EGT2
ENGINEERING TRIPOS PART IIA

Thursday 26 April 2018 2 to 3.40

**Module 3C8**

**MACHINE DESIGN**

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Attachment: Module 3C8 data sheet (9 pages).
Engineering Data Book

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

1 Figure 1 shows a simple thrust bearing consisting of five equally spaced steel balls rolling between two flat steel plates. The balls are of radius 10 mm, the contact modulus is 115 GPa, and the bearing carries a central load $W$ of 1 kN.

(a) State the conditions which must be satisfied for a contact to be described by the classical Hertzian equations. [15%]

(b) What is the maximum contact pressure, and what is the relative displacement of the two plates? [25%]

(c) The bearing is inadvertently assembled with *one* ball having a radius of 10.01 mm. Assuming that only three of the balls carry load, find the maximum contact pressure. [25%]

(d) Investigate whether the assumption in part (c) is correct. [35%]

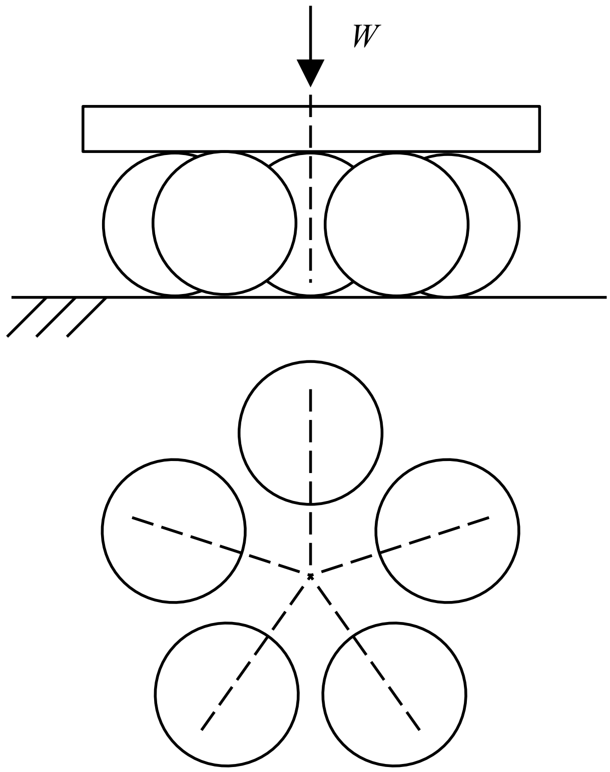



Fig. 1

2 Figure 2 shows a cross-section of the front axle assembly of a bicycle, with a pair of opposing angular contact ball bearings. The hub with integral outer races rotates about the axis of the axle with an angular velocity $\Omega_2$. The forks, shaft and inner races do not rotate. The radius of the balls is $r$. The points of contact $O_1$ and $O_2$ between the ball and the races are at radii $R_1$ and $R_2$ from the rotation axis of the hub. A line passing through the points of contact $O_1$ and $O_2$ makes an angle of 45° to the rotation axis of the hub.

(a) There is no sliding at $O_1$ and $O_2$. For the purpose of analysis, an angular velocity is superimposed on the assembly in order to make the centres of the balls stationary.

(i) The angular velocity of a ball about an axis perpendicular to the axis $O_1O_2$ is $\omega_t$, as shown in Fig. 2. Express $\omega_t$ in terms of $\Omega_2$ and the geometric variables. [25%]

(ii) The angular velocity of a ball about $O_1O_2$ is $\omega_n$, as shown in Fig. 2. Express the spin velocities at $O_1$ and $O_2$ in terms of $\Omega_2$, $\omega_n$ and the geometric variables. [25%]

(iii) Comment on the expressions for the spin velocities and their implication for possible values of the angular velocity $\omega_n$. [20%]

(b) The total vertical force on the front axle of the bicycle is 500 N. Each bearing contains eight equally-spaced balls. One ball in each bearing is at the lowest possible height. The balls in the upper half of each bearing carry no force. Make an appropriate assumption about how the force is shared between all of the balls in the lower half of each bearing, and estimate the forces at the contacts. [30%]

