ENGINEERING TRIPOS IIA, MACHINE DESIGN, 3C8

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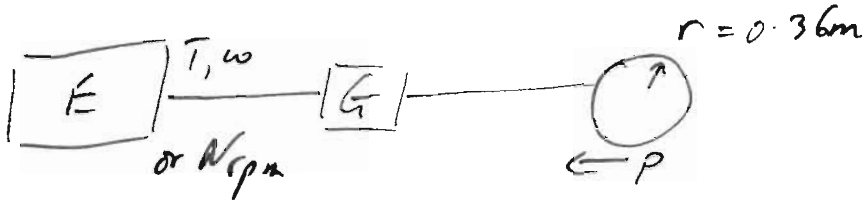
Q1.



Road force is P, resistance $F = 0.3V^2$

Assuming no losses in transmission $PV = T\omega$

But $V = \frac{r\omega}{G}$, $\omega = \frac{N \times 2\pi}{60}$

(a) For max. vehicle speed need max allowable power.

Likely to be at max N, but need to check.

N (rpm)	4000	5000	6000	This step often omitted.
Torque (Nm)	99	98	93	
$N \times$ Torque (Nm.rpm)	396×10^3	490×10^3	558×10^3	

So $T\omega = 93 \times \frac{6000 \times 2\pi}{60} = 0.3V^3$

steady speed so $P = F \Rightarrow V = \sqrt[3]{196.8 \times 10^3} = 58.0 \, ms^{-1}$

Required $G = \frac{r\omega}{V} = \frac{0.36 \times 6000 \times 2\pi}{60 \times 58.0} = 3.90$

1(b) If $G = 6 \Rightarrow V = \frac{0.36}{6} \cdot \frac{N.2\pi}{60} = 0.002\pi N$

Resistive force $F = 0.3V^2$ gives a resistive torque T at the engine of $\frac{Fr}{G}$, $T = 0.3.(0.002\pi N)^2 \times \frac{0.36}{6} = 7.11 \times 10^{-7} N^2$

N rpm	1000	2000	3000	4000	5000	6000
T Nm	0.71	2.84	6.4	11.4	17.8	25.6

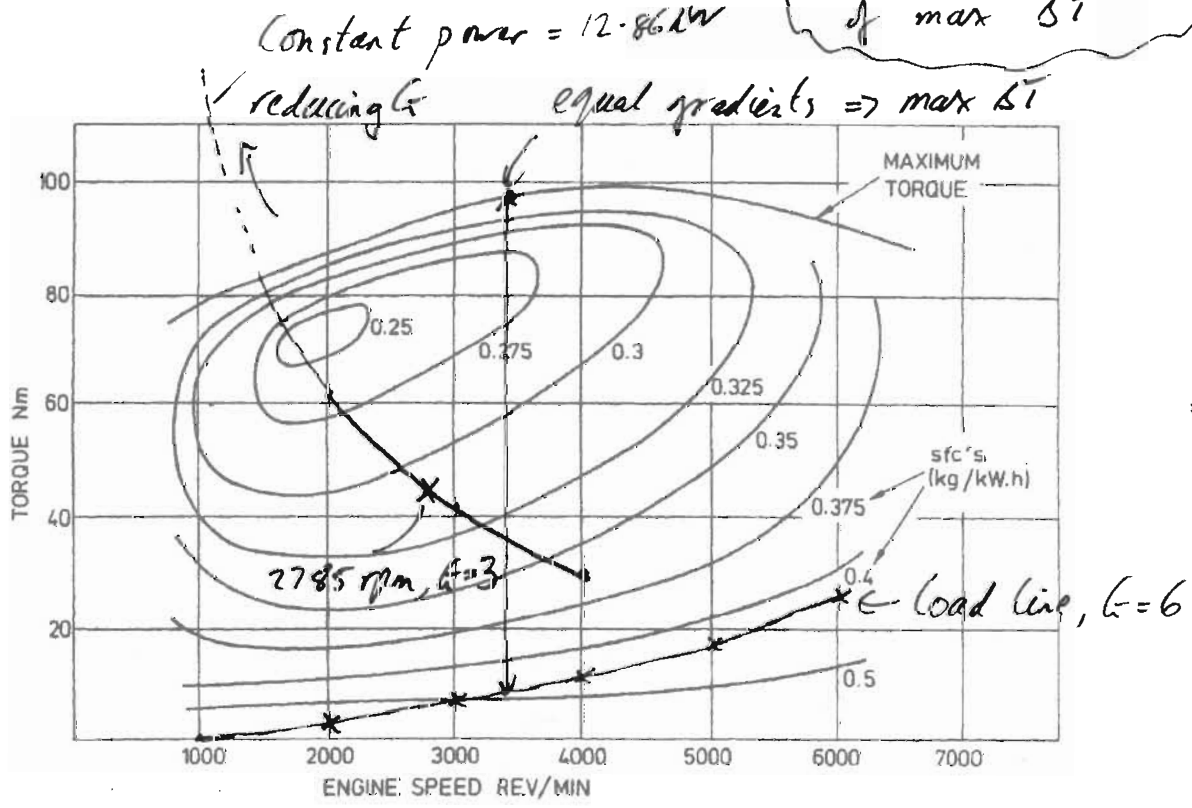
$\Delta T_{max} \simeq 89\,Nm$ @ 3400 rpm

