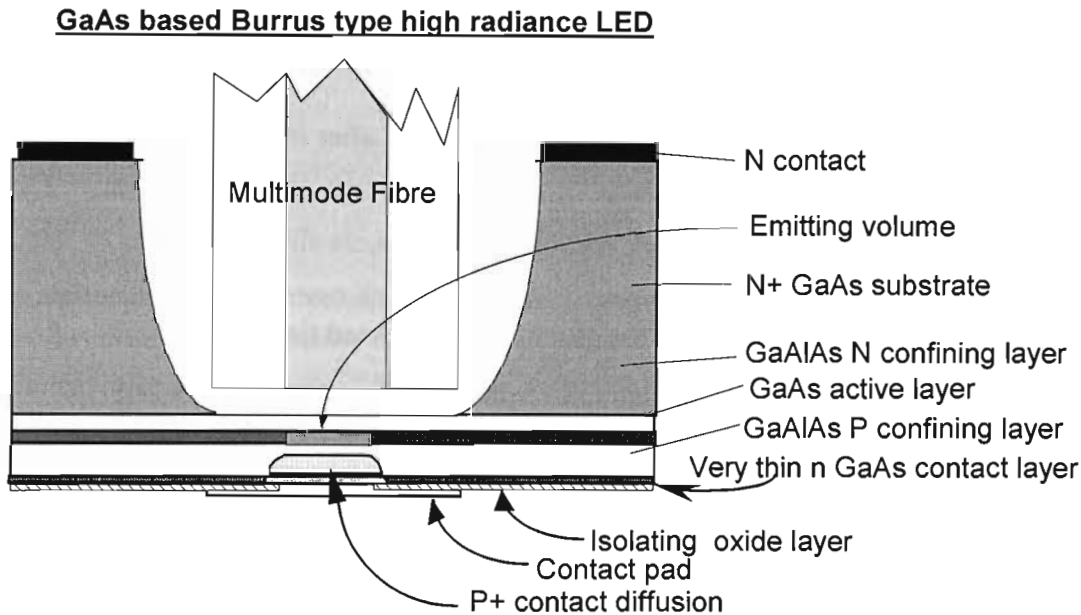


2014 Examination Cribs

Q1 (a) This is largely a bookwork section. A good answer would include a detailed description of the structure of a Burrus/SELED (with perhaps a detailed diagram as below) and discuss its operation, for example including details on current confinement and causes of optical loss. Answers should give details of typical range of colours with predicted numerical performance parameters, materials used and potential applications.



In terms of thermal management, the SELED has much of the substrate etched away: this allows high coupling efficiency into a multimode fibre, and sometimes a spherical micro-lens is interposed as well. In addition the heat generation is close to the p surface which can be bonded directly to a heatsink, and the contact metal also reflects some light back upwards into the fibre. [30%]

(b) (i) The optical power from the LED is given by, $P = \eta I hc / e \lambda$. The current, I , however is set by the voltage dropped across the voltage source resistance and the internal impedance of the SELED, so that,

$$I = (V - V_f) / (R_s + R_{LED}) = (V - (hc / e \lambda)) / R_{tot} \text{ where } R_{tot} = R_s + R_{LED}$$

$$\Rightarrow P = \eta hc (V - (hc / e \lambda)) / e \lambda R_{tot}$$

$$\Rightarrow \eta = e P \lambda R_{tot} / ((hc (V - (hc / e \lambda))) = \underline{0.07} \quad [30\%]$$

(ii) The overall quantum efficiency is given by $\eta = \eta_{int} \eta_{ext} \Rightarrow \eta_{int} = \eta / \eta_{ext} = 0.7$

However $\eta_{int} = (1 / \tau_{rr}) / (1 / \tau_{rr} + 1 / \tau_{nr})$

$$\Rightarrow \tau_{nr} = \eta_{int} \tau_{rr} / (1 - \eta_{int}) = 4.67 \text{ ns} \quad \Rightarrow \tau_c = 1 / (1 / \tau_{rr} + 1 / \tau_{nr}) = \underline{1.4 \text{ ns}} \quad [10\%]$$

(iii) The variation of LED power with temperature is given by

$$P(T)/P(T_1) = \exp[-(T - T_1)/T_0] = 0.44 \text{ for a temperature change from } 20^\circ\text{C to } 85^\circ\text{C}.$$

$$\Rightarrow P(85^\circ\text{C}) = 2.2 \text{ mW} \Rightarrow \text{Change in power} = \underline{2.8 \text{ mW}}$$

