V}\int_{E_c}^{\infty} g(E)\, f(E)\, dE$$

$$= \frac{1}{2\pi^2\hbar^3}(2m_e^*)^{3/2}\int_{E_c}^{\infty}\sqrt{E-E_c}\,.\,e^{\left(\frac{-(E-E_F)}{kT}\right)}\,dE$$

$$= \frac{1}{2\pi^2\hbar^3}(2m_e)^{3/2}\int_{E_c}^{\infty}\sqrt{E-E_c}\,.\,e^{\left(\frac{-(E-E_c)+(E_F-E_c)}{kT}\right)}\,dE$$

Change of variables
$x = \sqrt{E - E_c}$
$2x\,dx = dE$

$$= \frac{(2m_e)^{3/2}}{2\pi^2\hbar^3}\int_{x=0}^{\infty} x\,.\,e^{-\frac{x^2}{kT}}\,.\,e^{\frac{E_F-E_c}{kT}}\,.\,2x\,dx$$

$$= \frac{(2m_e)^{3/2}}{\cancel{2}\pi^2\hbar^3}\,e^{\frac{(E_F-E_c)}{kT}}\,\cancel{2}\int_0^{\infty} x^2 e^{-\frac{x^2}{kT}}\,dx$$

$$= \frac{(2m_e)^{3/2}}{\pi^2\hbar^3}\,e^{\frac{E_F-E_c}{kT}}\left(\frac{1}{4}\sqrt{\pi(kT)^3}\right)$$

$$= \frac{(2m_e)^{3/2}\,\pi^{\frac{1}{2}}\,(kT)^{3/2}}{4\pi^2\hbar^3}\,e^{\frac{E_F-E_c}{kT}}$$

$$= \frac{(m_e kT)^{3/2}}{\sqrt{2}\,\pi^{3/2}\,\hbar^3}\,e^{\frac{E_F-E_c}{kT}} \qquad \Rightarrow N_c = \left(\frac{m_e kT}{\pi\hbar^2}\right)^{3/2}\frac{1}{\sqrt{2}}$$

$$\Rightarrow B = \left(\frac{m_e k}{\pi\hbar^2}\right)^{3/2}\frac{1}{\sqrt{2}}$$

(b) In an intrinsic semiconductor, $n = p$

So

$$N_c \exp\left(\frac{E_{Fi}-E_c}{kT}\right) = N_v \exp\left(\frac{E_v - E_{Fi}}{kT}\right)$$

Rearranging gives $kT\ln\left(\frac{N_v}{N_c}\right) = E_{F_i} - E_c - E_V + E_{F_i}$

$$\therefore \quad E_{F_i} = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln\left(\frac{N_v}{N_c}\right)$$

$$\Rightarrow \quad A = \frac{kT}{2}$$

Usually, kT is very small, much less than the band gap energy, so the intrinsic Fermi level is approximately at mid gap.

(c) We have $\quad n = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$

and $\quad n_i = N_c \exp\left(\frac{E_{F_i} - E_c}{kT}\right)$

$$\Rightarrow \quad \frac{n}{n_i} = \frac{\exp\left(\frac{E_F - E_c}{kT}\right)}{\exp\left(\frac{E_{F_i} - E_c}{kT}\right)}$$

$$\therefore \quad \frac{n}{n_i} = \exp\left(\frac{E_F - E_{F_i}}{kT}\right)$$

$$\therefore \quad n = n_i \exp\left(\frac{E_F - E_{F_i}}{kT}\right)$$

(d) Assume $E_{F_i} = \frac{E_g}{2} = \frac{1.12}{2} = 0.56 \text{ eV}$

Assume all dopants ionized

$$n = N_D = 1.0 \times 10^{17} \text{ cm}^{-3}$$

$$E_F = E_{F_i} + kT \ln\left(\frac{n}{n_i}\right)$$

$$= 0.56 + 0.026 \ln\left(\frac{1.0 \times 10^{17}}{1.0 \times 10^{10}}\right)$$

$= 0.98$ eV, closer to CB edge than E_{F_i}

At 600 K, we need to determine a new value for n_i.