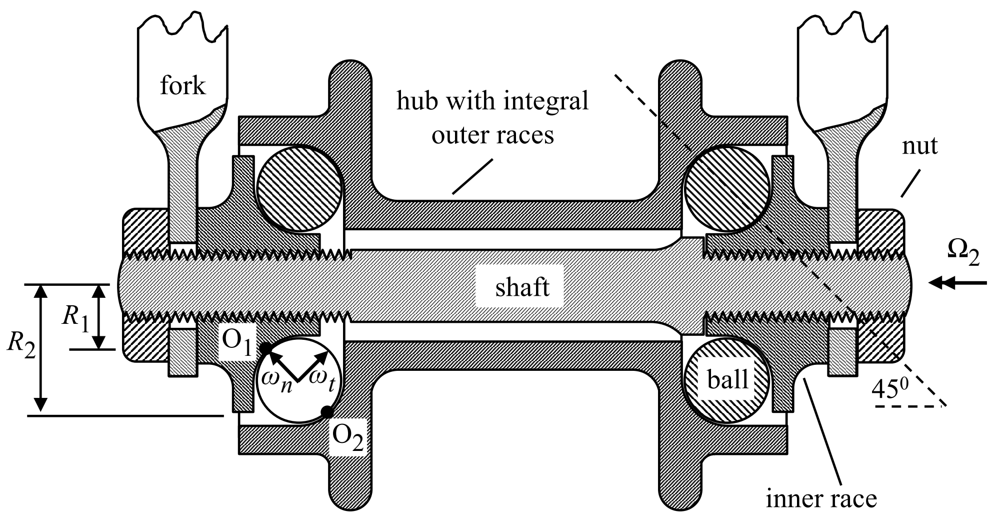



Fig. 2

3 Figure 3 shows the powertrain of an electric vehicle. It consists of two electric motors (labelled $M_A$ and $M_B$), and two sets of epicyclic gears (numbered 1 and 2). The output shaft drives the road wheels via a reduction gear (not shown). Each motor can rotate in either direction, and has a maximum power of 12 kW and a maximum torque of 80 N m as shown schematically in Fig. 4. Motor $M_A$ is connected to annulus A1 and carrier C2. Motor $M_B$ is connected to sun S1 and sun S2. Carrier C1 is either braked or free to rotate. Carrier C2 holds pairs of equally-sized planet wheels so that when carrier C2 is stationary, sun S2 and annulus A2 rotate in the same direction. Teeth numbers are shown in Fig. 3. There are no losses.

(a) Consider epicyclic 2 in isolation and show that the speed rule is

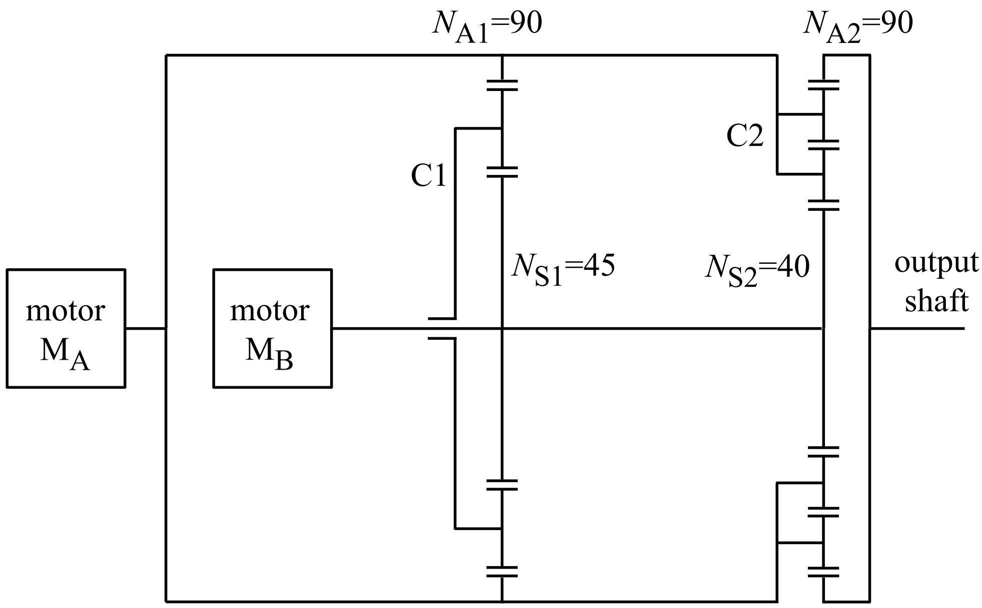
$$\omega_{S2} = \omega_{A2} R_2 + \omega_{C2}(1 - R_2)$$

where $R_2 = N_{A2}/N_{S2}$. [20%]

