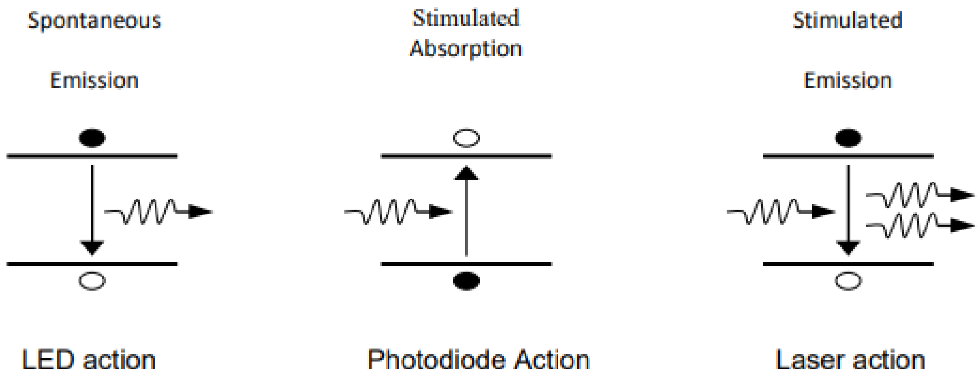
3B6 Crib 2024

Q1 (a) Bookwork : answer should include at least



- **Spontaneous Emission:** An electron in a high energy level falls, losing energy which is emitted as a photon – the basis of operation of a light emitting diode.

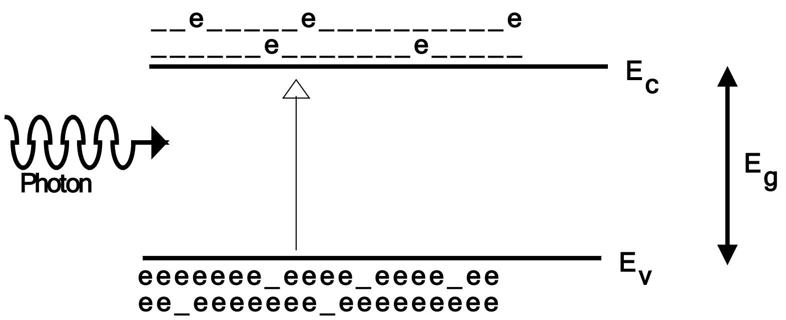
- **Stimulated Absorption:** An incident photon is absorbed in a material, causing the excitation of an electron to a higher energy level – the basis of operation of a photodiode.

- **Stimulated Emission:** A photon, incident upon an electron in a higher energy level, causes the electron to fall to a lower level thus generating a second photon. This is, therefore, an amplifying action. Two photons are generated from one and, in turn, they can cause the generation of two further photons. Using this method, high optical powers can be generated and this operation is the basis of lasing action. The generated photon has the same frequency and phase as the incident photon and, therefore, very pure monochromatic and coherent light is generated.

In respect of direct and indirect bandgap materials, both emission and absorption processes are affected, with direct bandgap materials being more efficient in all cases:

Very good answers might include some of the below: not all would be expected to get full marks

Absorption in the temporal domain causes excitation of a valence band electron to a vacant state in the conduction band in a semiconductor material.



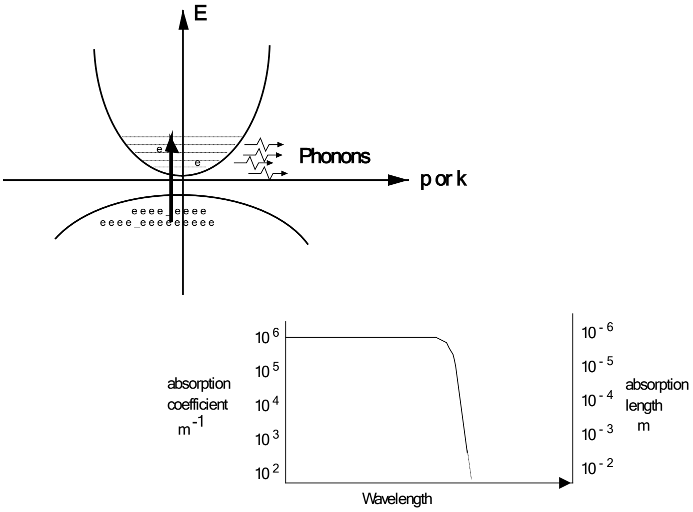
Photon energy:

E_{photon} = hc/λ > E_g

The absorption process itself depends upon the nature of the material being illuminated. In semiconductors, for example, absorption within direct bandgap materials can readily occur without requiring momentum changes for the excited carriers. In indirect materials, phonons are frequently involved, allowing substantial momentum changes. This results in different absorption properties.

