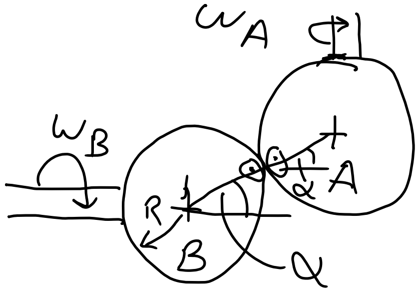
3C8 2023

(1)

a)

- connect shaft A to transmission between internal combustion engine and road wheels, possibly via a clutch, to form a 'parallel' hybrid.
- aim to run i.c. engine at most efficient operating point.
- adjust speed ratio to flywheel, to store energy when power required by the vehicle is less than that provided by the engine
- adjust speed ratio to deliver energy from flywheel when vehicle requires more power.

b,i)



$$\omega_B R \sin\alpha = \omega_A R \cos\alpha$$

$$\frac{\omega_A}{\omega_B} = \tan\alpha$$

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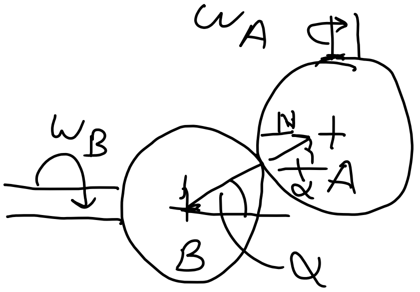
b,ii)



component of ω_B in $\underline{N}$ direction is $+\omega_B \cos\alpha$

component of ω_A in $\underline{N}$ direction is $+\omega_A \sin\alpha$

$$\text{spin speed} = \omega_B \cos\alpha - \omega_A \sin\alpha$$

$$= \omega_B \cos\alpha - \omega_B \tan\alpha \sin\alpha$$

$$= \omega_B (\cos\alpha - \tan\alpha \sin\alpha)$$

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b,iii) $\bar{\underline{u}} = \frac{1}{2}\left[(\underline{u}_1 - \underline{u}_c) + (\underline{u}_2 - \underline{u}_c)\right]$ ← Both from Lecture notes

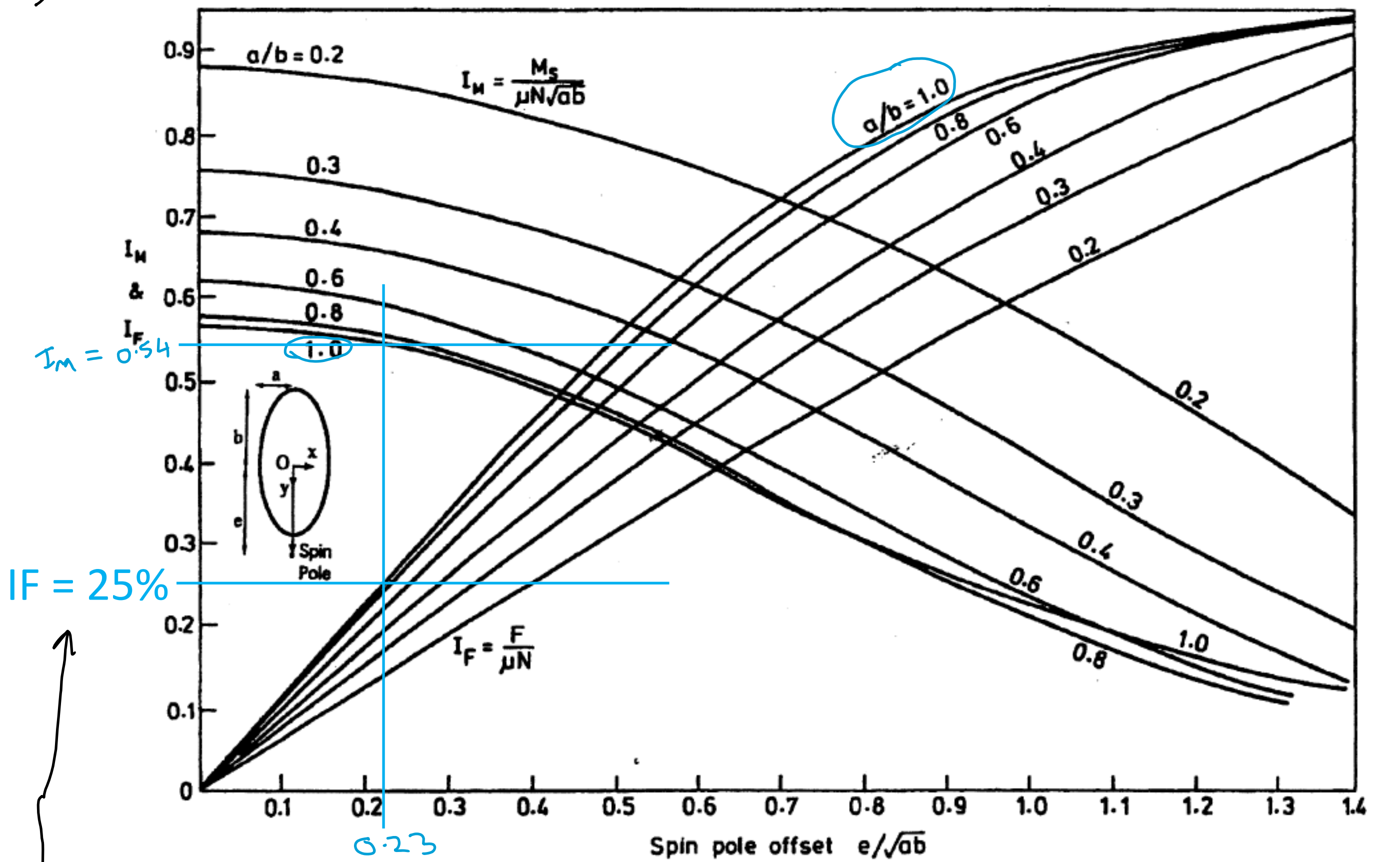
$\underline{u}_c = 0 \qquad \underline{u}_1 = \underline{u}_2$

