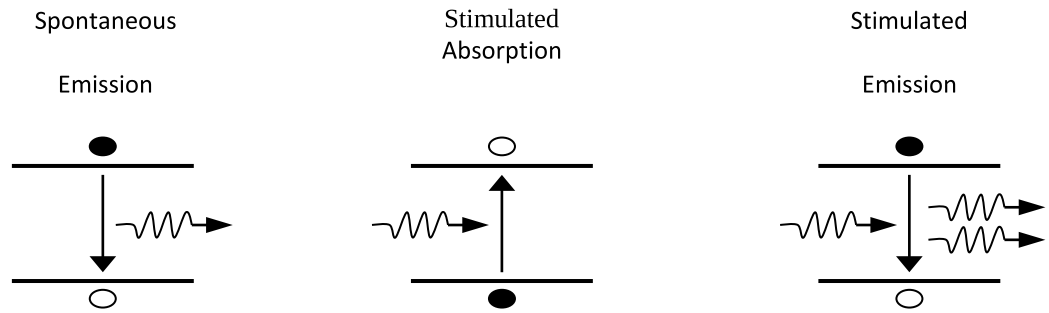
3B6 2018 Cribs to Examinations Paper

Q.1 (a) This is mainly a bookwork question. A good answer will include: There are three major types of electron/photon interactions in materials.



LED action | Photodiode Action | Laser action

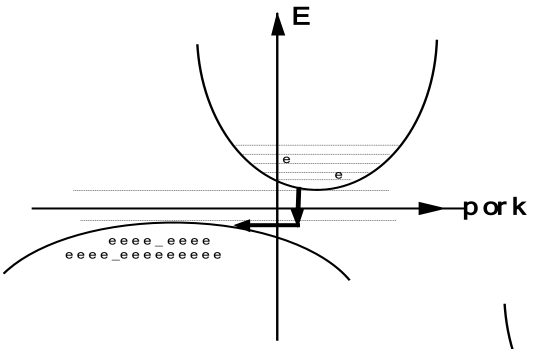
- **Spontaneous Emission:** An electron in a high energy level falls, losing energy which is emitted as a photon – the basis of operation of a light emitting diode.

- **Stimulated Absorption:** An incident photon is absorbed in a material, causing the excitation of an electron to a higher energy level – the basis of operation of a photodiode.

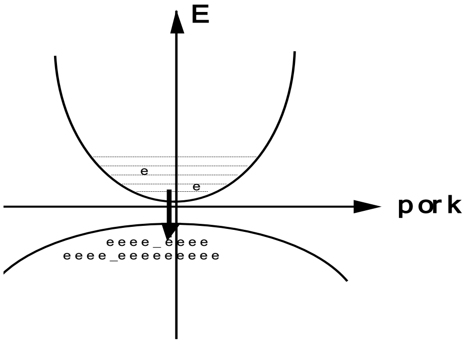
- **Stimulated Emission:** A photon, incident upon an electron in a higher energy level, causes the electron to fall to a lower level thus generating a second photon. This is, therefore, an amplifying action. Two photons are generated from one and, in turn, they can cause the generation of two further photons. Using this method, high optical powers can be generated and this operation is the basis of lasing action. The generated photon has the same frequency and phase as the incident photon and, therefore, very pure monochromatic and coherent light is generated.

In respect of direct and indirect bandgap materials, both emission and absorption processes are affected, with direct bandgap materials being more efficient in all cases:

In repect of **light emission** in semiconductors, this is usually by direct injection of electrons. This leads to the generation of photons when the electrons recombine with holes at a junction. Whilst absorption is achieved at many wavelengths, with energies greater than the bandgap, spontaneous emission only occurs at energies close to the bandgap corresponding to the likely energy separations of holes and electrons. As no photons are required