[10%]

(iv) The power dependence on time is given as $P(t) = P(0)e^{-\beta t}$ where $\beta = \beta_0 e^{-\frac{E_a}{kT}}$

If the lifetime, t_l , is defined as $0.5 = \exp(-\beta t_l)$, after some manipulation

$$t_l(T_1)/t_l(T_2) = \exp\{(-E_a/k)(1/T_2 - 1/T_1)\} = R \text{ say}$$

$$\Rightarrow E_a = -k \ln(R) / (1/T_2 - 1/T_1) = \underline{4 \times 10^{-20} \text{ J}} = \underline{0.25 \text{ eV}} \text{ after inserting values} \quad [20\%]$$

Q2 (a) (i) This is primarily a bookwork section. Good answers should include:

- A brief description of the three major types of electron/photon interactions in materials, namely Spontaneous Emission, Stimulated Absorption and Stimulated Emission.
- An overall explanation of the use of a pn junction at which stimulated emission occurs, with good answers describing the operation of semiconductor heterostructures.
- An explanation of the use of cleaved facets to allow optical feedback necessary for lasing to be achieved.
- Finally good answers should indicate how stripes, ridges, mesas or buried heterostructures can be used to confine light and carriers in the transverse direction, hence enhancing the device. [15%]

(ii) Again, a bookwork section, where a very brief explanation should be given of the need for direct bandgap materials, with good answers also commenting on the role of heterostructure materials. [5%]

(b) (i) A good answer should state that the rate equations make the following assumptions:

- the carrier, photon and current densities are constant in the diode laser throughout its volume,
- that the laser generates purely monochromatic light in one mode,
- that the amplification of light by stimulated emission is linear with carrier concentration and,
- that temperature effects are negligible.

In terms of the electron rate equation:

$$\frac{dn}{dt} = -\frac{n}{\tau_s} + \frac{I}{eV} - g(n - n_0)P$$

The LHS of the equation concerns the net rate of change of carrier concentration. The RHS has the following terms in order: (i) spontaneous emission, which causes a depletion of carriers per unit time), (ii) current injection (which causes an increase in carrier concentration), and (iii) net stimulated emission and absorption which causes carrier depletion and enhancement respectively.

$$\frac{dn}{dt} = g(n - n_0)P + \beta \frac{n}{\tau_s} - \frac{P}{\tau_p}$$

The LHS of the equation concerns the net rate of change of photon density of the lasing mode. The RHS has the following terms in order: (i) stimulated emission, which causes a growth in photon density, (ii) spontaneous emission (which causes an increase in photon density, but which is diluted by the spectral overlap of the stimulated and spontaneous emission), and (iii) loss of photons from facets and through scattering, as defined by a lifetime. **[10%]**

(ii) Assume that the laser is in steady state, $dn/dt = dP/dt = 0$ and assume that β is very small. Below threshold, when $P = 0$ (there is no lasing light generated), the electron rate equation becomes simply

$$0 = -n/\tau_s + I/eV \Rightarrow n = I\tau_s/eV$$

so that the overall operation of the laser can be understood. The carrier concentration in the laser increases linearly with current until threshold, when it saturates to a constant value.

$$(\Rightarrow I_{th} = eVn_{th}/\tau_s)$$

Rewriting the photon rate equation,

$$0 = g(n - n_0)P - P/\tau_p \Rightarrow P\{g(n - n_0) - 1/\tau_p\} = 0$$

As P may have values greater than 0 (and not less!),

$$g(n - n_0) - 1/\tau_p = 0 \Rightarrow n = n_0 + 1/(g\tau_p)$$

However all the terms on the right hand side of the equation are constants. Maintaining a steady state for all values of lasing photon density greater than zero, the carrier constant in the laser is constant. Let this value be called the threshold carrier density, n_{th} .

$$\Rightarrow I_{th} = (eV/\tau_s) \cdot (n_0 + 1/g\tau_p)$$

$$\text{Considering the electron rate equation, } 0 = -g(n - n_0)P - n/\tau_s + I/eV$$