Knowing $$n_i = N_c \exp\left(\frac{E_{Fi} - E_c}{kT}\right)$$

$$\frac{n_{i,600K}}{n_{i,300K}} = \frac{N_{c,600K}}{N_{c,300K}} \cdot \exp\left(\frac{-E_g/2}{k.600} + \frac{E_g/2}{k.300}\right)$$

$$= \left(\frac{600}{300}\right)^{3/2} \exp\left(\frac{0.56 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}}\left(\frac{-1}{600} + \frac{1}{300}\right)\right)$$

$$= 4 \exp(10.82)$$

$$= 2.0 \times 10^5$$

$$\therefore n_{i,600K} = 2.0 \times 10^5 \times n_{i,300K}$$

$$= 2.0 \times 10^{15} \text{ cm}^{-3}$$

$$E_F = \frac{E_g}{2} + kT \ln\left(\frac{10^{17}}{2 \times 10^{15}}\right)$$

$$= 0.56 + 0.052 \ln\left(\frac{10^{17}}{2 \times 10^{15}}\right)$$

$$= 0.76 \text{ eV}$$

NB. As expected, Fermi energy moves towards E_i as the temperature increases.

Examiner's comments:
Most popular question. Generally well done.
(b) Some missed the estimation of the value for A.
(d) (ii) Many did not know what to do.

Question 3

(a)(i) The particle's wavefunction is the solution to the Schrödinger equation.

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V\Psi = E\Psi$$

In region $x \leq 0$, $V = 0$

The general solution is combination of forward and backward travelling waves:

$$\Psi = A\left(\exp(jk_1x) + B\exp(-jk_1x)\right)$$

where $k_1 = \sqrt{\frac{2mE}{\hbar^2}}$

In the region $x > 0$, $V = V_1$

There is only a forward travelling wave:

$$\Psi = AC\exp(jk_2x)$$

where $k_2 = \sqrt{\frac{2m(E-V_1)}{\hbar^2}}$

(a)(ii) Apply boundary conditions

① $\Psi(0)_{left} = \Psi(0)_{right}$

$$\therefore A(1 + B) = AC$$

$$\therefore 1 + B = C$$

② $\frac{d\Psi}{dx}(0)_{left} = \frac{d\Psi}{dx}(0)_{right}$

$$\therefore A(jk_1 - jk_1B) = ACjk_2$$

$$\therefore k_1(1 - B) = k_2C$$

$$\therefore k_1(1 - B) = k_2(1 + B)$$

$$\therefore k_1 - k_2 = B(k_1 + k_2)$$

$$\therefore B = \frac{k_1 - k_2}{k_1 + k_2}$$

$$C = 1 + B$$
$$= 1 + \frac{k_1 - k_2}{k_1 + k_2}$$
$$= \frac{2k_1}{k_1 + k_2}$$

(a)(iii) Transmission coefficient = 1 - Reflection coefficient.

$$\text{Reflection coefficient} = B^2$$
$$= \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$$

$$\text{Transmission coefficient} = 1 - B^2$$
$$= 1 - \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$$
$$= \frac{(k_1 + k_2)^2 - (k_1 - k_2)^2}{(k_1 + k_2)^2}$$
$$= \frac{4k_1 k_2}{(k_1 + k_2)^2}$$

(b) When the particle reaches $x = d$, some of the wave is transmitted and some of the wave is reflected:

For $0 < x < d$ $\quad \Psi = D(e^{jk_2 x}) + Ee^{-jk_2 x}$

where $k_2 = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$ and D and E are constants.

This is quite different from the classical situation. In the classical situation, if $E > V_1$, the particle travels over the barrier and is not reflected at either $x = 0$ or at $x = d$. In the quantum mechanical situation, the wave is partially reflected at both $x = 0$ and at $x = d$.

(c) (i) $\Psi = Fe^{jkx} u(x)$

where $u(x)$ is a function with the same periodicity as the lattice, that is,

$$u(x) = u(x + nL)$$

where n is an integer.

(ii) When $k = \frac{\pi}{L}$,

$$\frac{2\pi}{\lambda} = \frac{\pi}{L}$$

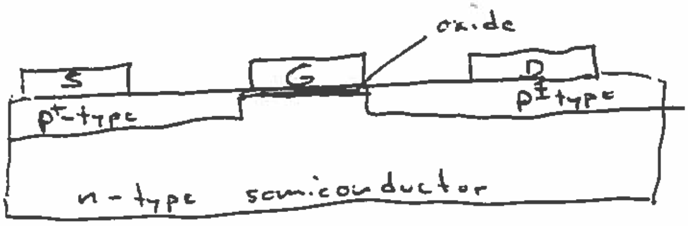
$$\therefore \lambda = \frac{L}{2}$$

and we meet the Bragg condition. The particle is Bragg-reflected and a standing wave results. The electron cannot propagate as a travelling wave.