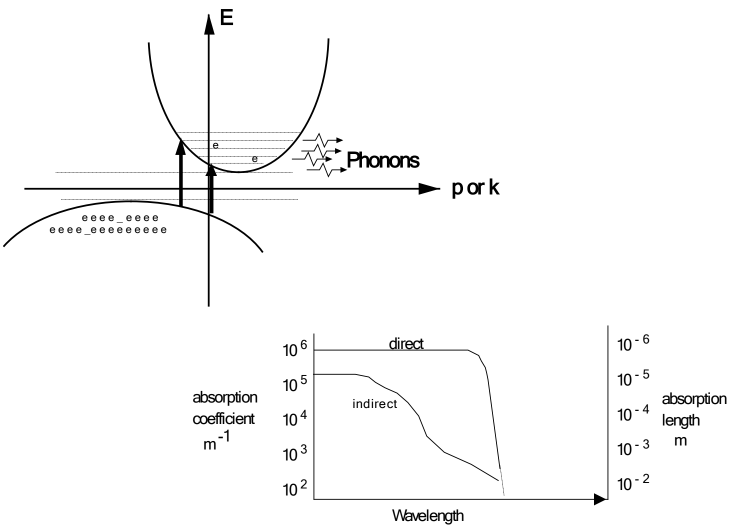
This part was generally well answered.

Direct bandgap semiconductor (example GaAs and InGaAsP)

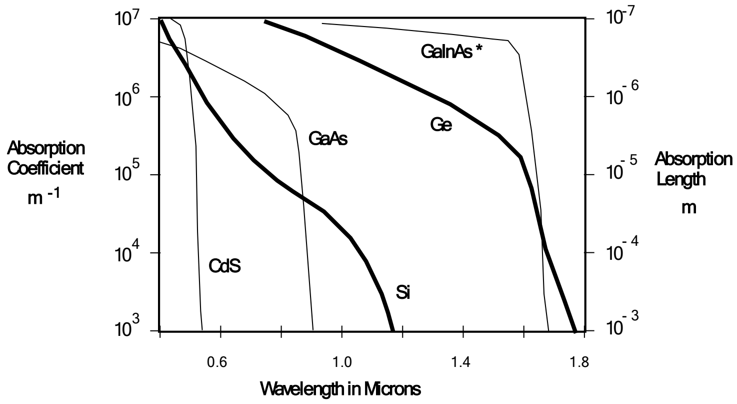


$$E_{ph} \geq E_g \quad => \quad \lambda_{ph} \leq \lambda_g$$

Indirect bandgap semiconductor (e.g. Silicon)

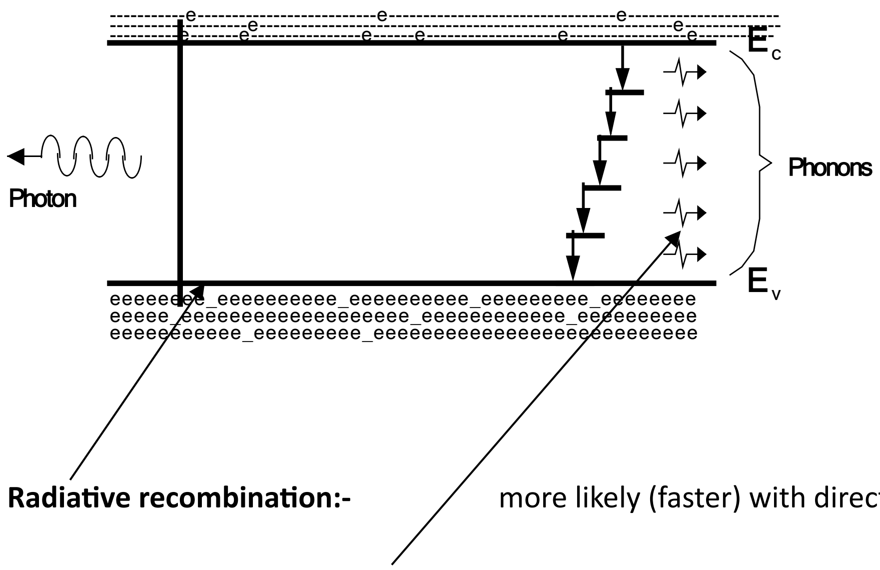


Absorption Characteristics of Common Semiconductors:



(NB GaAs and GaInAs are direct bandgap materials whereas Ge and Si are indirect.)