But $n = n_{th}$ for all $P > 0$, so in this regime,

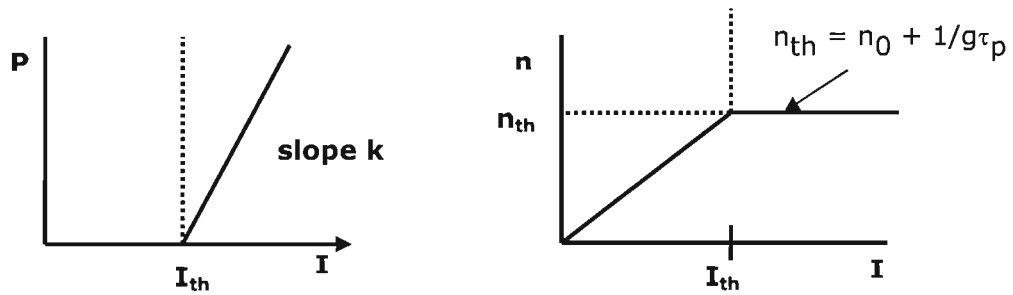
$$P = \frac{\{I/eV - n_{th}/\tau_s\}}{g(n - n_0)}$$

Letting $I_{th} = eVn_{th}/\tau_s$,

$$\Rightarrow P = \frac{\{I - I_{th}\}}{eV g(n_{th} - n_0)} = k(I - I_{th})$$

where the slope, the differential efficiency, $k = 1/eVg(n_{th} - n_0)$ **[20%]**

(iii)



[10%]

(c) From (b), $I_{th} = (eV/\tau_s) \cdot (n_0 + 1/g\tau_p) = (ewhL/\tau_s) \cdot (n_0 + 1/g\tau_p)$

$\Rightarrow L = I_{th}\tau_s / (ewh(n_0 + 1/g\tau_p))$

But $\tau_p = n_f / [c \{ \alpha + (1/L) \ln(1/R) \}]$

$\Rightarrow L = (I_{th}\tau_s - ewhc \cdot \ln(1/R)/gn) / (ewhn_0 + (ehwac/gn)) = \underline{508 \mu m}$

[20%]

(d) $\Delta\lambda = \lambda^2 / 2nL = \underline{0.62 \text{ nm}}$

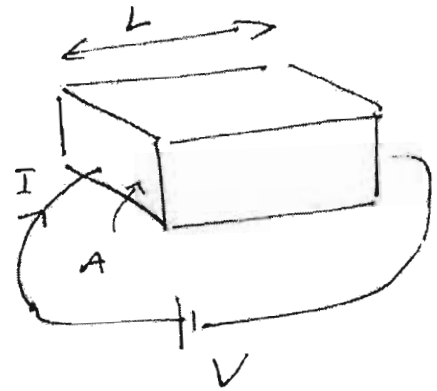
[20%]

3(a). To detect mid-IR wavelengths in a photodiode, it is necessary to have a semiconductor with a very small bandgap. Such materials are difficult to handle and don't produce good diode junctions, as well as resulting in a lot of leakage.

Photoconductors use a donor level inside the bandgap of a high quality semiconductor such as Si



(b) (i)
$$I = \frac{VA}{L} (n q_p \mu_n + p q_p \mu_p)$$



The term in the bracket, by analogy to Ohm's law, is the conductivity.

$n + p$ are the electron + hole charge densities

q_p is the electronic charge

$\mu_n + \mu_p$ are the electron + hole mobilities

(ii) Assuming the conductivity of the s/c without illumination is given by the above expression, under illumination the conductivity will increase due to the increase in electron + hole densities

hence
$$\Delta \sigma = q_p (\mu_n \Delta n + \mu_p \Delta p)$$

But increase in n is equal to the rate of photogeneration of electrons \times recombination time τ_n (per unit volume) + will be the same as the hole generation rate

Assuming electron & hole recombination times are the same
(an approximation but a reasonable one)

$$\Delta n = \Delta p = g \tau_r$$

Now
$$\Delta I_{\text{photocond}} = \frac{VA \Delta \sigma}{L}$$

$$= \frac{VA q \tau_r g (\mu_e + \mu_h)}{L}$$

But the external current in a photodiode is simply the photo arrival rate of ^{absorbed} photons/volume multiplied by the volume

$$\Delta I_{\text{photodiode}} = g q h \nu A \quad (\text{assuming } 100\% \text{ quantum efficiency})$$