Road force $\Delta P = \frac{89 \times 6}{0.36} = 1483\,N$ due to extra torque ΔT

But $\Delta P = mg \sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{1483}{9.81 \times 1000}\right) = 8.7^\circ \simeq 9^\circ$

Road speed $V = \frac{r\,N.2\pi}{G.60} = \frac{0.36 \times 3400 \times 2 \times \pi}{6 \times 60} = 22.6\,ms^{-1}$

Some variability in road speed expected as error in estimating location of max ΔT



1 (c)(i)

Highest gear corresponds to high wheel speed, ie. $G = 15$

Now $T = 0.46 \times 10^{-3} N^2$ due to road resistance.

Near the maximum torque of 99 Nm at 4200 rpm road torque = 0.8 Nm, so $\Delta T = 98\,\text{Nm}$ close to this peak.

ΔP available for acceleration $= 98 \times \frac{G}{r} = 4083\,N$

$$\text{accn} = \frac{\Delta P}{m} = 4.1\,\text{m s}^{-2}$$

(c)(ii) Plot line of constant power on characteristic

At $35\,\text{m s}^{-1}$ $F = 0.3V^2 = 367.5\,N$, Power $= FV = 12.86\,\text{kW}$

$$T = \frac{\text{Power}}{\omega} = \frac{12.86 \times 10^3}{N \times 2\pi} \times 60 = \frac{122.8}{N/1000}$$

In principle we could get close to the minimum fuel consumption with small enough G, but we are limited to $G = 3$, corresponding to $N = \frac{3 \times 35}{0.36} \times \frac{60}{2\pi} = 2785$ rpm

So best fuel consumption ≈ 0.31 kg/kWh

$$= \frac{0.31 \times 12.86}{0.75}\ \text{l/hr}$$

This step often omitted

$$1\ \text{hour} = \frac{35 \times 3600}{1000}\ \text{km} = 126\ \text{km}$$

$$\therefore \text{consumption} = \frac{0.31 \times 12.86}{0.75 \times 1.26} = 4.22\ \text{l/100 km}$$

(~ 67 mpg)

2 (a) See notes.

(b) From datasheet $p_0 = \frac{1}{\pi}\left(\frac{6PE^{*2}}{R^2}\right)^{\frac{1}{3}}$

$$= \frac{1}{\pi}\left(\frac{6E^{*2}}{R^2}\right)^{\frac{1}{3}} P^{1/3} = k P^{1/3} \quad (1)$$

$$k = \frac{1}{\pi}\left(\frac{6E^{*2}}{R^2}\right)^{\frac{1}{3}}$$

Area $A = \pi a^2 = \pi\left(\frac{3PR}{4E^*}\right)^{2/3} = k_1 P^{2/3}$ (2)

$$\Rightarrow k_1 = \pi\left(\frac{3R}{4E^*}\right)^{2/3}$$

~~&R~~ $\delta P = 2\pi r\, p(r)\, \delta r$

$$= 2\pi r\, p_0 \sqrt{1 - r^2/a^2}\; \delta r$$

(c) Load w per asperity $= \dfrac{\text{Pressure (Force/area)}}{\text{No. asperities per unit area}}$

$$= \frac{p_0}{m}\sqrt{1 - r^2/a^2}$$

$$= \frac{k}{m} P^{1/3} \sqrt{1 - \frac{r^2}{a^2}}$$

Q 2 (d)

Area of each small contact $\alpha = K_1' w^{2/3}$

where $K_1 = \pi\left(\frac{3R_1}{4E^*}\right)^{2/3}$

Within the anulus there are q points of contact, where $q = m.\underbrace{2\pi r\delta r}_{\text{area of annulus}}$

Summing up to give total true contact area

$$A_1 = \int_{r=0}^{a} \alpha\, m.2\pi r dr = m\int_{r=0}^{a} K_1'\left(\frac{KP^{1/3}}{m}\right)\left(1-\frac{r^2}{a^2}\right)^{1/3}.2\pi r dr$$

$$= \lambda P^{2/9}\int_0^a\left(1-\frac{r^2}{a^2}\right)^{1/3} r dr$$

where λ = constant.