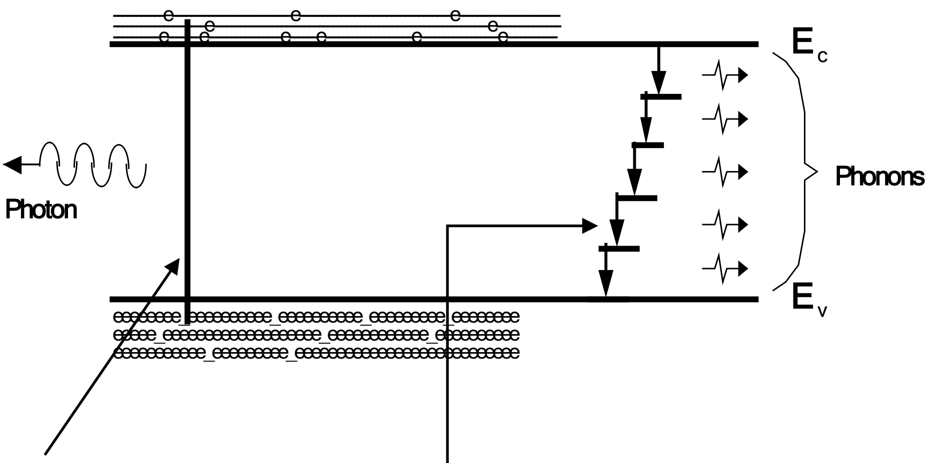


Indirect semiconductor: needs both phonon and photon of correct energy – not very likely



Direct bandgap semi-conductor: needs only a photon of correct energy – much more likely

An answer might also indicate the impact on non-radiative recombination:



Radiative recombination:- more likely (faster) with direct gap.

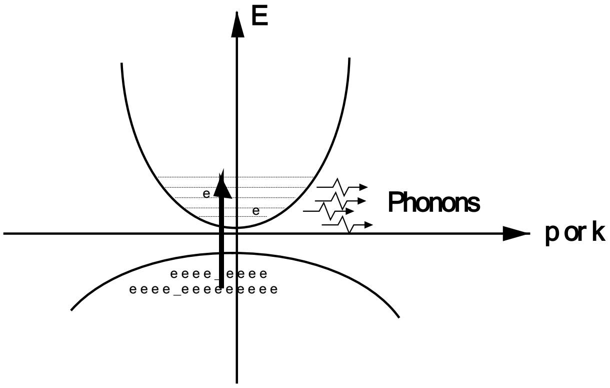
Non-radiative recombination:- doesn't depend much on band structure. More likely (faster) with imperfect and/or impure crystals.

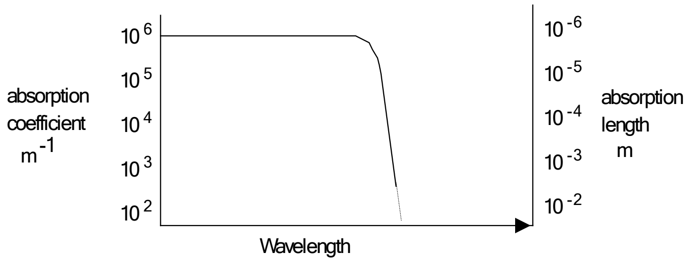
A good answer might also give examples of efficiencies, e.g.:

	Bandgap	**(nm)**	**Direct/Indirect**	**η_{max}(%)**
GaAs	1.44 eV	860	direct	10.00
GaP	2.26 eV	549	indirect	0.10
$Ga_{0.6}As_{0.4}P$		650	direct	0.20
$Ga_{0.15}As_{0.85}P$		589	indirect	0.05
InGaAsP		1300	direct	>1.00
GaN		~400	direct	15.00

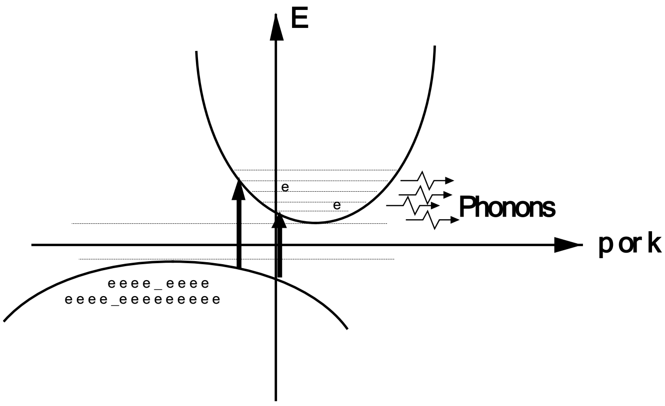
Absorption also depends upon the nature of the material being illuminated. In semiconductors, for example, absorption within direct bandgap materials can readily occur without requiring momentum changes for the excited carriers. In indirect materials, phonons are frequently involved, allowing substantial momentum changes. This results in different absorption properties.