$|\bar{\underline{u}}| = \omega_B \cdot R \sin\alpha$

↓

Mean entraining velocity $\bar{V} = \dfrac{V_1 + V_2}{2}$

c) From Page 2 of Data Book



25% of available friction used

$\frac{a}{b} = 1.0$ as circular contact patch between two spheres

$$\text{Input power} = F \cdot \bar{v} = F \cdot \omega_B \cdot R \cdot \sin\alpha = 0.432 F \cdot \omega_B \cdot R \quad (\alpha = 25^\circ)$$

$$\text{Power lost} = \omega_{spin} \cdot M + V_{slip} \cdot F$$

R = ?

a = ?

$M = 0.54\mu N a$ (from plot)

$F = 0.25\mu N$ (from question)

$V_{slip} = e \cdot \omega_{spin} = 0.23 a \omega_{spin}$ (from plot)

$\omega_{spin} = \omega_B(\cos\alpha - \tan\alpha \sin\alpha)$ (from earlier in question)

$= \omega_B(\cos 25^\circ - (\tan 25^\circ)(\sin 25^\circ)) = 0.709\,\omega_B$

$$1 - \eta = \frac{\text{Power lost}}{\text{Input power}} = \frac{(\omega_{spin})(0.54\mu N a) + (0.23 a \omega_{spin})(0.25\mu N)}{0.432 F \omega_B R}$$

$\omega_{spin} = 0.709\,\omega_B$

$$1-\eta = \frac{\text{Power lost}}{\text{Input power}} = \frac{(\omega_{spin})(0.54\mu Na) + (0.23a\,\omega_{spin})(0.25\mu N)}{0.432F\omega_B R}$$

$$= \frac{0.709\omega_B\left[0.54\mu Na + (0.23a)(0.25\mu N)\right]}{0.432F\omega_B R}$$

$$= 0.981\,\mu Na / FR$$

$$= 0.981\,\mu Na / 0.25\mu NR$$

$$= 3.924\,a/R$$

From Data Book

Effective curvature $\dfrac{1}{R^*} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

$$= \frac{1}{0.05\,m} + \frac{1}{0.05\,m} = 40\,m^{-1}$$

$$\Rightarrow R^* = 0.025\,m$$

Semi contact width or contact radius*

Circular contact

diameter $2a$; load P

$E^* = 115$ GPa for steel on steel

$$a = \left\{\frac{3PR}{4E^*}\right\}^{1/3}$$

$$\Rightarrow a = \sqrt[3]{\frac{3\times 400N \times 0.025m}{4\times 115\,GPa}} = \sqrt[3]{6.522\times 10^{-11}\,m^3}$$

