Version SH/4

EGT2
ENGINEERING TRIPOS PART IIA

Friday 2 May 2014 9.30 to 11

Module 3B5

SEMICONDUCTOR ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 The *spherically-symmetric* potential well around the nucleus of a hydrogen atom in free-space is given by

$$V = \frac{-e^2}{4\pi\varepsilon_0 r}$$

where e is the electronic charge, ε_0 is the permittivity of free space and r is the distance from the nucleus.

(a) Explain the physical significance of the *time-independent Schrödinger equation*, which for a particle with a wavefunction Ψ is given by

$$\frac{-\hbar^2}{2m}\nabla^2\Psi + V\Psi = E\Psi \ .$$ [10%]

(b) Explain the origin and physical meaning of the three quantum numbers n, l and m_l that are associated with the solution to the time-independent Schrödinger equation for the hydrogen atom. [20%]

(c) The wavefunction of an electron in the 1s state of a hydrogen atom is given by

$$\Psi_{1s} = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a}\right)^{3/2}\exp\left(\frac{-r}{a}\right)$$

(i) Show that $a = 5.29\times10^{-11}$ m if this wavefunction is a valid solution to the time-independent Schrödinger equation. Calculate the energy of an electron in this state. [40%]

(ii) Sketch how both the Ψ_{1s} wavefunction and the probability of finding the electron in the 1s state at a particular radius vary with radial distance from the nucleus. [10%]

(iii) Calculate the radius at which the probability of finding the electron in the 1s state is greatest. [20%]

NOTE: In spherical polar coordinates r, θ, ϕ

$$\nabla^2\Psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2\Psi}{\partial\phi^2}$$

2 (a) The *Fermi function f*(E) is given by

$$f(E) = \frac{1}{\exp\left(\frac{E - E_F}{kT}\right) + 1}$$

where E_F is the Fermi energy and k is the *Boltzmann constant.*

(i) Why must the Fermi function be used to describe the occupation of quantum states by electrons? [15%]

(ii) Explain why the Fermi function would be expected to tend to a Boltzmann function for $E >> E_F$. [15%]

(b) The electron energy density of states $g(E)$ for a metal of volume V given by the *Free Electron Theory* is

$$g(E)\mathrm{d}E = \frac{V}{2\pi^2\hbar^3}(2m)^{3/2} E^{1/2}\mathrm{d}E$$

where m is the free electron mass.

(i) Show that the total energy E_t of all the free electrons added together in a metal at a temperature of 0 K is given by

$$E_t = \frac{V(2m)^{3/2}}{5\pi^2\hbar^3} E_F^{5/2}$$ [25%]

(ii) Hence, by determining an expression for the number density of free electrons, show that the average energy E_a of all the free electrons in the metal at 0 K is related to the Fermi energy by

$$E_a = \frac{3E_F}{5}$$ [25%]

(iii) Calculate the average energy of the free electrons in aluminium at 0 K. Aluminium is in group III of the *periodic table* and the number density of atoms in aluminium is 5.303×10^{28} m^{-3}. [20%]

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3 (a) Figure 1 schematically shows the channel region of a *junction field-effect transistor* (JFET). Sketch how the *drain-source* current varies with drain-source voltage for a number of different *gate-source* voltages. Explain the terms *pinch-off voltage* and *saturation region* with the aid of Fig. 1. [20%] 4

(b) Starting from the Poisson equation, show that the pinch-off voltage V_p for the JFET shown in Fig. 1 is given by

$$V_p = \frac{-t^2 e N_D}{8\varepsilon_0 \varepsilon_r}$$

where ε_r is the relative permittivity of the *n-type* semiconductor. Assume that the doping density in the n-type channel region N_D is much lower than that in the *p-type* gate region N_A, and that the *built-in potential* is negligible. [35%] 7

(c) Explain why current saturation can occur without pinch-off for a JFET with a short channel length. [15%] 3

(d) For a high frequency circuit application, the pn junctions in Fig. 1 are replaced by *Schottky barrier* junctions forming a *metal-semiconductor field-effect transistor* (MESFET). N-type GaAs with a donor density N_D of 10^{22} m^{-3} is used for the channel region of the MESFET. GaAs has a *band gap* of 1.4 eV, an *electron affinity* of 4.07 eV and an effective *density of states* in the conduction band of 4.7×10^{23} m^{-3}. Assume $t = 1\,\mu$m, $\varepsilon_r = 13.1$ for the n-type GaAs and room temperature operation.

