HEMH/5

EGT2
ENGINEERING TRIPOS PART IIA

Friday 2 May 2014 9.30 to 11

Module 3C5

DYNAMICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C5 Dynamics and 3C6 Vibration data sheet (6 pages).
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A circular cylindrical rotor of mass m and radius a spins freely on a light shaft BG. The centre of mass of the rotor is at G. The shaft is fixed at B to a light string OB of length L as shown in Fig. 1. The string is fixed at O. The distance BG is b and BG is aligned with the axis of the rotor. The rotor is spinning at a constant *fast* rate ω and its polar moment of inertia can be taken as $C = \frac{1}{2} ma^2$. In steady-state precession the shaft BG remains horizontal and the angle α between OB and the vertical is constant as shown in the figure. The plane OBG remains vertical and rotates at a steady rate Ω.

(a) Draw a free-body diagram of the shaft and rotor and hence show that, given certain assumptions, the precession rate $\Omega \approx \sqrt{\dfrac{g\alpha}{b}}$, where g is the acceleration due to gravity. What assumptions have been made? [:

(b) Find an expression for the string angle α in terms of g, a, b and ω. [60%]

(c) Verify that the small-α assumption is consistent with the assumption of fast spin. [10%]

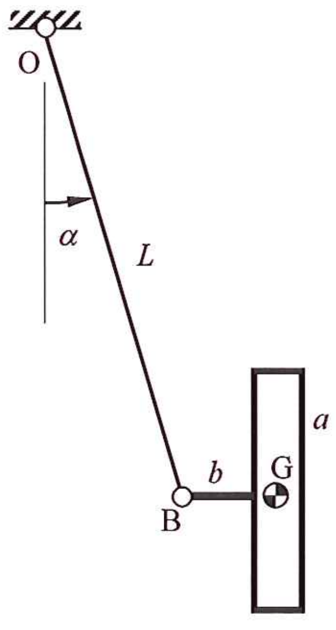

Fig. 1

2 An unbalanced ball comprises a solid uniform hemisphere of mass m and radius a fixed inside a light rigid spherical shell of the same radius as shown in Fig. 2. The centre of mass is at G, a distance $\frac{3}{8}a$ from the centre of the ball which is at O. The ball rolls without slip on a horizontal plane.

There is a steady-state motion in which the centre of the ball moves at a constant speed v while following a circular path of radius $R \gg a$, centred on BB'. The line OG remains horizontal and directed at the centre of the circle as shown in the figure.

(a) Use a no-slip condition to find an expression for the angular velocity of the ball during steady state motion [30%]

(b) Find an expression for the path radius R in terms of v and g, where g is the acceleration due to gravity. [50%]

(c) Show that the coefficient of friction between the ball and table must exceed $\frac{15}{56}$. [20%]

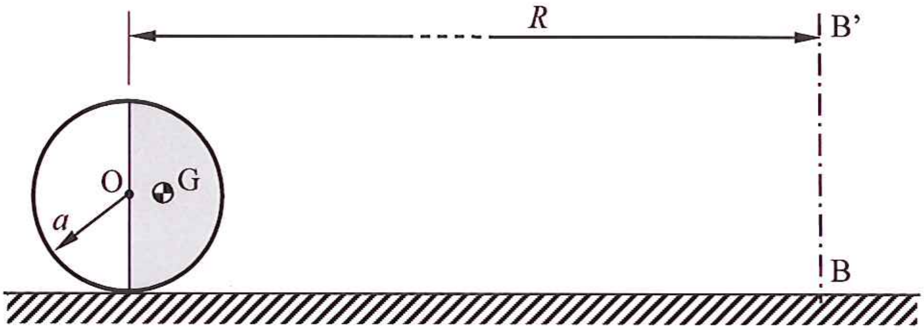

Fig. 2

3 (a) The puzzle shown in Fig. 3(a) is in the form of a cube of mass m and side a. It is assembled from 27 smaller solid cubes each of mass $m/27$ and side $a/3$.