$$= 4.025\times 10^{-4}\,m$$

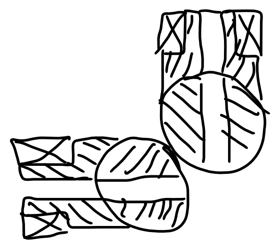
$$1-\eta = 3.924\,a/R = 3.924\times 4.025\times 10^{-4}\,m / 0.05\,m$$

$$= 0.0316$$

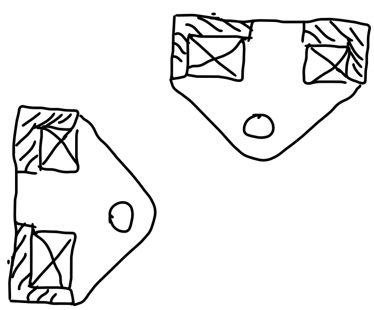
$$\Rightarrow \eta = 1 - 0.0316 = 0.968 = 96.8\%$$

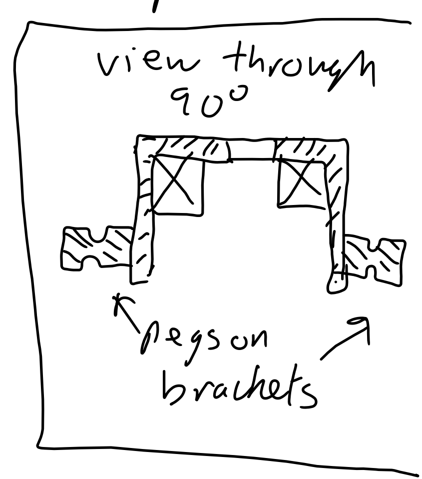
d) The top half of sphere A and the left half of sphere B are not involved in the contact for the $0^\circ \leq \alpha \leq 90^\circ$ required.

A mating cup and bearing arrangement ~~that~~ uses these surfaces could be produced



Attached to the outside of these bearings could be a bracket extending to the spheres' centres





A pair of springs between the pegs (coincident with the spheres' centres) could then allow the normal contact force between the spheres to be maintained. (A further slotted linkage could be added to prevent unwanted rotation or twisting of the springs.)

2(a) From the data sheet

$$\omega_s = (1+R)\,\omega_c - R\,\omega_a \quad \text{where } R = \frac{A}{S}$$

$$\Rightarrow \begin{cases} S_1 = (1+R_1)C_1 - \cancel{R_1 A_1} \to 0 \\ S_2 = (1+R_2)C_2 - R_2 A_2 \end{cases}$$

Also $\omega_i = C_1 \qquad \omega_0 = S_2$

$A_1 = 0 \qquad C_1 = A_2$

$S_1 = C_2$

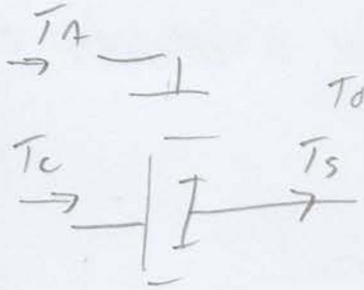
$S_1 = C_2 = (1+R_1)\,\omega_i$ from (1)

$\omega_0 = (1+R_2)\,S_1 - R_2\,\omega_i$ from (2)

$$\omega_0 = (1+R_2)(1+R_1)\,\omega_i - R_2\,\omega_i$$

$$\frac{\omega_0}{\omega_i} = (1+R_2)(1+R_1) - R_2 = 1 + R_1R_2 + R_1$$

(b)



Torque on shafts second epicyclic

Overall power assuming no losses $\Rightarrow -T_S\,\omega_0 = T_i\,\omega_i$

from epicyclic speed rule

Virtual power $\Rightarrow T_C\,\omega_C' + T_S\,\omega_S' + T_A\,\omega_A' = 0$

Put $\omega_A' = 0 \Rightarrow \quad T_C = -T_S \left.\frac{\omega_S'}{\omega_C'}\right|_{\omega_A'=0} = -T_S\,(1+R_2)$

$$= T_i\,\frac{\omega_i}{\omega_0}\,(1+R_2) = T_i\,\frac{1+R_2}{1+R_1R_2+R_1}$$

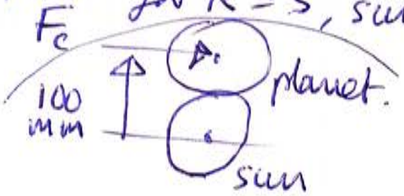
2 (c) $R_1 = R_2 = 3 \quad \therefore \quad \dfrac{\omega_o}{\omega_i} = 1 + R_1 + R_1 R_2 = 1 + 3 + 3.3 = 13 = \dfrac{\omega_{s2}}{\omega_i}$

$$\left|\frac{T_s}{T_i}\right| = \frac{1+R_2}{1+R_1+R_1R_2} = \frac{1+3}{13} = \frac{4}{13}$$