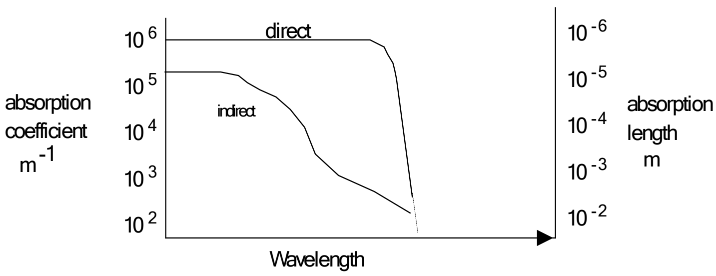
Direct bandgap semiconductor (example GaAs and InGaAsP)





Indirect bandgap semiconductor (e.g. Silicon)





(b) (i) Consider circuit with an ideal voltage source, V, of 5 V, and an internal resistance, R_{int}, of 1 Ω, a series resistance, R, in series with the LED which has a forward voltage V_f. (= hc/λ) where h is Plank's constant, λis the wavelength and c is the free space speed of light.

The quantum efficiency of the LED is η_{ext}. $\eta_{int} = \eta_{ext}.((1/\tau_{rr})/((1/\tau_{nr})+ (1/\tau_{rr})))$

As a result the output power, P, = $\eta_{ext}.((1/\tau_{rr})/((1/\tau_{nr})+ (1/\tau_{rr}))). (V_f I/e)$

where I is the current and e is the electronic charge

However $I = (V – V_f)/(R + R_{int})$, so

P, = $\eta_{ext}.((1/\tau_{rr})/((1/\tau_{nr})+ (1/\tau_{rr}))). (V – hc/\lambda)/(R + R_{int}) . (hc/\lambda /e)$

Inserting known values => **R = 23.8 Ω**

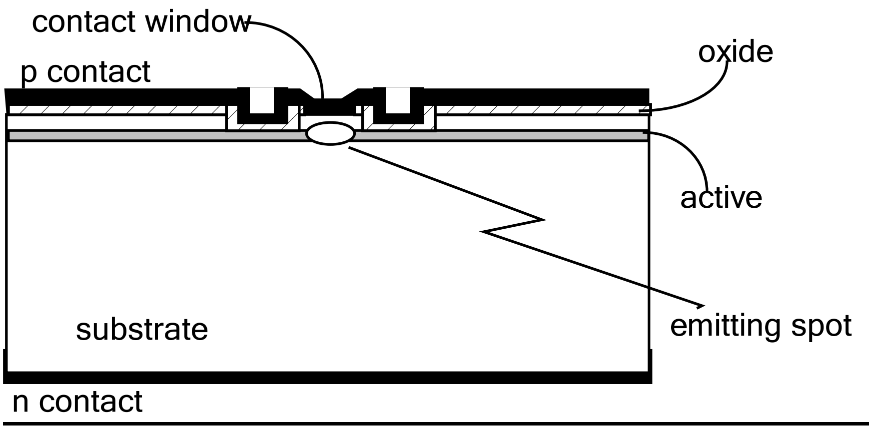
(ii) $T_o = (T – T_1) . \ln (P_T/P_{T1})$ = **87 k**

(iii) Using data sheet, $\beta_{T1}t_{l1} = \beta_{T2}t_{l2}$ => $t_{l1/}\ t_{l2} = \beta_{T2/}\ \beta_{T2} = \exp(-E_a/k(1/T_1-1/T_2))$ = **0.0177**

(c) A good answer should indicate how doping can be used to pin excitons and allow materials to exhibit direct bandgap performance. Separately the recombination rate can be approximated to $R = B.N_a.n_{inj}$ and as a result, doping can be used to increase bandwidth.

