EGT2
ENGINEERING TRIPOS PART IIA

Monday 26 April 2021 9.00 to 10.40

Module 3B5

SEMICONDUCTOR ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet and at the top of each answer sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 15 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 A Metal-Semiconductor Field-Effect Transistor (MESFET) is fabricated using n-type GaAs with a donor density of $N_D = 10^{21}\,\mathrm{m}^{-3}$ as channel material. GaAs has a band gap of 1.43 eV, an electron affinity of 4.07 eV and an effective density of states in the conduction band of $4.7 \times 10^{23}\,\mathrm{m}^{-3}$. Assume room temperature (300 K) operation.

(a) Outline the requirements for the metal work function to make the source and drain contacts ohmic. Calculate the maximum possible metal work function. Sketch an equilibrium band diagram for such an ohmic contact. [20%]

(b) Starting from the Poisson equation, show that the unbiased depletion width, w, in the n-type GaAs at the interface to the gate metal is given by

$$w = \left(\frac{2\epsilon_0\epsilon_r V_0}{eN_D}\right)^{\frac{1}{2}},$$

where ϵ_r is the relative permittivity of GaAs and V_0 is the built-in potential. [20%]

(c) The drain-source current I_{DS} of the MESFET can be derived to be

$$I_{DS} = \frac{dhN_D e\mu_e V_p}{L}\left[\frac{V_{DS}}{V_p} - \frac{2}{3}\left(\frac{V_{GS}}{V_p}\right)^{\frac{3}{2}} + \frac{2}{3}\left(\frac{V_{GS}-V_{DS}}{V_p}\right)^{\frac{3}{2}}\right],$$

where d is the depth, h the height and L the length of the gated channel, and μ_e is the electron mobility in the n-type GaAs.

(i) Explain the term pinch-off voltage, V_p, and state the limit of V_{DS} up to which the equation for I_{DS} is valid. Draw a schematic of the depletion region across the gate at the pinch-off point. [15%]

(ii) Derive an expression for the small signal mutual transconductance, g_m, in the saturated region. Sketch the transfer characteristic of the MESFET. [25%]

(d) Figure 1 shows the output characteristic of a modern MESFET. Discuss how this deviates from the equation in part (c) and give an explanation for the observed behaviour. What does this imply for the choice of channel material? [20%]

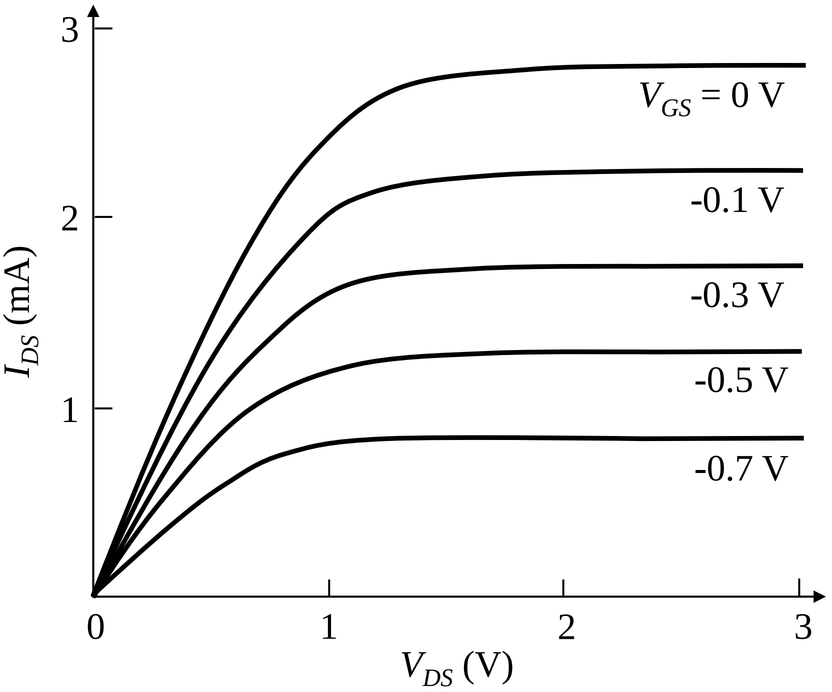

Fig. 1

2 The electronic density of states in the conduction band of a semiconductor is given by

$$g(E)dE = \frac{V}{2\pi^2\hbar^3}\left(2m_e^*\right)^{3/2}\left(E - E_C\right)^{1/2} dE,$$

where m_e^* is the effective mass of electrons and E_C is the energy at the bottom of the conduction band. The Fermi-Dirac function is given by

$$f(E) = \frac{1}{\exp\left(\frac{E-E_F}{kT}\right) + 1},$$

where E_F is the Fermi level.