$P_i = T_i \cdot \omega_i \quad \therefore \quad T_i = \dfrac{19\,kW}{1\ rad/s} = 19\ kNm$

$\therefore \ |T_s| = \frac{4}{13} \cdot 19\ kNm = 5{\cdot}85\ kNm$

Assume single planet per carrier initially.
Note that for $R=3$, sun and planet are same PCD, and $\frac{1}{3}$ of annulus PCD



$T_s = F_c \cdot 0{\cdot}1m$

$\therefore F_c = \dfrac{5{\cdot}85 \cdot 10^3}{0{\cdot}1} = 58{\cdot}5\ kN$

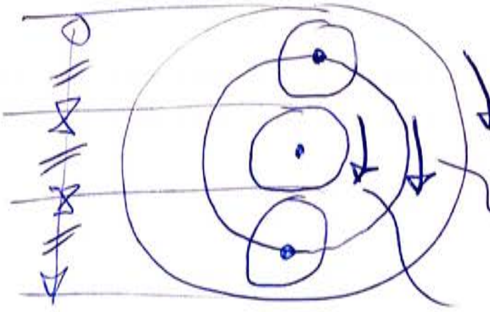
$\therefore P = \dfrac{F_c}{6} = 9{\cdot}75\ kN$ (two brgs. per planet, three planets)

Find speed of carrier $\omega_{c2} = \omega_{s1}$

$\omega_i = 1\ rad/s \qquad \omega_{s1} = (1+R_1)\,\omega_{c1} - R_1\,\omega_{A1}$

$= (1+3) \cdot 1 - 3 \cdot 0$

$\omega_{s1} = 4\ rad/s = \omega_{c2}$



$\omega_{A2} = 1\ rad/s$

$\omega_{c2} = 4\ rad/s.$

$\omega_{s2} = 13\ rad/s$

subtract 4 rad/s from system to bring carrier stationary

then speed of sun is 13-4 rad/s, which is also the speed of a planet relative to the carrier.
Number of revolutions of planet relative to carrier:

$$\frac{2\,years \times 365\,days \times 24\,hours \times 60\,mins \times 60\,s \times 9\ rad/s}{2\pi\ rad/rev}$$

$= 90 \cdot 10^6$ revs

$\therefore \ L = 90.$

life equation

$$L = a_1 \left(\frac{C}{P}\right)^p$$

$a_1 = 0.12$ (1% failure prob.)

$p = \frac{10}{3}$ (roller bearing)

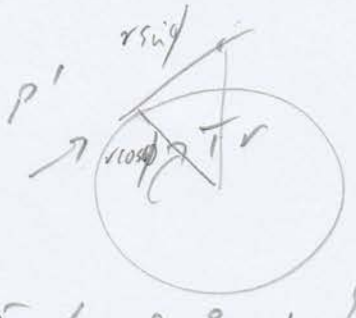
$$C = \left(\frac{L}{a_1}\right)^{\frac{1}{p}} P$$

$$= \left(\frac{90}{0.21}\right)^{\frac{3}{10}} 9.75 \cdot 10^3$$

$$\underline{\underline{C = 60.1 \text{ kN}}}$$

3(a)

$r = \frac{mN}{2}$



$T = r\cos\phi \cdot w \cdot p'$

For pinion $m = 3\,mm$ $N = 25$, $\phi = 20°$ p' $P'_T = p'\cos\phi$
$w = 10\,mm$.

(i) $\sigma_b = \frac{p'_T}{Jm} = \frac{p'\cos\phi}{Jm} \Rightarrow p' = \frac{\sigma_b Jm}{\cos\phi}$

Using chart $J \simeq 0.25$ using load applied at tip, $N = 25$

$\Rightarrow T = \left(\frac{mN}{2}\right)\cancel{\cos\phi} \cdot w \cdot \frac{\sigma_b Jm}{\cancel{\cos\phi}}$

$= \frac{3\times10^{-3}}{2} \times 25 \times 10\times10^{-3} \times 300\times10^{6} \times 0.25 \times 3\times10^{-3}$ $m^3 Nm^{-2}$