Hence photoconductive gain

$$G = \frac{\Delta I_{\text{photocond}}}{\Delta I_{\text{photodiode}}} = \frac{\frac{g \tau_r q V A (\mu_e + \mu_h)}{L}}{g q h \nu A}$$

$$= \frac{\tau_r V}{L^2} (\mu_e + \mu_h)$$

Substituting $\frac{V}{L} = E \tau$ & $v_e = \mu_e E$ & $v_p = \mu_p E$

& $\frac{v_e}{L} = \frac{1}{\tau_{et}}$, $\frac{v_p}{L} = \frac{1}{\tau_{pt}}$ (electron & hole transit times)

$$G = \frac{\tau_r}{\tau_{et}} + \frac{\tau_r}{\tau_{pt}}$$

Can be a large number (e.g. for Cds $\sim 10^5$)

(c). TIA has $R_f = 1k\Omega$
 $B = 7.5MHz$

Data rate = ~~10 Gbps~~ 6 Gbps \Rightarrow Bit period = 100ps

- (i) pin photodiode - made from InGaAs
- relatively simple to fabricate
 - simple to bias (just need to ensure full depletion - few V)
 - fairly low noise
 - gain of 1

APD

- made from Ge or InGaAs
- quite difficult to fabricate
- difficult to control bias at optimum bias (which will be high - 10sV).
- avalanche gain
- high excess noise factor ($\alpha = \sim 1$)
 so benefits of avalanche gain to SNR are not as great as for Si at $< 1\mu m$ operating wavelengths

(ii) If thermal noise is dominant process at low incident optical power (a good approximation)

$20dB = 100 \rightarrow$

$$SNR = \frac{\left(\eta \frac{e\lambda}{hc} P \right)^2}{4kTB/R}$$

(can ignore shot noise due to $I_{ph} + I_d$)

$$(\text{Sensitivity})^2 = P^2 = \frac{100 \times 4kTB/R}{\left(\eta \frac{e\lambda}{hc} \right)^2}$$

$$= \frac{100 \times 4 \times 1.38 \times 10^{-23} \times 308 \times 7.5 \times 10^9 / 1000}{\left(\frac{0.9 \times 1.6 \times 10^{-19} \times 1.55 \times 10^{-6}}{6.63 \times 10^{-34} \times 3 \times 10^8} \right)^2}$$

$$\left(\frac{0.9 \times 1.6 \times 10^{-19} \times 1.55 \times 10^{-6}}{6.63 \times 10^{-34} \times 3 \times 10^8} \right)^2 \leftarrow 1.122^2$$

(c) (ii) cont $P^2 = 1.32 \times 10^{-11} \text{ W}^2$

$$P = 3.17 \mu\text{W}$$

$$\begin{aligned} \text{In dBm } P &= 10 \log \left(\frac{3.17 \mu\text{W}}{1 \text{ mW}} \right) \\ &= -25.0 \text{ dBm.} \end{aligned}$$

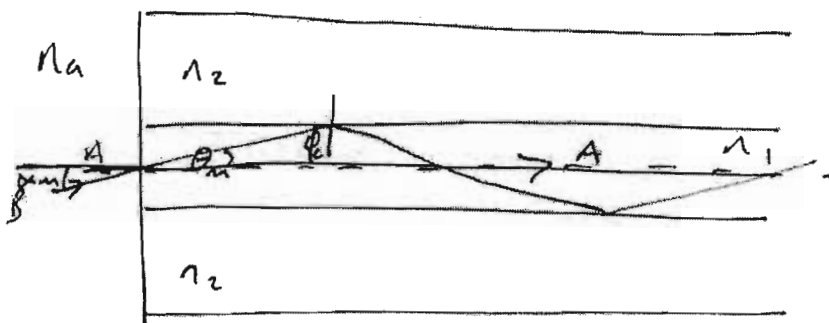
(iii) For APD
can no longer
assume thermal
noise limited.

$$\begin{aligned} \text{SNR} &= \frac{M^2 \left(\frac{q e \lambda}{h c} P \right)^2}{M^{2+x} \left(\frac{q e \lambda}{h c} P + I_d \right)^2 + \frac{4 k T B}{R}} \\ &= \frac{30^2 \left(1.122 \times 3.17 \times 10^{-6} \right)^2}{30^{2.5} \left(1.122 \times 3.17 \times 10^{-6} + 10 \times 10^{-9} \right)^2 + \frac{4 \times 1.38 \times 10^{-23} \times 506 \times 7.5 \times 10^9}{1000}} \\ &= 268 \text{ (or } 24.3 \text{ dB)} \end{aligned}$$

- an improvement of 4.3 dB over pin case so APD receiver will be more sensitive.