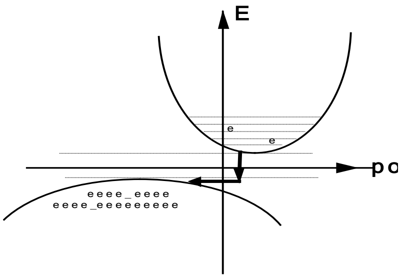
Light Emission from Semiconductors: Carrier Recombination



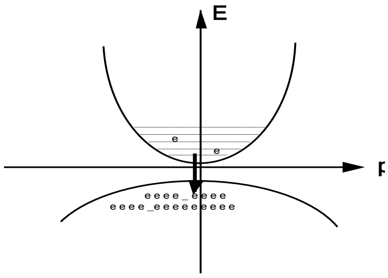
Radiative recombination:- more likely (faster) with direct gap.

Non-radiative recombination:- doesn't depend much on band structure. More likely (faster) with imperfect and/or impure crystals.

Light emission in semiconductors is usually by direct injection of electrons. This leads to the generation of photons when the electrons recombine with holes at a junction. Whilst absorption is achieved at many wavelengths, with energies greater than the bandgap, spontaneous emission only occurs at energies close to the bandgap corresponding to the likely energy separations of holes and electrons.



Indirect semiconductor: needs both phonon and photon of correct energy – not very likely



Direct bandgap semi-conductor: needs only a photon of correct energy – much more likely

b) i) $E_g = hc/e\lambda$ -> inserting values, bandgap = 2.75 eV

ii) Hence 2.75V is dropped across the LED in forward bias.

5V = 2.75V + (2+R)I => I = (5 – 2.75)/(2 + R)

Output power = $\eta_i \eta_e$ (hc/λ) I/e

$\eta_i = 1/\tau_{rr} / (1/\tau_{nr} + 1/\tau_{rr}) = 0.667$

Substituting values, I = 109/1 mA

Rearranging voltage equation and substituting values, R = 18.6 Ω

Few students correctly answered the part on electrical to optical power conversion efficiency

iii) $$\frac{P(T)}{P(T_1)} = e^{-\left(\frac{T-T_1}{T_0}\right)}$$

Rearranging and taking natural log

$T_0 = (T_1 - T)/\ln(P(T)/P(T_1)) = 54$ K

iv)

$$P(t) = P(0)\,e^{-\beta t}$$
$$\beta = \beta_0\, e^{-\frac{E_a}{kT}}$$

=> at half power $0.5 = \exp(-\beta_1 t)$, => $\beta_1 = 1.386 \times 10^{-4}$ hr^{-1}

From half power equation $\beta\, t_{1/2} = \beta_1 t_{1\,1/2}$ => $t_{1/2} / t_{1\,1/2} = \beta_1 / \beta = \exp - E_a/k(1/T_1 - 1/T) = 3597$

So half power lifetime at 20 °C = 18,160 hours (2.07 years)

c) A good answer should indicate how doping can be used to pin excitons and allow materials to exhibit direct bandgap performance. Separately the recombination rate can be approximated to $R = B.N_a.n_{inj}$ and as a result, doping can be used to increase bandwidth and hence reduce risetime.

Most students were able to explain how doping could reduce the spontaneous lifetime, and hence the rise time, but few could answer how it could be used to improve efficiency.

Q2 a) Bookwork

Most candidates could describe the basic requirements for lasing (gain and feedback) but didn't provide a very good description of the resulting relatinve performances of LEDs and lasers.

In describing *operating* conditions for diode lasing, a typical answer should indicate that two main conditions must be achieved: (i) stimulated amplification must be stronger than absorption so that any optical signal is rapidly amplified in power, and (ii) some form of optical feedback must be provided so that the lasing light generated, can in part be fed back so that stimulated amplification can continue to occur, thus causing sustained emission and hence lasing output.

In order to achieve good power efficiency *(i.e. steps)*, good confinement of current injection and minimisation of unwanted optical losses must be achieved. A good answer will list some structures that can be used in laser diodes to achieve this.