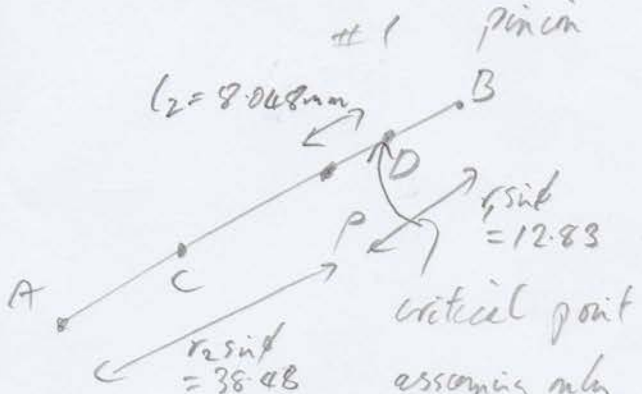
Pinion N

$= \underline{84.4\ Nm}$

(ii)

from data sheet formula, $N = 75$

$\frac{L_2}{m} = 2.682 \Rightarrow L_2 = 8.048\,mm$



$R_1 = 12.83 - 8.048 = 4.782\,mm$

$R_2 = 38.48 + 8.048 = 46.53\,mm$

$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = 4.336\,mm$

Hertz for line contacts: $p_0 = \left(\frac{p'E^*}{\pi R}\right)^{\frac{1}{2}}$

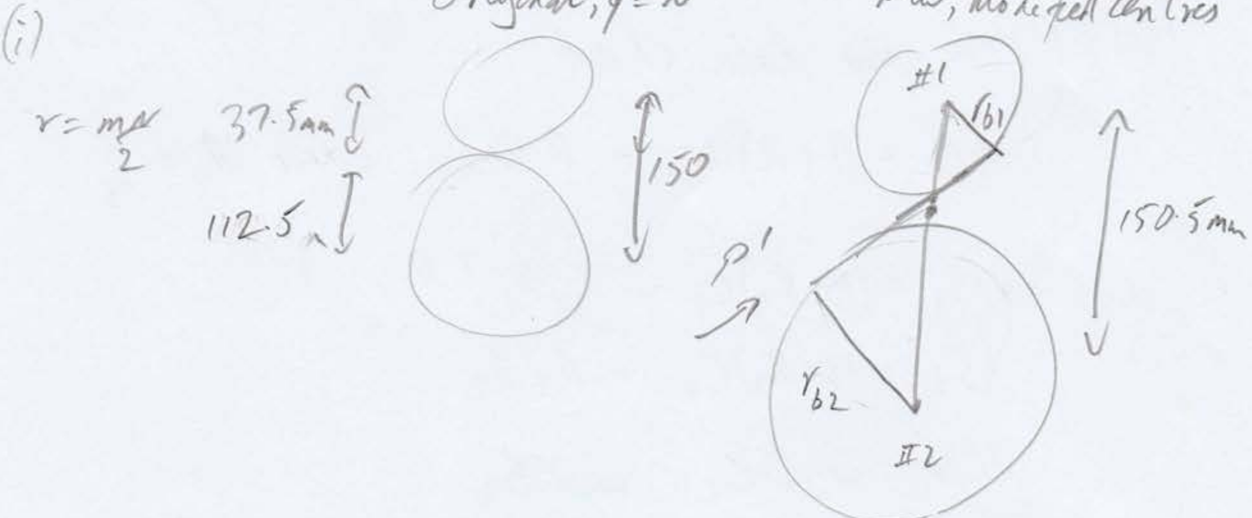
$\Rightarrow p' = \frac{p_0^2 \pi R}{E^*} \Rightarrow T = rw\cos\phi\, p'$

$= \frac{mN}{2} \cdot w \cdot \cos\phi \cdot p_0^2 \cdot \pi \cdot R / E^*$

$= \frac{3\times10^{-3}}{2} \cdot 25 \cdot 10\times10^{-3} \cdot \cos 20 \cdot (1100\times10^{6})^2 \cdot \pi \cdot 4.336\times10^{-3} / 115\times10^{9}$

$= \underline{50.5\ Nm}$

(b) (i)