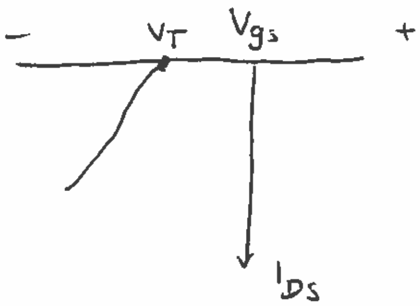
Examiner's comments:
Also a popular question. Generally well done.
(a) (iii) Most didn't square it for probability.
(b) Many confused with the case of E < V1.
(c) (i) Not many managed to show u(x) and u(x)=u(x+nL).

Question 4

2) a) p-channel enhancement MOSFET



V_T is the voltage that needs to be applied to gate to induce strong inversion in semiconductor interface to oxide.



At strong inversion the channel opens and conduction occurs from S to D. See the transfer characteristics.

b) Resistance of small length of channel is given by

$$\delta R = \frac{\rho\, \delta x}{t_{inv} W}$$

where t_{inv} is thickness of inverted channel. If density of inverted holes is p_{inv}, then

$$\delta R = \frac{\delta x}{p_{inv} e \mu_{HFE} t_{inv} W}$$

$$Q_F = p_{inv} e\, t_{inv} \longrightarrow \delta R = \frac{\delta x}{Q_f \mu_{HFE} W}$$

V drop across small length of channel is

$$\delta V(x) = I_{DS}\, \delta R$$

b) Hence $I_{DS}\, \delta(x) = Q_f \mu_{nFE} W \delta V(x)$

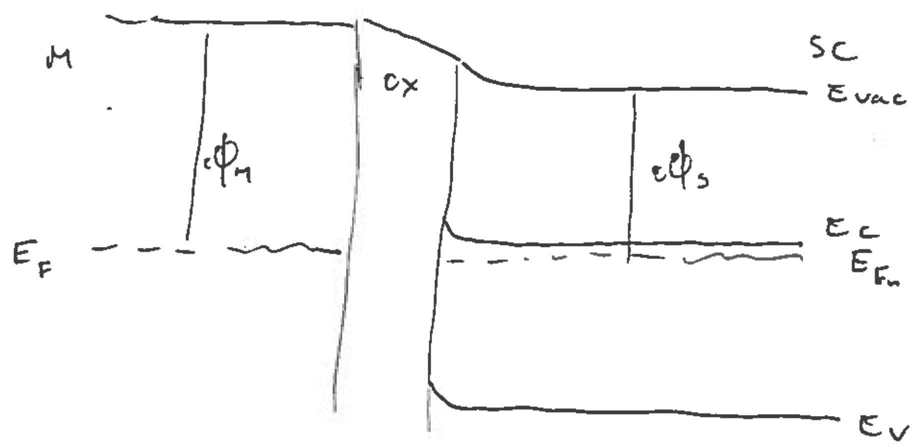
Substituting for Q_f from given equation and integrating from S to D gives:

$$\int_0^L I_{DS}\, \delta x = - \int_0^{V_{DS}} C_i \left[V_{GS} - V_T - V(x) \right] \mu_{nFE} W \, \delta V(x)$$

$$\longrightarrow \quad I_{DS} = - \frac{C_i \mu_{nFE} W}{L} \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

c) V_T is defined via strong inversion condition. There is some drain conduction below this threshold, ref. to as sub-threshold conduction, due to weak inversion in channel. This is key for transistor switching as so called subthreshold slope highlights what change in V_{GS} will change I_{DS} by an order of magnitude.

d) $\phi_M > \phi_S$



e) Si is average semiconductor but its key advantage is its native oxide and low defect density at $Si : SiO_2$ interface. This is key for MOSFET design and hence CMOS.

Channel conductance

$$\sigma = p_{inv}\, e\, \mu_{hFE} = N_D\, e\, \mu_{hFE} = 10^{22}\, m^{-3}\, e\ 0.025\ \frac{m^2}{Vs}$$

def of strong inversion $\quad = 40\ \frac{1}{\Omega m}$

f) In constant field scaling fields are kept constant, because breakdown fields are material constants.

So $\quad E^* = \frac{V^*}{L^*} = \frac{kV}{kL} = E \quad$ (k scale factor)

This means that gate oxide thickness becomes very thin, and tunnelling through it can occur.
A solution is to replace SiO_2 by a material with higher dielectric constant $\varepsilon = \varepsilon_r \varepsilon_0$. This means overall gate Capacitance $C = \frac{\varepsilon}{d}$ can be kept constant while d (thickness) can be larger. These materials are called "high-k" dielectrics. These materials should also have adequate breakdown strength and make good interface to Si / SiO_2.

Examiner's comments:
(a) Some put p-type substrate for p-channel device.
(f) Would be good to show the scale factor.