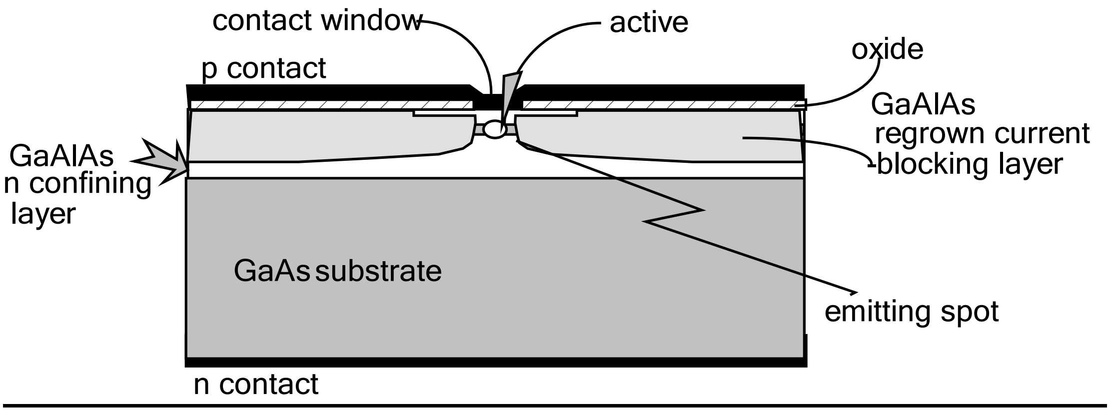
Q.2 (a) This is primarily a bookwork question. A good answer might include the following points:

Ridge laser schematic end view



The ridge laser has reasonable current confinement, and the shaped surface provides a weak but adequate lateral waveguide. This design is very widely used for telecommunications.

Buried heterostructure laser schematic end view



The Buried Heterostructure laser has excellent current confinement and waveguiding, but is difficult and expensive to make, and is not quite as reliable as the ridge laser. The current confining layers are of $Ga_{0.4}Al_{0.6}As$ doped with germanium, which gives a high resistivity. These structures are actually more popular with InP / GaInAsP materials, where good quality semi-insulating semiconductor cannot be grown, so in this case the current blocking layer consists of a p-n-p sandwich of InP, which together with the n InP of the substrate gives an n-p-n-p or "thyristor" structure which always has one reverse biased junction and blocks current under low bias conditions.

Overall ridge lasers have excellent single transverse mode performance up to high optical powers, are very reliable and relatively simple to construct. Buried heterostructure devices show superior confinement properties and hence can exhibit very low threshold currents and high modulation rates. Their fabrication is more complex than for ridge laser diodes. A good answer will include typical performance values.

(b) The rate equations below describe the dynamic properties of a diode laser. In this case, the assumptions made are (1) the carrier, photon and current densities are constant in the diode laser throughout its volume, (2) that the laser generates purely monochromatic light in one mode, (3) that the amplification of light by stimulated emission is linear with carrier concentration and, (4) that temperature effects are negligible.

The parameters below are as follows: n is the carrier concentration in the laser, P is the photon density in the lasing mode, g is a gain constant, n_o is the transparency carrier density (where gain = loss), τ_s is the spontaneous recombination time of carriers, τ_p is the photon lifetime in the cavity (ie the effective time for which the photon remains in the cavity after generation before either leaving or being reabsorbed), β is the coupling coefficient (that ratio of spontaneous emission at the lasing wavelength to that generated totally, typically small ~ 10^{-4} or 10^{-5}), V is the laser active region volume, e is the electronic charge and I is the laser current.

The physical representations of the different terms are as below.

$$\underbrace{\frac{dn}{dt}}_{\text{Nett change of carrier concentration in laser/s}} = -\underbrace{\frac{n}{\tau_s}}_{\text{Depletion of carriers through generation of photons by spontaneous emission}} + \underbrace{\frac{I}{eV}}_{\text{Rate of increase of carrier cone by current injection}} - \underbrace{g(n-n_o)P}_{\text{Depletion of carriers through generation of photons by stimulated emission}}$$

$$\frac{dP}{dt} = g(n - n_o)P + \beta\frac{n}{\tau_s} - \frac{P}{\tau_p}$$

Nett change in photon density of lasing mode/s

Increase in photon density by stimulated emission

Increase in photon density of lasing mode by spontaneous emission

Decrease due to losses in laser and from facets

(c) Consider the steady state situation, dn/dt = dP/dt = 0 and assume that $\beta = 0$. Rewriting the photon rate equation,

$$0 = g(n - n_o)P - P/\tau_p$$

$$=> \qquad P\{g(n - n_o) - 1/\tau_p\} = 0$$

As P may have values greater than 0 (and not less!),

$$g(n - n_o) - 1/\tau_p = 0$$

$$=>$$

$$n = n_o + 1/(g\tau_p)$$

However all the terms on the right hand side of the equation are constants. Maintaining a steady state for all values of lasing photon density greater than zero, the carrier constant in the laser is constant. Let this value be called the threshold carrier density, n_{th}.