Base circle is unchanged (defines involute) $= 37.5 \cos 20°$

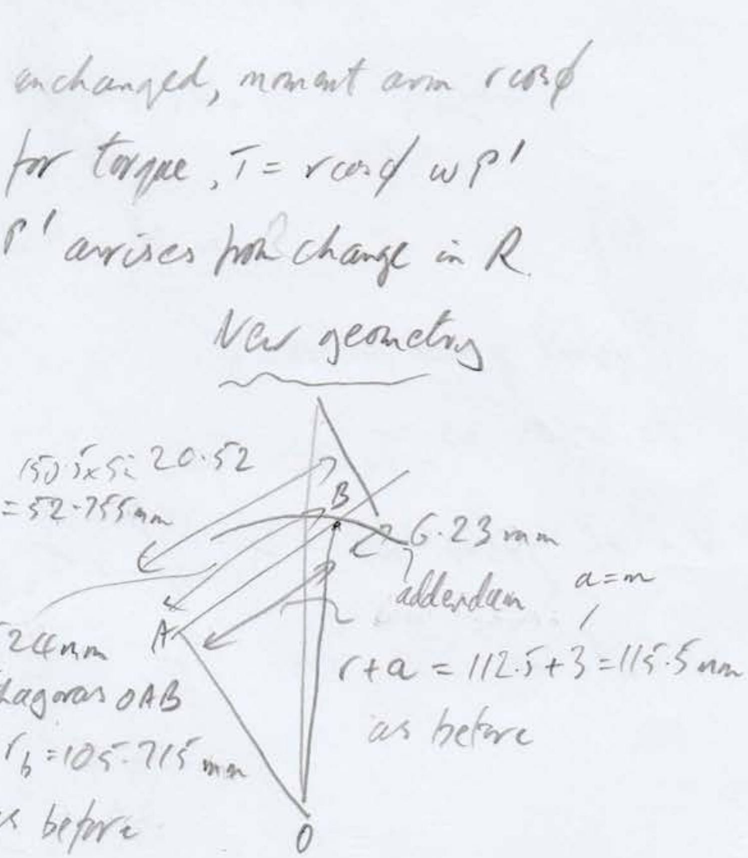
New pitch circle for pinion $= \dfrac{37.5 \times 150.5}{150} = 37.625\text{ mm}$

So $37.625 \cos\phi = 37.5 \cos 20$

$\phi = 20.52°$

(ii) As base circle is unchanged, moment arm $r\cos\phi$ is unchanged in formula for torque, $T = r\cos\phi\, w P'$

Difference in allowable P' arises from change in R



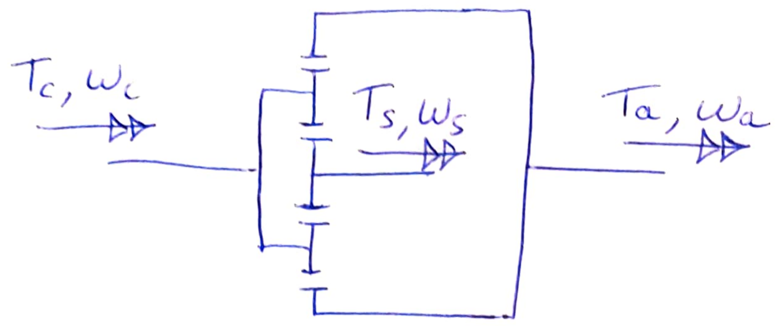
$\frac{1}{R} = \left(\frac{1}{6.23} + \frac{1}{46.524}\right)$

$\Rightarrow R = 5.494\text{ mm} = 27\%$ increase

As $p_0 \sim \left(\frac{P'}{R}\right)^{\frac{1}{2}}$

An increase in R of 27% corresponds to a 27% increase in allowable torque.

4 a)



data sheet $\omega_s = (1+R)\omega_c - R\omega_a$ $\quad R = \frac{A}{S} = \frac{90}{20} = \frac{9}{2}$

power conservation, virtual speeds

$$T_a\omega_a' + T_c\omega_c' + T_s\omega_s' = 0$$

let $\omega_a' = 0$

$$\frac{T_s}{T_c} = -\frac{\omega_c'}{\omega_s'}\bigg|_{\omega_a'=0} = -\frac{1}{1+R} = \underline{\underline{-\frac{2}{11}}}$$

torque equilibrium

$$T_a + T_c + T_s = 0$$

$$\frac{T_a}{T_c} + 1 + \frac{T_s}{T_c} = 0$$

$$\frac{T_a}{T_c} = -\frac{T_s}{T_c} - 1 = \frac{2}{11} - 1 = \underline{\underline{-\frac{9}{11}}}$$

b) Max vehicle speed requires max traction force at wheel, therefore max torque from annulus. Epicyclic torque ratios are fixed so max torque on annulus means max torque on carrier. Hence operate engine at max torque, 100 Nm @ 4300 rpm = 450.3 rad/s = ω_c.