As a result of these requirements, in a typical laser system, much more care must be taken to ensure that the light does not scatter or "leak" out of the lasing region. It is also important to ensure that an optical cavity is bounded by reflectors, so that a lasing filament is formed which oscillates back and forth within the cavity, and that the generated light is confined to cause further stimulated emission. By using partial reflectors, some of the light is emitted from the cavity as the output from the laser.The formation of such a cavity, however, has a major effect on the form of optical spectrum generated.

b) Most students could calculate the required current above threshold, but many didn't realise they needed to add to the threshold current.

i) $P = \eta\, hc/e\lambda\, (I - I_{th})$

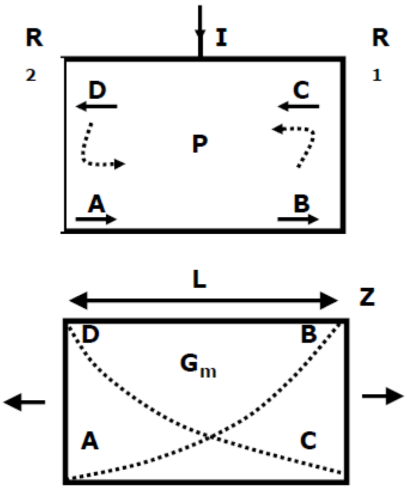
Rearranging and substituting in values, I = 32.4 mA

ii) Most made a good attempt at the proof for differential efficiency, but didn't explain the bottom line of the ratio.

$dP/dz = G_m P$

=>

$P(Z) = P(o)\exp(G_m z)$



The photon lifetime of the laser cavity can be readily determined by considering the amplification of laser light as it propagates along the laser cavity.

Assume that stimulated emission encounters a gain per unit length (due to stimulated amplification), G, and a loss per unit length due to scattering and absorption, α, as it passes along the laser. The gain G in practice creates extra photons to compensate for those photons lost as the signal travels over a distance of unit length.

Therefore the stimulated light A starting at one facet will be incident on the opposite facet with an optical power

$$B = \exp\{(G - \alpha)L\}\, A$$

At that point part of the signal is reflected with a coefficient R and the signal then passes back amplified by 1 the same amount as above and again reflected by the initial facet. Lasing action will occur in the net round trip gain of the signal is unity i.e. if

$$A.\ \exp\{(G - \alpha)L\}.R_1 \exp\{(G - \alpha)L\}.R_2 = A$$

=> $G = \alpha + (1/2L)\ln(1/(R_1R_2))$ *[N.B. Gain/unit length]*

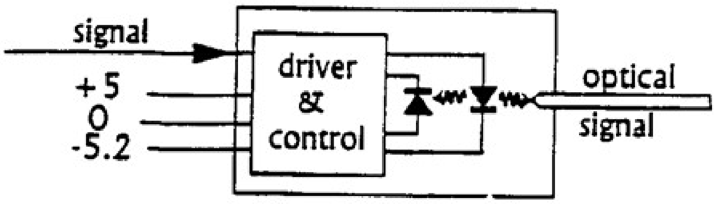
Effective gain needed to overcome facet losses

Above threshold the differential efficiency is simply the proportion of photons leaving the cavity through the facets over the total number of photons (those lost to the cavity at the facets plus those lost to scattering), ie

$$\eta_D = \frac{\ln(1/(R_1R_2))/(2L)}{\alpha + \ln(1/(R_1R_2))/(2L)}$$

Substituting the given values, R = 0.571

iii)



Assume η_D doesn't degrade with temperature

$$J_{th}(T) = J_0\, e^{\frac{T}{T_0}}$$

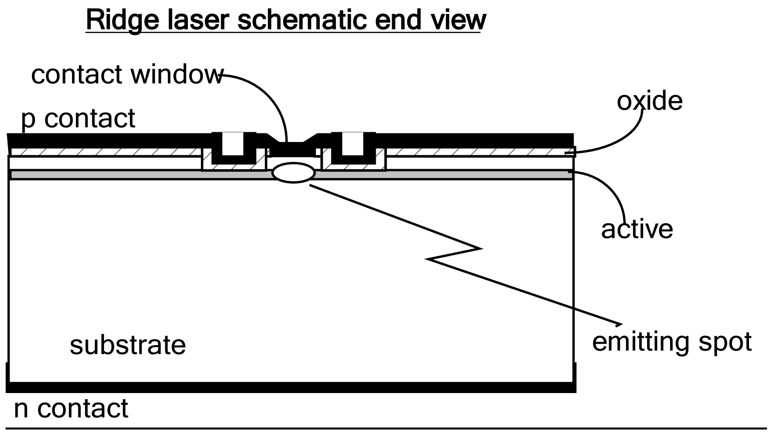
Current above threshold at room temperature is therefore the same as at the high temperature, i.e. 8.4 mA, so at this temp, I_{th} = 41.6 mA

So 41.6/24 = exp (T_{max}/T_0) / exp (T/T_0)