Considering the electron rate equation,

$$0 = -g(n - n_o)P - n/\tau_s + I/eV$$

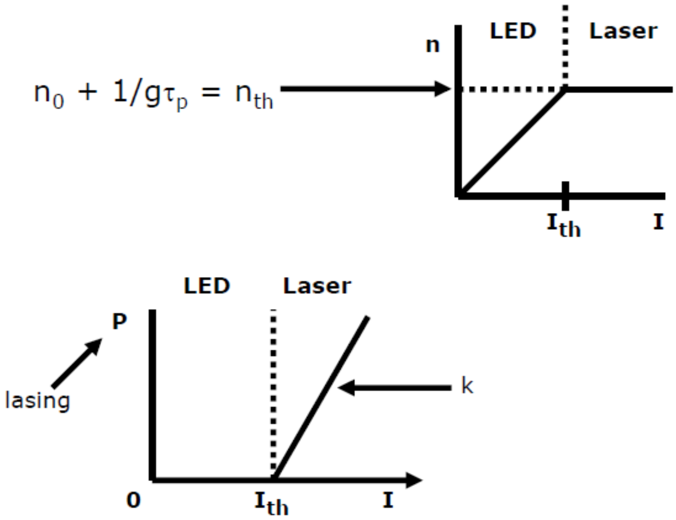
But $n = n_{th}$ for all P>0, so in this regime,

$$P = \frac{\{I/eV - n_{th}/\tau_s\}}{g(n - n_o)}$$

Let $I_{th} = eVn_{th}/\tau_s$,

$$=> \qquad P = \frac{\{I - I_{th}\}}{eV\,g(n_{th} - n_o)} = k(I - I_{th})$$

As a result the optical power generated by the laser may be shown to be proportional to the current above the threshold current. Below threshold, when P = 0 (there is no lasing light generated), the electron rate equation becomes simply, $n = I\tau_s/eV$, so that the overall operation of the laser can be understood. The carrier concentration in the laser increases linearly with current until threshold, when it saturates to a constant value. Below lasing, no stimulated emission is emitted but above, the light increases linearly with current.

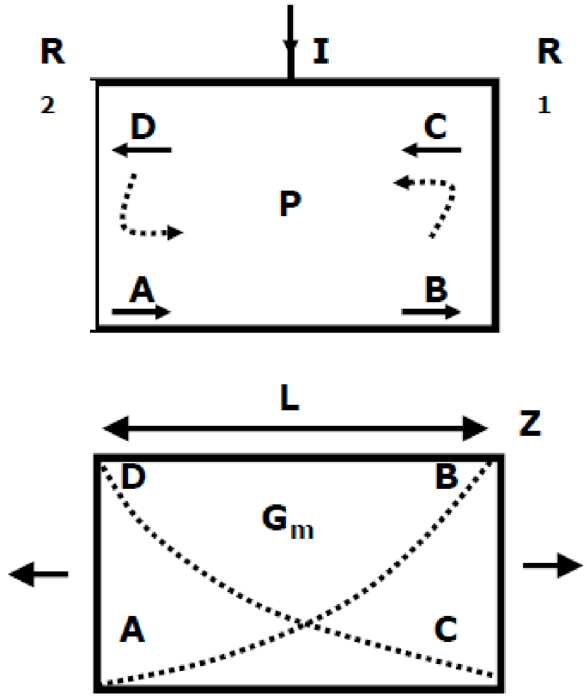


(d) To determine the total output power from the laser diode, consider the optical cavity.

$$dP/dz = G_m P$$

$$=>$$

$$P(Z) = P(o)\exp(G_m z)$$



The photon lifetime of the laser cavity can be readily determined by considering the amplification of laser light as it propagates along the laser cavity. Assume that stimulated emission encounters a gain per unit length (due to stimulated amplification), G, and a loss per unit length due to scattering and absorption, α, as it passes along the laser. The gain G in practice creates extra photons to compensate for those photons lost as the signal travels over a distance of unit length.