4(a) POF is very cheap to make, terminate & handle, though does suffer from (relatively) high attenuation & dispersion, so ideal applications are where data rates are low & distances not too great. The main ones are in vehicles (especially in cars). There is a lot of effort devoted to developing the technology for ~~low~~ home networks. They have a low loss window in the red, so we ideal for using red LED sources, which are extremely cheap. Glass fibres are preferred for long distances & high data rates because their loss & dispersion is extremely low. They are also compatible with high bandwidth AlGaAs & InGaAsP optical sources and Si / InGaAs detectors, which mean that high performance links are readily ~~now~~ available.

(b)



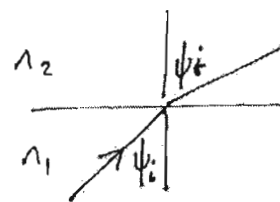
Rays are guided in core by ~~Snell's law~~
Total Internal Reflection (TIR)

Snell's law states

$$n_1 \sin \phi_i = n_2 \sin \phi_r$$

when $\phi_r = 90^\circ$ (and hence no refraction is possible), ϕ_i is called the critical angle ϕ_c

$$\sin \phi_c = \frac{n_2}{n_1}$$



For the ray B $\theta_m = \frac{\pi}{2} - \psi_c$

so the angle of incidence of the incoming ray on the end face of the fibre must be less than a certain value α_m

By Snell's Law again

$$n_a \sin \alpha_m = n_1 \sin \theta_m = n_2 \cos \psi_c$$

From above, since $\sin \psi_c = \frac{n_2}{n_1}$

$$\cos^2 \psi_c = 1 - \frac{n_2^2}{n_1^2}$$

$$\cos^2 \psi_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

$$\text{So } n_a \sin \alpha_m = \sqrt{n_1^2 - n_2^2}$$

(usually $n_a = 1$) + ~~so~~ this value is a measure of the light gather power of the fibre and is called the numerical aperture

$$\text{So } \sin \alpha_m = NA = \sqrt{n_1^2 - n_2^2}$$

(formally $NA = n_a \frac{\sqrt{n_1^2 - n_2^2}}{n_a}$ but since n_a usually = 1, this is usually ignored).

(c) Fibres are single mode if normalised frequency $V < 2.405$.

$$V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2}$$

core radius

$$\text{So } a < \frac{2.405 \lambda}{2\pi \sqrt{n_1^2 - n_2^2}} < \frac{2.405 \times 1.55 \times 10^{-6}}{2\pi \sqrt{1.45^2 - 1.44^2}} < 3.49 \mu\text{m}$$

so core diameter $< 2a$
 $< 6.98 \mu\text{m}$.

Coupling efficiency

Lambertian source has power/unit solid angle radiated

$$I(\theta) = I_0 \cos\theta$$

Total power in forward dir =

$$P_0 = \int_0^{\pi/2} 2\pi I_0 \cos\theta \sin\theta d\theta$$
$$= \pi I_0$$

Power collected by fibre

$$P = \int_0^{\alpha_m} 2\pi I_0 \cos\theta \sin\theta d\theta$$
$$= \pi I_0 \sin^2 \alpha_m$$

so collection efficiency (using α_m from (b))

$$\frac{P}{P_0} = \frac{\pi I_0 \sin^2 \alpha_m}{\pi I_0} = n_1^2 - n_2^2$$
$$= 1.45^2 - 1.44^2$$
$$= 0.029$$

(d) i) Single mode fibre so no intermodal dispersion

$$\Delta t = D \sigma h$$

$$= 15 \times 0.2 \times L = 3L \text{ (units of km)}$$

$$\Delta t_{out}^2 = \Delta t_{in}^2 + \Delta t^2$$

$$t_{in} = 400 \text{ ps} \Rightarrow t_{out} = 600 \text{ ps}$$

$$600^2 = 400^2 + (3L)^2$$

$$3L = \sqrt{600^2 - 400^2}$$

3

$$= 149 \text{ km}$$

ii) Input power = 2 mW = +3 dBm.

$$\begin{aligned} \text{Losses: } 149 \times 0.3 \text{ dB} &= 44.7 \text{ dB} && \text{(fibre)} \\ &+ 2 \text{ dB} && \text{(coupling)} \\ &= 46.7 \text{ dB} \end{aligned}$$

$$\begin{aligned} \text{Budget} &= \text{losses} + \text{margin} \\ &= 46.7 + 3 = 49.7 \text{ dBm} \end{aligned}$$

So Receiver sensitivity must be better than
 $+3 \text{ dBm} - 49.7 \text{ dB} = -46.7 \text{ dBm}$