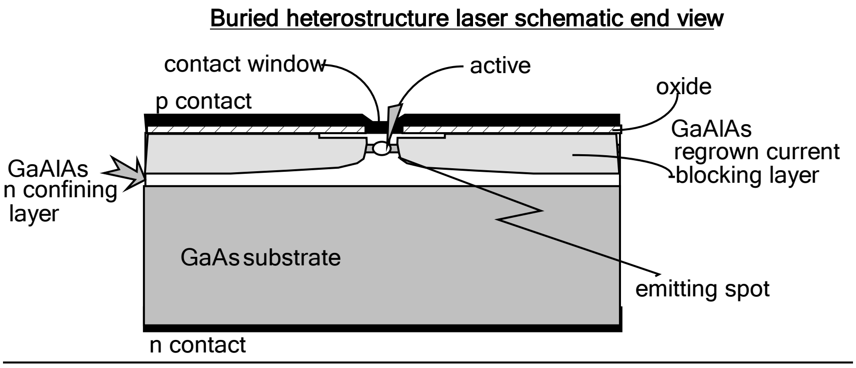
Substituting, T_{max} = 69.5 °C

iv) Increasing the reflectivity reduces the required threshold current as more photons are recycled in the cavity but also results in fewer leaving each round trip and hence the slope efficiency (external quantum efficiency is reduced). So there is a trade off been the efficiency of achieving lasing (better with high reflectivity) with the efficiency of extracting the lasing photons once they are created. For a particular required output power and laser active region design, there will be an optimum reflectivity. For a slightly increased cost it is possible to have different reflectivities at either end of the cavity – so if you only want to extract light at one of the cavity, use 100% relectivity at the other end and reduced reflectivity at the other end to maximise useful light extraction for a given R_1R_2 product.

c) Most could describe the structure and relative merits of the ridge and buried heterostructure lasers.



The ridge laser has reasonable current confinement, and the shaped surface provides a weak but adequate lateral waveguide. This design is very widely used for telecommunications.



The Buried Heterostructure laser has excellent current confinement and waveguiding, but is difficult and expensive to make, and is not quite as reliable as the ridge laser. The current confining layers are of $Ga_{0.4}Al_{0.6}As$ doped with germanium, which gives a high resistivity. These structures are actually more popular with InP / GaInAsP materials, where good quality semi-insulating semiconductor cannot be grown, so in this case the current blocking layer consists of a p-n-p sandwich of InP, which together with the n InP of the substrate gives an n-p-n-p or "thyristor" structure which always has one reverse biased junction and blocks current under low bias conditions.

Q3 (a) Answers must include the two categories: (a) those which result from the distortion of the fiber from the ideal straight line configuration (extrinsic losses) and (b) those that are inherent in the fiber itself (intrinsic losses).

A lot of students missed the extrinsic losses.

Very good answers might include some of the below: not all would be expected to get full marks

Extrinsic Losses - Bend Loss

When a fibre is bent into an arc of a circle, the loss is found to be strongly dependent on the radius of curvature, and a semiempirical expression that is sometimes used to describe the loss is

$$\alpha_B = C \exp\left(-\frac{r}{r_c}\right)$$

where αB is the absorption coefficient due to the bend, r the bend radius and rc a constant which depends on the fibre parameters. Quite considerable losses can be observed when bend radii of the order of millimetres or less are encountered. In practice, such deformations are avoided.

Intrinsic Losses

Losses intrinsic to silica fibres have two main sources, scattering and absorption losses.

Scattering Losses

So far we have assumed in our discussion of light propagation in fibres that the material is homogeneous. However, silica is an amorphous material and thus suffers from structural disorder: that is, the same basic molecular units are present throughout the material but these are connected together in an essentially random way. This results in a fluctuation in the refractive index through the material with each irregularity acting as a point scattering centre. The scale of the fluctuations is of the order of $\lambda/10$ or less, and the scattering from these centres is known as Rayleigh scattering and characterised by an absorption coefficient that varies as λ^{-4}.

Absorption Losses

Absorption losses in the visible and near-infrared regions arise mainly from the presence of impurities, particularly traces of transition metal ions (e.g. Fe3+, CU2+) or hydroxyl ions (OH-). The latter are responsible for absorption peaks at 0.95μm, 1.24μm and 1.39μm. Most of the dramatic successes in reducing fiber losses came about because of better control of impurity concentrations.

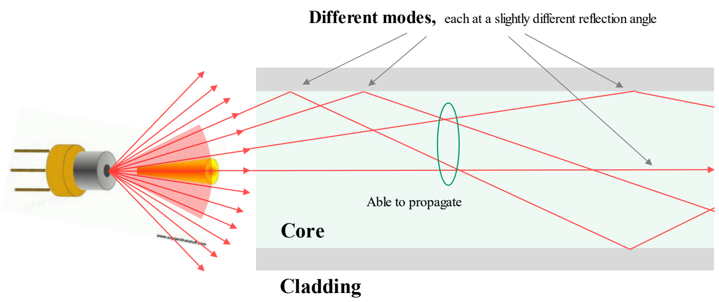
At wavelengths greater than about 1.6 μm, the main losses are due to transitions between vibrational states of the lattice itself. Although the actual fundamental absorption peaks occur at wavelengths well into the infrared (in SiO_2, for example, the main peak is at 9 μm), an incoming photon can simultaneously excite two or more fundamental lattice vibrations (or phonons). Thus, a number of strong absorption bands extend all the way down to about 3μm with appreciable absorption still occurring below 2μm.