Therefore the stimulated light A starting at one facet will be incident on the opposite facet with an optical power

$$B = \exp\{(G - \alpha)L\}\ A$$

At that point part of the signal is reflected with a coefficient R and the signal then passes back amplified by 1 the same amount as above and again reflected by the initial facet. Lasing action will occur in the net round trip gain of the signal is unity i.e. if

$$A.\ \exp\{(G - \alpha)L\}.R_1\exp\{(G - \alpha)L\}.R_2 = A$$

$$=> \quad G = \alpha + (1/2L)\ln(1/(R_1R_2))$$

This value of G is equal to the ratio of photons lost as the signal travels a unit length. Hence the proportion of photons lost per unit time is simply the gain G times the speed of light in the laser material, v_g (i.e. gain/length x length/time). As a result the average time for which one photon will remain in the cavity is given by $\tau_p = 1/Gv_g = 1/\{v_g\{\alpha + (1/2L)\ln(1/R_1R_2)\}\}$

The total output power from the laser may be determined readily in terms of the laser cavity photon density, P, noting that the light output from the laser is that equivalent to a loss per unit length of $(1/2L)\ln(1/(R_1R_2))$.

As a result the proportion of photons leaving the cavity per unit time is given by

$$(1/2L)\ln(1/(R_1R_2)).v_g$$

=> the total output power is given by,

$$P_{out} = (hc/\lambda)(PV) \quad (v_g/2L).\ln(1/(R_1R_2))$$

Energy of

one photon

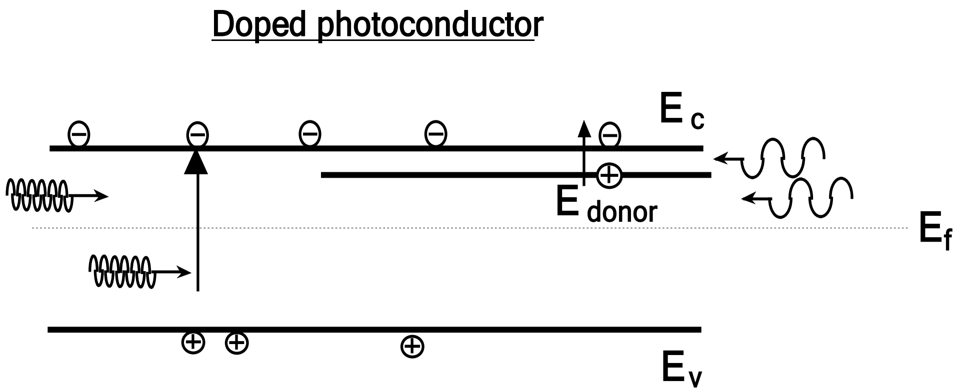
Total number of Photons in laser

Density x volume

$$=> P_{out} = hcPVv_g\ln(1/R^2)/(2\lambda L) \qquad \text{if} \quad R_1 = R_2 = R$$

3 a) Bookwork but an answer might include:

In order to detect wavelengths >about 1.6μm, we have to use a semiconductor with a bandgap less than the normal Ge or GaInAs/InP. Materials for these wavelengths are difficult to handle and produce good junctions, and in any case junction leakage becomes large. Therefore, instead of exciting electrons all the way across the bandgap in a piece of Silicon, we can consider introducing a donor level inside the bandgap, with energy gap between it and the conduction band slightly less than the photon energy to be detected. Thermal exitation will ionise a significant proportion of the donor atoms if the energy gap is less than about 5kT. Using the relationship λ = 1.24/E this implies we can use this technique for wavelengths out to about 10μm at room temperature, but cooling will improve sensitivity. Beyond 10μm cooling becomes almost essential.



b) i) If the conductivity of the photoconductor material is σ, then the current in the block of material will be:

$$I = V.A.\sigma / L$$

and hence the increase in current due to any increase in conductivity caused by photogenerated carriers is:

$$\Delta I = V.A.\Delta\sigma / L$$

But the increase in conductivity,

$$\Delta\sigma = q\,(\,\mu_e.\Delta n \;+\; \mu_h \Delta p\,)$$

But the increase in electron numbers, Δn , is equal to the rate of *generation* of electrons, g , multiplied by the recombination time for electons, τ_r.The hole generation rate will be the same, and we can almost always use the same recombination time τ_r ,, so:

$$\Delta n \;=\; \Delta p \;=\; g\,.\tau_r$$

so substituting in the equations above, the increase in current due to optical absorption in the photoconductor is:

$$\Delta I = \frac{g.\tau_r.e.V.A}{L}\left(\mu_e + \mu_p\right).$$

but the current generated in a photodiode would be simply the arrival rate of photons integrated over the active volome. ie.