(i) Find the principal moments of inertia of the cube puzzle at the vertex A shown in Fig. 3(a). [25%]

(ii) The "top face" of the puzzle (comprising 9 cubes) is rotated through an angle of 45° as shown in Fig. 3(b). Find the principal moments of inertia of the face-rotated puzzle at point B. [25%]

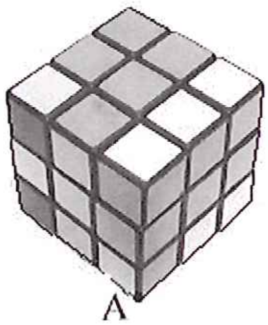

Fig. 3(a)

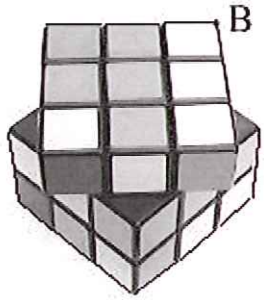

Fig. 3(b)

(b) (i) Explain briefly the conditions under which Lagrange's equations can lead to conservation laws. [10%]

(ii) A GPS satellite of mass m orbits the Earth. Its position is described in terms of spherical polar coordinates (r, θ, ϕ) where r is distance from the centre of the Earth, θ is polar angle measured downwards from the axis through the North pole, and ϕ is angle of longitude. The gravitational potential of the Earth, allowing for the effect of the equatorial bulge, can be written

$$V = -\frac{GMm}{r}\left[1 - \frac{J_2 a^2}{2r^2}(3\cos^2\theta - 1)\right]$$

where a is the radius of the Earth and J_2 is a dimensionless constant.

Find an expression for the kinetic energy of the satellite and use Lagrange's equations to obtain a conservation law. Explain its physical interpretation. [40%]

4 A Porter governor, an early feedback device used on steam engines, is shown in Fig. 4. A mass M can slide freely on a massless vertical shaft OB, which rotates with constant angular velocity Ω in response to an applied torque Q. Two equal masses m are joined to the mass M and to the top of the shaft by four massless bars of length a, freely hinged at all joints and inclined to the shaft by angle θ as shown.

(a) Find expressions for the total kinetic and potential energies of the Porter governor. [20%]

(b) Use Lagrange's equation to.show that the equation of motion governing $\theta(t)$ is

$$(m+2M\sin^2\theta)\ddot{\theta}+2M\sin\theta\cos\theta\,\dot{\theta}^2-m\sin\theta\cos\theta\,\Omega^2+(m+M)\frac{g}{a}\sin\theta=0\ .$$ [30%]

(c) Find all possible equilibrium positions with θ=constant, and investigate the stability of each. For any stable positions, find the frequency of small oscillations about the equilibrium. [50%]

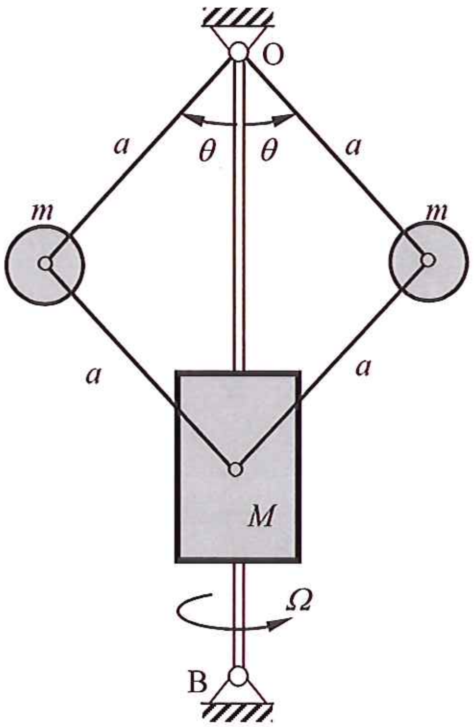

Fig. 4

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