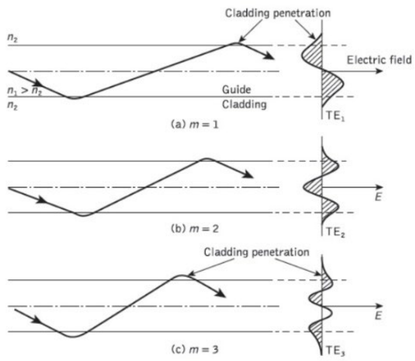
Therefore, the fibre loss cannot be reduced to zero.

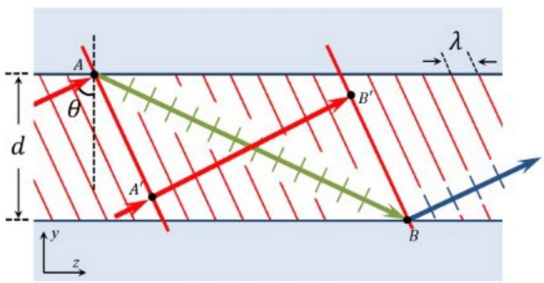
(b) (i) If the fibre is used to transmit 850 nm signals, it will become multi-mode. Therefore, the intermodal dispersion will dominate.

(ii) If the fibre is used to transmit 1550 nm signals, it will stay in single-mode. However, its material dispersion will increase and thus the single-mode dispersion will still increase.

(c) In any optical waveguide, the radiation is restricted to travel down the guide in certain "modes" (multiple light rays). Mode: a light wave whose field distribution does not change as it propagates, except for phase, and each mode corresponds to a different electromagnetic field distribution in the fibre.







$$\frac{2\pi}{\lambda}\overline{AB} + 2\varphi_r - \frac{2\pi}{\lambda}\overline{A'B'} = 2\pi \cdot m$$

$$\frac{2\pi}{\lambda}\overline{AB} + 2\varphi_r - \frac{2\pi}{\lambda}\overline{A'B'} = 2\pi \cdot m$$

m is the mode number
φ_r is the phase change associated with the internal reflections

(d) The number of modes in a step index circular waveguide is determined by a V parameter, often called the normalised frequency. V is a function of the fibre radius (a), the wavelength of light (λ) and the refractive indices of the core and cladding respectively.

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^{\ 2} - n_2^{\ 2}}$$

The n_1 and n_2 refers to the refractive indices of core and cladding respectively, which are constants derived from a step index fibre.

The fibre can only support one mode if V<2.405 and is then called a single mode fibre (SMF).

(e) (i) V=2.49

As V is slight higher than 2.405, according to the Fig. 1, there are only 2 LP modes: LP_{01}, LP_{11}.

Note: The number of modes cannot be estimated using $V^2/2$ as V is NOT far larger than 1.

(ii) If only the fundamental mode of the fibre is excited, it will not suffer from intermodal dispersions. Only waveguide dispersion and chromatic (material) dispersion occur. Thus, it's dispersion penalty will be much improved.

(f) V=1.99, therefore, it is now a single-mode fibre at 1550nm wavelength.

For the dispersion factor to be maintained within ±10 ps/nm-km, the wavelength range has to be from 1240nm to 1490nm.

However, to maintain a single-mode fibre, the V has to be smaller than 2.405, and so the wavelength needs to be over 1.284nm. Thus, the allowed laser wavelength range is between 1284nm to 1490nm.

Note that a number of students did not consider the single-mode impact on the wavelength range.

Q4 (a) Bookwork:

Drift is, by definition, charged particle motion in response to an applied electric field. When an electric field is applied across a semiconductor, the carriers start moving, producing a current.

Carrier diffusion is due to the thermal energy which causes the carriers to move at random even when no field is applied. Because it is this thermal energy which drives the diffusion process, at T = 0 Kelvin there is no diffusion.