(b) Carrier C1 is free to rotate. Motors $M_A$ and $M_B$ rotate in the same direction at 600 rad $s^{-1}$. A torque of 36 N m is generated at the output shaft. Calculate the speed of the output shaft and calculate the torque applied by motor $M_A$ and the torque applied by motor $M_B$. [30%]

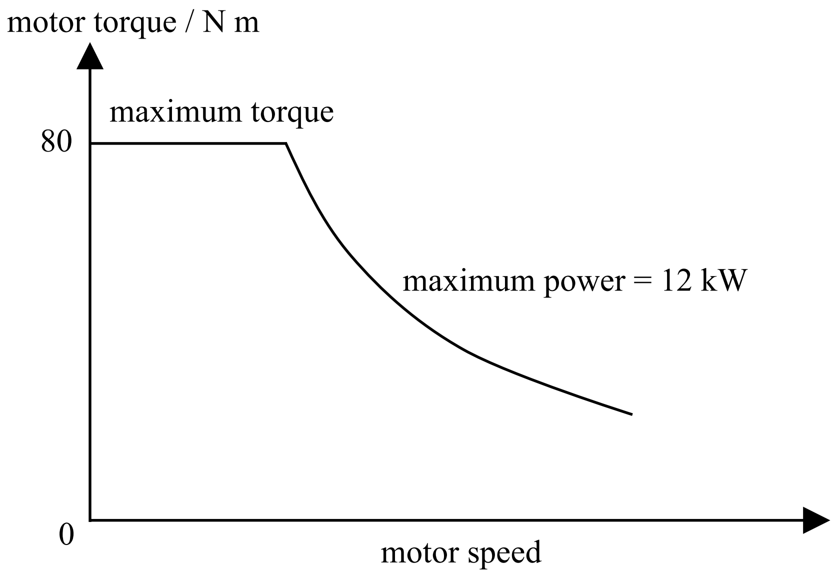
(c) Carrier C1 is braked. The output shaft rotates at 60 rad $s^{-1}$. Motors $M_A$ and $M_B$ operate at their maximum output. Calculate the speed of motor $M_A$ and the speed of motor $M_B$, and calculate the torque generated at the output shaft. [40%]

(d) Explain the benefit of the arrangement shown in Fig. 3 compared to an equivalent single electric motor (maximum power 24 kW and maximum torque 160 N m) driving the output shaft directly. [10%]



$N_{A1}$=90
$N_{A2}$=90
C1
C2
$N_{S1}$=45
$N_{S2}$=40
motor $M_A$
motor $M_B$
output shaft


Fig. 3



motor torque / N m
maximum torque
80
maximum power = 12 kW
0
motor speed


Fig. 4

4 A rack and pinion is used to raise or lower the platform of an offshore drilling rig. The teeth have an involute curve. The module is 100 mm and the face width is 127 mm. The rack is required to generate a force of 5 MN in the direction of the rack's travel. The contact modulus is 115 GPa.

(a) State the benefits of the involute curve when applied to gears. [20%]

(b) Three designs are to be considered, with the aim of minimising the pinion torque.

(i) In the first design the pressure angle is $20^\circ$ and the addendum of the rack and the pinion are equal to the module. Show that the minimum number of teeth on the pinion to avoid interference is 18. For this number of teeth find the pinion torque and the maximum contact pressure at the pitch point. Assume a single contact. [40%]

(ii) In the second design the pressure angle is $25^\circ$ and the addendum of the rack and the pinion are equal to the module. Find the minimum number of teeth to avoid interference and for this number of teeth find the pinion torque and the maximum contact pressure at the pitch point. Assume a single contact. [10%]

(iii) In the third design the pressure angle $20^\circ$. The addendum of the rack is two-thirds of the module and the addendum of the pinion is four-thirds of the module. Find the minimum number of teeth to avoid interference and for this number of teeth find the pinion torque and the maximum contact pressure at the pitch point. Assume a single contact. [15%]

(c) Discuss the problem of specifying the pressure angle and rack addendum to achieve the required rack force with minimum pinion torque. Mention any constraints that should be accounted for. [15%]

**END OF PAPER**