(i) Calculate the pinch-off voltage V_p for the MESFET. 2

(ii) Calculate the work function of the n-type GaAs. 2

(iii) Draw a *band diagram* of the Schottky barrier junction. State whether a metal with a lower or higher work function than in (ii) above is required. 2 [30%] 6

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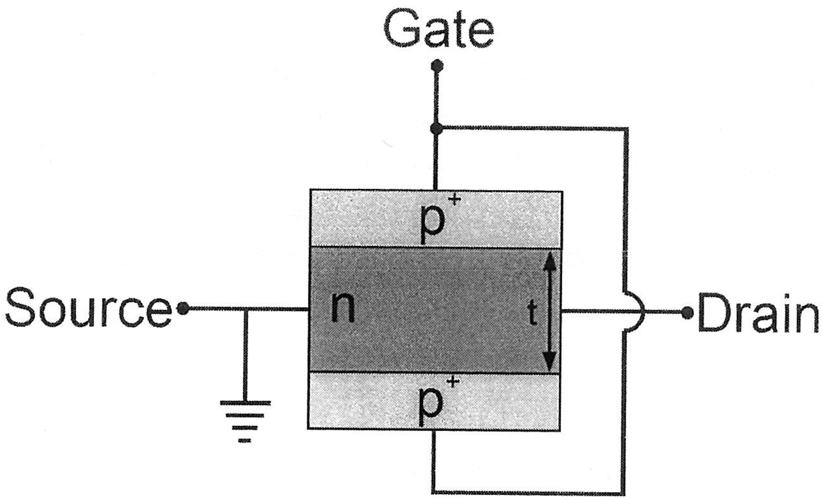

Fig. 1

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4 (a) A plate of *n-type* silicon is continuously illuminated on one face with light. The photon energy is larger than the *band gap* and the light is absorbed over a very short length. There is no applied electric field. Find an expression for the excess concentration of holes as a function of distance away from the illuminated face in terms of properties of the carriers and the generation rate of electron-hole pairs at the surface. State all assumptions made. Sketch the variation of the excess concentration of holes in the silicon. [25%]

(b) Sketch the distribution of minority carriers either side of the depletion region in a p^+n *junction* for a positive bias applied to the *p-type* region with respect to the n-type region. Comment on how and why this compares to (a). [15%]

(c) Sketch how the excess hole concentration varies across the base region of a p^+np *bipolar junction transistor* (BJT) in the active mode for conditions $W_b > L_h$ and $W_b << L_h$. W_b is the undepleted width of the base and L_h is the diffusion length of holes in the n-type semiconductor. Comment on how the hole distribution compares to (b) and which condition will give improved transistor operation. [25%]

(d) In order to improve the frequency response of a p^+np BJT the doping density is varied exponentially across the base region. This results in an uniform electric field of magnitude 100 kVm^{-1} for attracting holes across a base width W_b = 1 μm. The average hole mobility across W_b is 0.05 $m^2V^{-1}s^{-1}$ and $W_b << L_h$.

(i) Calculate the *transit time* due to hole diffusion across the base.

(ii) Calculate the *drift transit time* of holes across the base due to the *built-in* field and compare your answer to (i). [20%]

(e) Explain how the emitter injection efficiency of a BJT can be improved by using different semiconductor materials and draw a band diagram of such a *heterojunction bipolar transistor*. [15%]

Note: the Continuity equation for holes is

$$\frac{\partial(\Delta p)}{\partial t} = -\frac{\Delta p}{\tau_h} - \mu_h \varepsilon \frac{\partial(\Delta p)}{\partial x} + D_h \frac{\partial^2(\Delta p)}{\partial x^2}$$

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