Mobility and Diffusion Coefficient: The Einstein Relationship

J=I/A, A, cross section area

The electron current in a semiconductor is given by:

$$J_n = e.\mu_n.n.E + e.D_n.\frac{dn}{dx} \qquad (1)$$

In equilibrium, this must be zero, so:

$$0 = n.\mu_n.\frac{dV}{dx} + D_n.\frac{dn}{dx} \qquad (2)$$

Electrons follow the Fermi–Dirac statistics/distribution, thus, in thermal equilibrium:

$$n = n_o \exp\left(\frac{-eV}{kT}\right) \qquad (3)$$

k is Boltzmann constant

and so:

$$\frac{dn}{dx} = \frac{-e}{kT}.n_o \exp\left(\frac{-eV}{kT}\right).\frac{dV}{dx} \qquad (4)$$

or

$$\frac{dn}{dx} = \frac{-e}{kT}.n.\frac{dV}{dx} \qquad (5)$$

Substituting (5) in (2)

$$0 = n.\mu_n.\frac{dV}{dx} + D_n.\frac{-e}{kT}.n.\frac{dV}{dx}$$

or:

$$\mu_n = D_n.\frac{e}{kT} \quad \text{or} \quad D_n = \mu_n.\frac{kT}{e}$$

Exactly similar relationships apply to holes. Therefore, compared with drift, diffusion is very slow.

(b) Answers would include quantum noise, shot noise, and thermal noise, though shot noise is used to quantify quantum noise in the intensity modulation system.

Quantum Noise

On a sufficiently short time scale the signal might be represented by the quantum fluctuation associated with electron photon interaction. This quantum noise is known as shot noise. In physics, quantum noise refers to the uncertainty of a physical quantity that is due to its quantum origin. In certain situations, quantum noise appears as shot noise; for example, most optical communications use amplitude modulation, and thus, the quantum noise appears as shot noise only. For the case of uncertainty in the electric field in some lasers, the quantum noise is not just shot noise; uncertainties of both amplitude and phase contribute to the quantum noise. This quantum noise leads to a fundamental limit for signal detection in a photodiode. If we assume no noise from any other source a statistical description of the noise process can be used to estimate the minimum number of photons required to detect a signal.

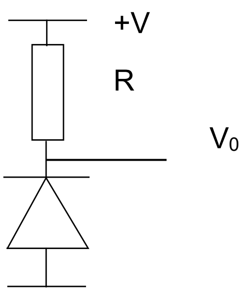
Shot Noise

This is the major inherent source of optical noise in detection systems. The noise originates in optical detection processes from the quantum nature of the fluctuations in the received signal. It is an effect not only observed in optical devices but also in all electrical devices where carriers interact with junctions or interfaces (e.g. at electrical diode junctions).

Thermal or Johnson Noise

Any dissipative element in a system introduces noise due to the random interactions involved in the dissipation process. Thus in an electronic circuit, any resistance gives rise to this (thermal or Johnson) noise as a consequence of the random thermal motions of the charge carriers. For further explanation, this is generated by the thermal agitation of the charge carriers, due to their random motion. This is caused by heat, in regardless of any applied voltage. This may be observed as current fluctuations in the resistive component or as corresponding voltage fluctuations across the terminals.

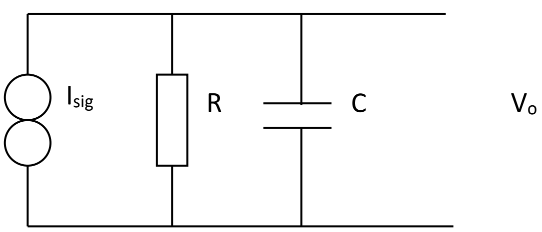
(c) The most basic photodiode bias circuit is shown below.



Here the photocurrent generated in the photodiode also passes through a resistor, thus generating a voltage

$$Vo = I_{sig}R$$

However, there is a capacitance associated with the depletion width of the photodiode and so, once a frequency higher than DC is applied we need to take this into account. It is possible to draw the small signal equivalent circuit for the circuit above.



Consequently the output voltage will be given by the signal current driving the parallel RC combination i.e.,

$V_o = I_{sig} \times (R \,||\, 1/j\omega C)$

$= I_{sig} R / (1 + j\omega CR)$

This gives a 3dB bandwidth of $\omega = 1/CR$ or $f = 1/2\pi CR$.