(a) Show that if $E_C - E_F >> kT$, the free electron concentration, n, is given by

$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right),$$

where N_C is the effective density of states in the conduction band, given by

$$N_C = BT^{\frac{3}{2}},$$

where B is a constant. Find an expression for the constant B in terms of m_e^*. [30%]

(b) Show that in an intrinsic semiconductor, the intrinsic Fermi level E_{Fi} is given by

$$E_{Fi} = \frac{E_C + E_V}{2} + A\ln\left(\frac{N_V}{N_C}\right),$$

where E_V is the energy at the top of the valence band and N_V is the effective density of states in the valence band. Calculate the value of the constant A at room temperature. Explain why it is acceptable to approximate the intrinsic Fermi level as lying in the middle of the band gap. [20%]

(c) Show that in an n-doped semiconductor, the density of electrons in the conduction band is given by

$$n = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right),$$

where n_i is the intrinsic carrier density. [20%]

(d) Silicon has a band gap of 1.12 eV and an intrinsic carrier density, n_i, of $1.0 \times 10^{10}\,\mathrm{cm}^{-3}$ at 300 K.

(i) Using the equation of part (c), calculate the position of the Fermi level at 300 K if the Si is doped with phosphorus atoms to a concentration of $1.0 \times 10^{17}\,\mathrm{cm}^{-3}$. State all assumptions. [10%]

(ii) Estimate the intrinsic carrier density at 600 K. Hence, calculate the position of the Fermi level if the Si is doped with a concentration of phosphorus atoms of $1.0 \times 10^{17}\,\mathrm{cm}^{-3}$ at 600 K. [20%]

Note:

$$\int_0^\infty x^2 \exp\left(-\alpha x^2\right) = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}$$

3 (a) A particle travelling from left to right is incident on a barrier of potential V_1, as shown in Fig. 2. The particle's energy E is greater than the barrier height, that is, $E > V_1$.

(i) Show that the wavefunction $\Psi(x)$ given by

$$\Psi(x) = \begin{cases} A\left[\exp^{jk_1 x} + B\exp^{-jk_1 x}\right] & x \leq 0 \\ AC\exp^{jk_2 x} & x \geq 0 \end{cases},$$

is a valid solution of the time-independent Schrödinger equation and find expressions for k_1 and k_2. [30%]

(ii) Solve for coefficients B and C in terms of k_1 and k_2. Do not attempt to solve for coefficient A. [20%]

(iii) Using coefficients B and C, find expressions for the probabilities of reflection and transmission, that is, the reflection and transmission coefficients of the particle at the barrier. [10%]

(b) A particle of energy $E > V_1$ is incident on a potential barrier that extends only between $0 < x < d$, as shown in Fig. 3. Suggest an expression for the particle's wavefunction in the region $0 < x < d$. Explain how the particle's quantum mechanical behaviour compares with its classically predicted behaviour at the interfaces at $x = 0$ and $x = d$. [20%]

(c) A particle with energy $E > V_1$ travels through a potential landscape of periodic barriers of potential V_1, as illustrated in Fig. 4.

(i) Write a general expression for the particle's wavefunction. [10%]

(ii) Describe the propagation of the particle in this potential landscape if the particle's wavenumber is given by $k = \frac{\pi}{L}$. [10%]