$T_c = 100\ \text{Nm}$ $\quad \frac{T_a}{T_c} = -\frac{9}{11}$ $\quad \therefore T_a = -\frac{9}{11} \cdot 100\ \text{Nm}$

$$\therefore T_{wheel} = \frac{9}{11} \cdot 100 \cdot 5 = 409.1\ \text{Nm}$$

$$F_{traction} = \frac{T_{wheel}}{r_{wheel}} = \frac{409.1\ \text{Nm}}{0.35\ \text{m}} = 1168.8\ \text{N}$$

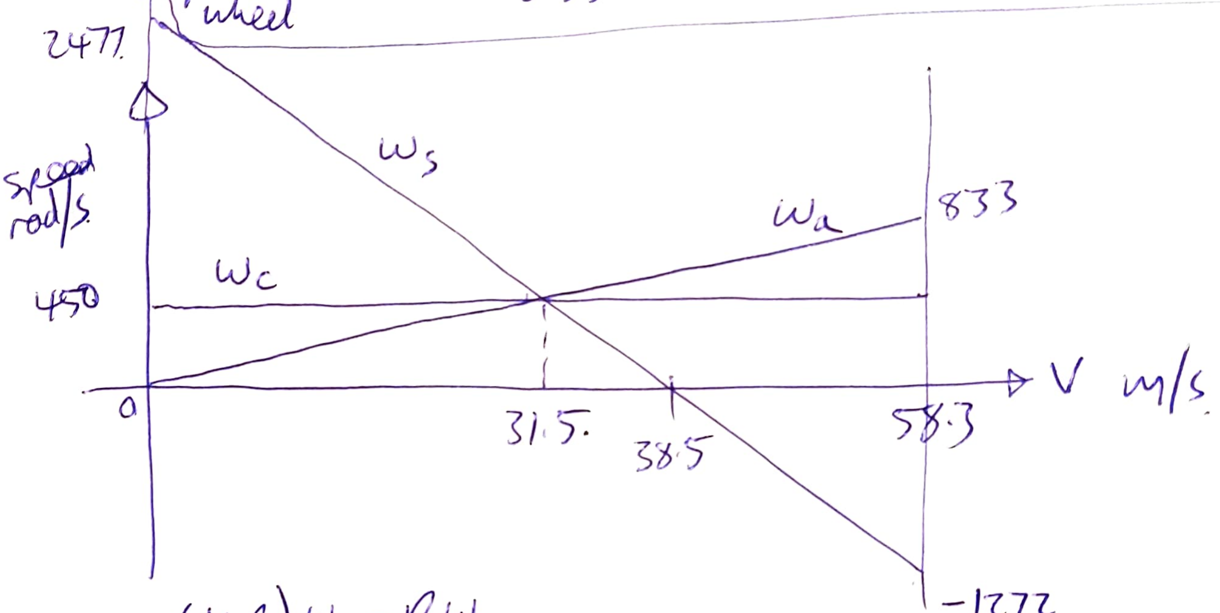
$$F_{traction} = 150 + 0.3V^2$$

$$V = \sqrt{\frac{1168.8 - 150}{0.3}}$$

$$\underline{\underline{V = 58.3 \text{ m/s}}}$$

$$\omega_a = \frac{V}{r_{wheel}} \cdot 5 = \frac{58.3}{0.35} \cdot 5 = \underline{\underline{832.9 \text{ rad/s}}}$$

c) i)



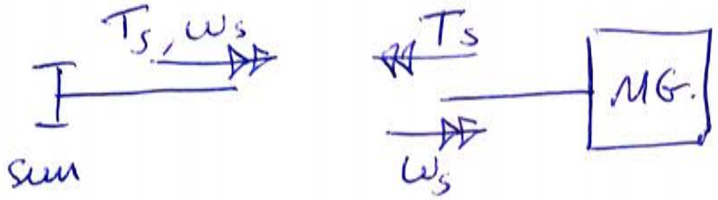
$$\omega_s = (1+R)\,\omega_c - R\,\omega_a$$