$$\Delta I_{photodiode} = g.e.L.A$$

And we define photoconductive Gain as:

$$G = \frac{\Delta I_{photoconductor}}{\Delta I_{photodiode}} = \frac{\frac{g.\tau_r.e.V.A}{L}\left(\mu_e + \mu_p\right)}{g.e.L.A} = \frac{\tau_r.V}{L^2}\left[\mu_e + \mu_p\right]$$

The equation above is not obvious in its meaning, but physical understanding can be gained from noting that:

$$\frac{V}{L} = E \quad and \quad v_e = E.\mu_e \quad and\ similarly \quad v_p = E.\mu_p$$

Substituting in (3.7) from (3.8) we then have:

$$G = \frac{\tau_r}{L}\left[v_e + v_p\right]$$

But

$$\frac{v_e}{L} = \frac{1}{\tau_{et}} \quad and \quad \frac{v_p}{L} = \frac{1}{\tau_{pt}}$$

where τ_{et} , τ_{pt} *are the electron and hole transit times respectively*, and so:

$$G = \frac{\tau_r}{\tau_{et}} + \frac{\tau_r}{\tau_{pt}}$$

ii) $G = \frac{\tau_{re}}{\tau_{te}} + \frac{\tau_{rh}}{\tau_{th}} = \frac{0.4}{1\times10^{-6}} + \frac{0.2}{3\times10^{-6}} = 4.66 \times 10^5$

c)

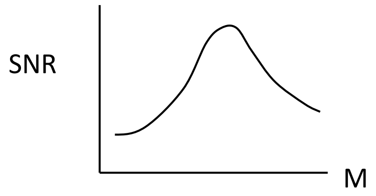
$SNR = \frac{I_{PD}^2 R_f}{<i_n^2> R_f} = \frac{\left(gP_{opt}\right)^2}{2e\left(gP_{opt}+I_d\right)BW+\frac{4kTBW}{R_f}} = 5.01, \quad g = \frac{e\eta\lambda}{hc} = 0.854\ A/W$

$\Rightarrow \left(gP_{opt}\right)^2 - 16.032 \times 10^{-15}\left(gP_{opt}\right) - 12.582 \times 10^{-21} = 0$

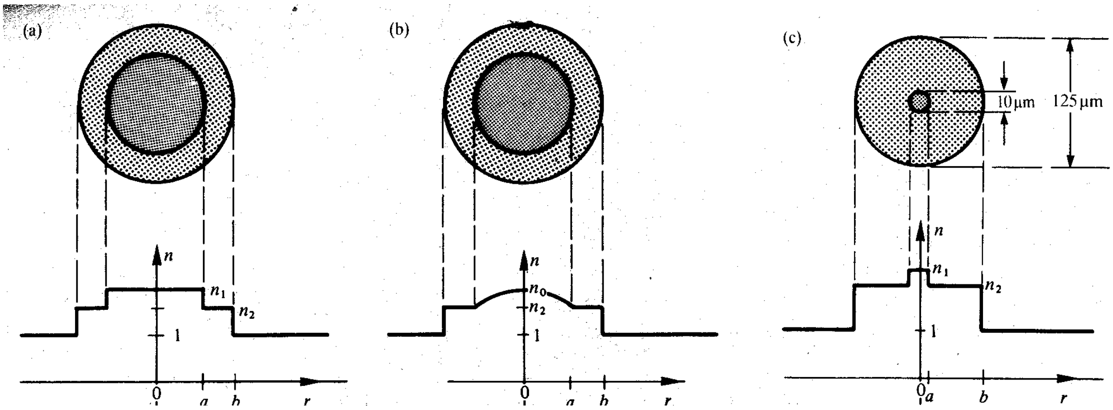
$\Rightarrow gP_{opt} \approx 1.12 \times 10^{-10} A \Rightarrow P_{opt} \approx 1.31 \times 10^{-10}\ W = 0.131\ nW = -68.8\ dBm$

d) i) Assuming the sensitivity of the circuit is limited by thermal noise, then the sensitivity can be improved by cooling. Going from approx 303K to 77k (liquid nitrogen) would improve the sensitivity by a factor of $\sqrt{(\frac{303}{77})}$ or approximately 2.

ii) Use an avalanche photodiode. As the avalanche gain is increased from M=1, the SNR initially rises since the signal power rises as a function of M^2 and the dominant thermal noise is unaffected. However, as a result of the excess noise factor x, the shot noise increases at a faster rate than the signal, as a function of increasing power. It is therefore apparent that there is an optimum multiplication factor for the lowest signal to noise ratio. This is easily determined by differentiating the signal to noise ratio with respect to the multiplication factor and setting the derivative to zero for the optimum operating point.