Note: The time-independent Schrödinger equation is given by

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} + V\Psi = E\Psi$$

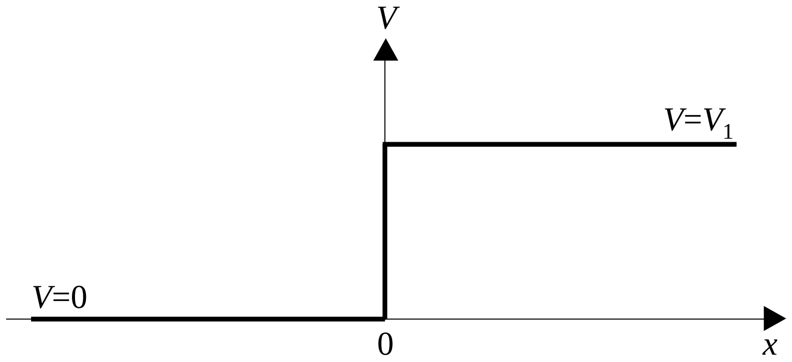

Fig. 2

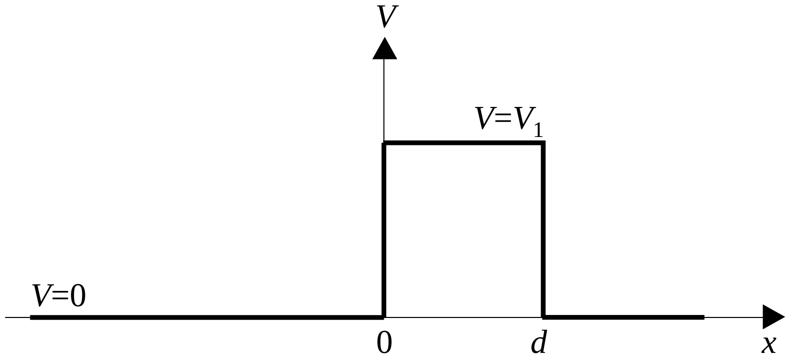

Fig. 3

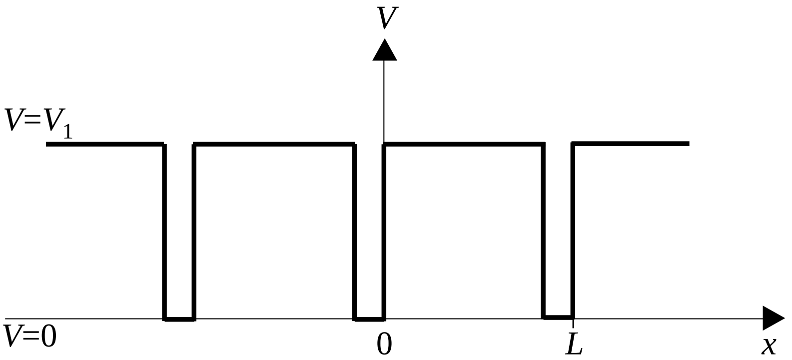

Fig. 4

4 (a) Sketch the design of a p-channel enhancement Metal-Oxide-Semiconductor Field-Effect Transistor (MOSFET). Explain the term threshold voltage, V_T, and indicate V_T in a sketch of the MOSFET transfer characteristics. [15%]

(b) Show that the drain-source current, I_{DS}, of the MOSFET can be expressed as

$$I_{DS} = -\frac{C_{ox}\mu_{hFE}W}{L}\left[(V_{GS} - V_T)\,V_{DS} - \frac{V_{DS}^2}{2}\right],$$

where μ_{hFE} is the field effect mobility of holes in the channel, C_{ox} is the capacitance per unit area of the gate oxide, W is the channel width, L is the channel length, V_{GS} is the gate-source voltage and V_{DS} is the drain-source voltage. You can use the fact that the free carrier density per unit area in the inverted region, Q_f, is given by

$$Q_f = -C_{ox}\left[V_{GS} - V_T - V(x)\right],$$

where $V(x)$ is the voltage induced across the channel by the drain-source voltage, V_{DS}. [30%]

(c) Explain what is meant by sub-threshold conduction and its importance for MOSFET switching. [10%]

(d) Draw an equilibrium band diagram of the MOS gate structure, assuming that the work function of the metal is greater than that of the doped semiconductor. Indicate the work functions and include the vacuum energy in your drawing. [15%]

(e) For a MOSFET design as in part (a) implemented in Si, calculate the channel conductivity near the Si-SiO$_2$ interface at strong inversion. Assume the Si body has a dopant density of 10^{22} m^{-3} and $\mu_{hFE} = 0.025\,\mathrm{m^2V^{-1}s^{-1}}$. State any further assumptions. Discuss why silicon has become the dominant material in the semiconductor industry. [15%]

(f) Explain the concept of constant field scaling. Discuss what such scaling means for the gate oxide thickness and why this had led to the replacement of SiO$_2$ as gate dielectric. Comment on the properties required of such new dielectrics. [15%]

END OF PAPER