Bookwork:

Response time can be reduced by reducing C_j

$$C_j = \frac{A}{2}(2e\varepsilon_0\varepsilon_r N_d)^{1/2} V^{-1/2}$$

This can be achieved by reducing the diode area A, the doping level N_d or increasing the bias level V

A can only be reduced so far before it is difficult to couple light into the photodiode

Reduction in N_d and increase in V results in increase in W_{dep}

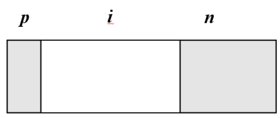
$$\tau_{drift} = \frac{W_{dep}}{v_{sat}}$$

A compromise must be reached therefore between reducing the depletion capacitance and keeping W_{dep} reasonably small

Very good answers might include some of the below: not all would be expected to get full marks

A p-i-n photodiode consists of an intrinsic "undoped" region sandwiched between heavily doped p+ and n+ regions. The depletion layer is almost completely defined by the intrinsic region.

Advantages of *pin* Photodiode



- Increasing the width of the depletion layer *increases the area available for capturing light* – high quantum efficiency
- Increasing the width of the depletion layer *reduces the junction capacitance* and *thereby the RC time constant*
- Yet, the transit time increases with the width of the depletion layer
- Reducing the ratio between the length of diffusion and the length of drift results in *a greater proportion of the generated current being carried by the faster drift process*.

(d) The terms in the bracket refers to conductivity,

$$\Delta\sigma = q\,(\mu_e.\Delta n + \mu_h\Delta p) \quad (3.3)$$

But the increase in electron numbers, Δn , is equal to the rate of *generation* of electrons, g , multiplied by the recombination time for electrons, τ_r. The hole generation rate will be the same, and we can almost always use the same recombination time τ_r ,, so:

$$\Delta n = \Delta p = g.\tau_r \quad (3.4)$$

so substituting in the equations above, the increase in current due to optical absorption in the photoconductor is:

$$\Delta I = \frac{g.\tau_r.e.V.A}{L}\left(\mu_e + \mu_p\right). \quad (3.5)$$

but the current generated in a photodiode would be simply the arrival rate of photons integrated over the active volume, ie.

$$\Delta I_{photodiode} = g.e.L.A \quad (3.6)$$

And we define photoconductive Gain as:

$$G = \frac{\Delta I_{photoconductor}}{\Delta I_{photodiode}} = \frac{\frac{g.\tau_r.e.V.A}{L}\left(\mu_e + \mu_p\right)}{g.e.L.A} = \frac{\tau_r.V}{L^2}\left[\mu_e + \mu_p\right] \quad (3.7)$$

The equation above is not obvious in its meaning, but physical understanding can be gained from noting that:

$$\frac{V}{L} = E \quad \text{and} \quad v_e = E.\mu_e \quad \text{and similarly} \quad v_p = E.\mu_p \quad (3.8)$$

Substituting in (3.7) from (3.8) we then have:

$$G = \frac{\tau_r}{L}\left[v_e + v_p\right] \quad (3.9)$$

But

$$\frac{v_e}{L} = \frac{1}{\tau_{et}} \quad \text{and} \quad \frac{v_p}{L} = \frac{1}{\tau_{pt}} \quad (3.10)$$

where τ_{et} , τ_{pt} *are the electron and hole transit times respectively*, and so:

$$G = \frac{\tau_r}{\tau_{et}} + \frac{\tau_r}{\tau_{pt}} \quad (3.11)$$

The gain corresponds then to the number of electrons that cross the photoconductor before an extra electron (or hole) recombines *plus* the number of holes crossing in the same period.

(e) The link dominated noise source is shot noise.

(e)(i) Shot noise $= 2eM^{2+x}(\mathcal{R}P + I_d)B$

even when $P = 0$ mW

Shot noise $= 2 \times 1.6 \times 10^{-19} \times 100^{2.5} \times 10 \times 10^{-9} \times 20 \times 10^{9}$

$= 6.4 \times 10^{-12}\ A^2$

Thermal noise $= \frac{4kTB}{R_f}$

$= \frac{4 \times 1.38 \times 10^{-23} \times 300 \times 20 \times 10^{9}}{10^3}$

$= 3.312 \times 10^{-13}\ A^2$

Thus Shot noise from dark current is already larger than thermal noise.

The link is shot noise dominated.

(ii) $SNR = \frac{(M\mathcal{R}P)^2}{2M^{2+x}e(\mathcal{R}P + I_d)B + \frac{4kTB}{R_f}}$

$\mathcal{R} = e\eta\lambda/hc = 0.95$

$P = -10\text{dBm} = 0.1\text{mW} = 10^{-4}\ W$

$SNR = \frac{(100 \times 0.95 \times 10^{-4})^2}{2 \times 100^{2.5} \times 1.6 \times 10^{-19} \times (0.95 \times 10^{-4} + 10 \times 10^{-9}) \times 20 \times 10^{9} + \frac{4 \times 1.38 \times 10^{-23} \times 300 \times 20 \times 10^{9}}{10^3}}$

$= 1484 = 31.7$ dB