Q4. a) There are three main types of optical fibre, namely step index (SI) multimode fibre, graded index multimode fibre and step index single mode fibre. The refractive index profiles are shown below. There are two main types of multimode fibre. Each of these has a core diameter typically in excess of 50μm. Step index multimode fibre is relatively cheap, easy to handle and to join together, but it does suffer from high dispersion, and therefore limited bandwidth. Graded index multimode fibre is the most expensive type but it combines the other advantages of SI multimode fibre with greatly reduced dispersion. This is the predominant fibre type used in in-building (up to 550m) applications. Finally, SI single mode fibre has a core diameter <10μm – resulting in only one mode being allowed. This means that dispersion is low but the fibres are quite difficult to handle. Because of the low dispersion, it is the only choice for long distance transmission (>2km – 10,000km).



SI MMF has extremely high dispersion and is therefore used for e.g. sensing or low bandwidth communications applications (e.g. car based networks, home networks). Wavelengths are usually those of low cost sources (.e.g. 650nm for POF and 850nm for glass SI-MMF)

GI MMF has moderate dispersion and loss. It is usually used for datacommunications applications over building scales (up to ~ 500m). Data rates can be as high as ~10Gb/s. Typical wavelengths used are 850nm or 1300nm for slightly higher performance applications.

SI-SMF has the lowest dispersion and loss. The lowest loss is at 1550nm which also happens to be the gain wavelength of the EDFA, so this is the most used wavelength regime. Dispersion is low but non-zero at this wavelength (but DCF is available). Sometimes used at the zero dispersion wavelength of 1300nm in non amplified applications.

(b)

(i) $\eta = NA^2 = n_{co}^2 - n_{clad}^2, n_{co} = 1.53, n_{clad} = 1.50 \Rightarrow \eta = 0.0909 \Rightarrow \sim 9\ \%$

(ii) $\Delta t = \frac{L}{c}\frac{n_{co}}{n_{clad}}(n_{co} - n_{clad}) = D \times L, n_{co} = 1.53, n_{clad} = 1.52, \Rightarrow\ D = 0.033\ ns/m$

(c)

(i) dispersion limit

$t_{out} = 1.3 \times t_{in}$, $t_{out}^2 = t_{in}^2 + \Delta t^2 \Rightarrow \Delta t_{max}^2 = 0.69\, t_{in}^2$, $\Rightarrow \Delta t_{max} = 0.83\, t_{in} = 8.3\, ns$

$L_{max} = \frac{\Delta t_{max}}{D} = 249m$

(ii) need attenuation limit ≥ dispersion limit

Received power at length Lmax

$$P_{Rx}(dBm) = P_{Tx}(dBm) - Loss\,(dB) - Margin\,(dB)$$

Link loss includes attenuation (1.3 dB/km × L), coupling and connector loss (1 dB)

$\eta = NA^2 = n_{co}^2 - n_{clad}^2, n_{co} = 1.53, n_{clad} = 1.52 \Rightarrow \eta = 0.0305 \Rightarrow Coupling\ loss = 15.16\, dB$

$P_{Rx}(dBm) = 0\, dBm - 1\, dB - 1.3 \frac{dB}{km} L_{max}\,(km) - 15.16\, dB - 3\, dB = -19.48\, dBm$

$$S_{Rx} \leq P_{Rx} \Rightarrow S_{Rx} \leq -19.5\, dBm$$

(iii) It is necessary to increase the bandwidth of the link. An LED is typically limited to a few 100 Mb/s at most so a laser (typically a VCSEL for MMF) would be used. Higher bandwidth fibre would be necessary also. GI-MMF of sufficient bandwidth would probably be the best choice from a practical point of view, though SMF would also work. The link loss would also need to be reduced, but coupling light from a laser is much more efficient than butt coupling an LED (Lamertian source).