$$\omega_s = \frac{11}{2}\omega_c - \frac{9}{2}\omega_a$$

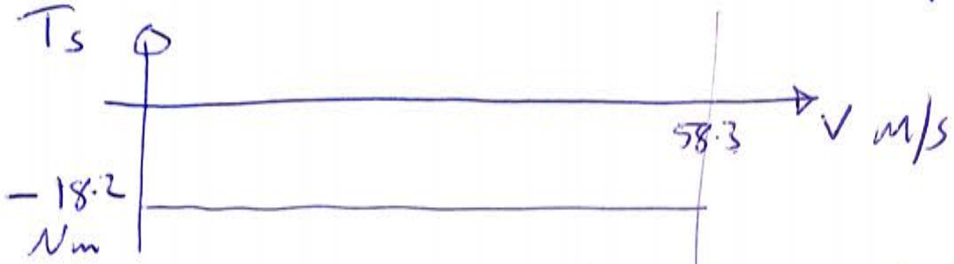
when $\omega_a = 0$, $\omega_s = \frac{11}{2}\omega_c = \frac{11}{2} \cdot 450 = 2477$ rad/s

when $\omega_a = 833$, $\omega_c = 450$, $\omega_s = \frac{11}{2} \cdot 450 - \frac{9}{2} 833 = -1272$ rad/s

c) ii)

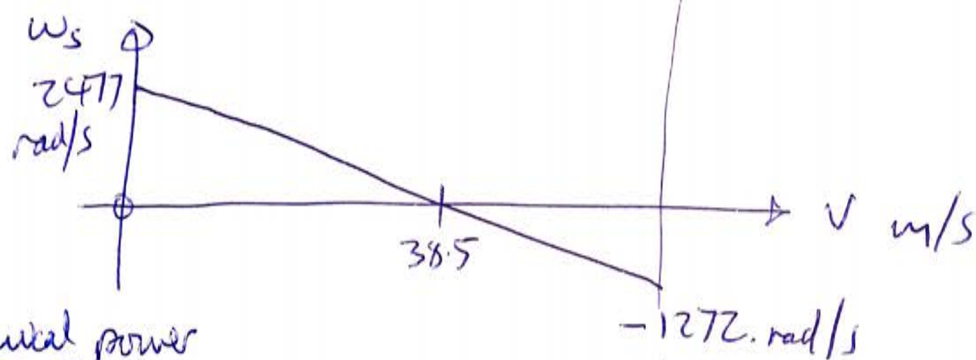


$P = T_s \omega_s$
+ve P is <u>out</u> of MG.

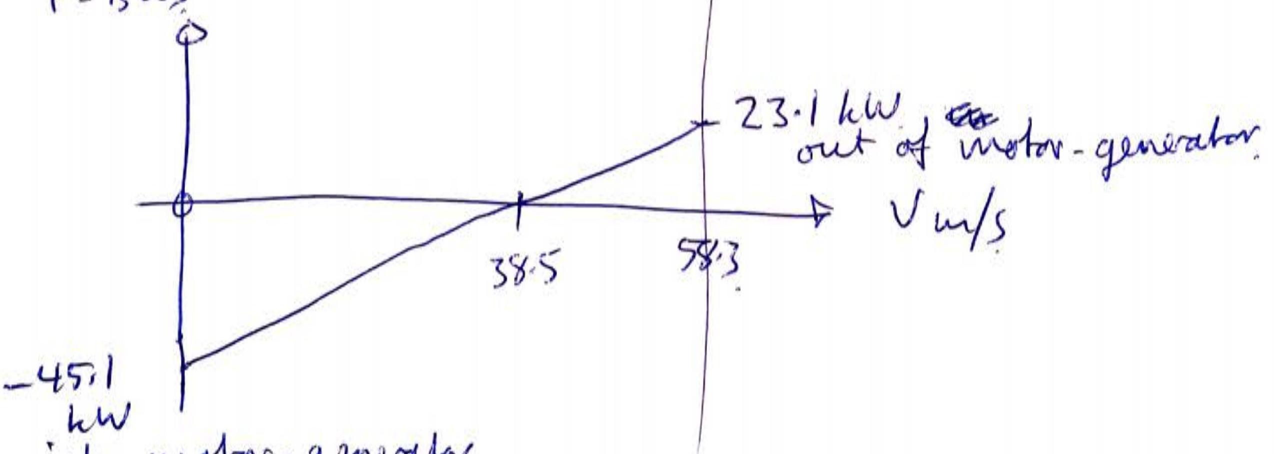


$$T_s = -\frac{2}{11} T_c = -\frac{2}{11} \cdot 100 = \underline{\underline{-18.2\ Nm}}$$

from (i)



mechanical power
$P = T_s \omega_s$



into motor-generator.

(equal to power from engine $T_c \omega_c = 100 \cdot 450$)
as expected