Put $r = a\sin\theta$

$dr = a\cos\theta\, d\theta$

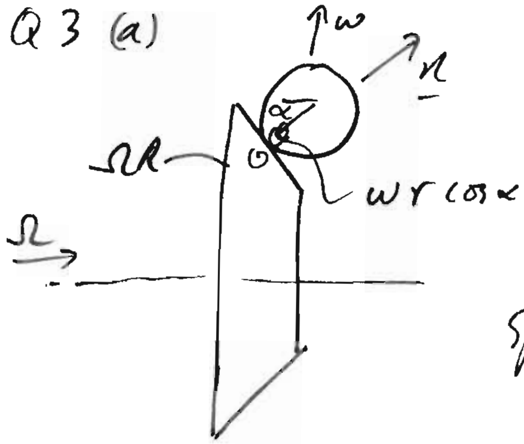
$$\Rightarrow A_1 = \lambda P^{2/9} a^2 \underbrace{\int_0^{\pi/2}\cos^{5/3}\theta\sin\theta\, d\theta}_{\text{another constant}}$$

$A_1 = \lambda' P^{2/9} a^2$ but $a^2 = k_1 P^{2/3}$

$\rightarrow A_1 \propto P^{2/9}.P^{2/3} = P^{8/9}$

(e) Real machine components have roughness of the type ~~mode~~ modelled, so analysis helps understand how contact area increases with load. Introduction of asperities gives ~~deper~~ nearly linear dependence on load, in contrast to smooth contact where $A \sim P^{2/3}$. But real asperities will wear in, and deform plastically so need to take this into account. Also consider lubrication.

Q 3 (a)



No slip $\Rightarrow \Omega r = w r \cos\alpha$

$$w = \frac{\Omega R}{r} \frac{1}{\cos\alpha}$$

Spin velocity $\omega_\gamma = (\underline{\omega}_1 - \underline{\omega}_2) . \underline{n}$

$$= \Omega \cos\alpha - \omega \sin\alpha$$

$$= \Omega \left(\cos\alpha - \frac{R}{r} \tan\alpha \right)$$

(b) Circular contact:

$$p_0 = \frac{1}{\pi} \left(\frac{6 P E^{*2}}{R^2} \right)^{\frac{1}{3}} \Rightarrow (1.2 \times 10^9) \pi = \frac{6 N . (115 \times 10^9)^2}{(15 \times 10^{-3})^2}$$

$$= 152 \text{ Newtons}$$

$$a = \left(\frac{3 P R}{4 E^*} \right)^{\frac{1}{3}} = 0.246 \text{mm}$$

(c) As the spin pole offset increases, slip in the contact tends to a limiting value $\tau = \mu p$ with all the shear stresses aligned in the direction of slip. In the limit $F = \mu N$ when gross slip occurs.

Then $Q = F R = \mu N R$

(d) Now $F = \frac{1}{3} \mu N \Rightarrow I_F = \frac{1}{3}$

For $\frac{a}{b} = 1$, $\frac{e}{a} = 0.29$ from charts and $I_m = 0.54$.

Power transmitted $= v F$

Power lost $= \omega_\gamma M + \Delta v F$

3 (d) (cont) But $\Delta v = e\,\omega_2$

$$v = \Omega R$$

$$\omega_2 = \Omega\left(\cos\alpha - \frac{R}{r}\tan\alpha\right)$$

$$\Rightarrow 1-\eta = \frac{\text{Power lost}}{\text{Power transmitted}} = \frac{|\omega_2 m| + \Delta v F}{v F} \quad \times \frac{1}{\Omega}\frac{1}{\mu N a}$$

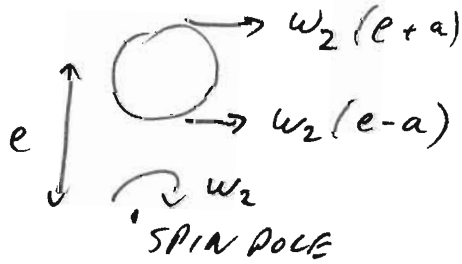
$$= \frac{\frac{\omega_2}{\Omega}\frac{m}{\mu N a} + \frac{\Delta v}{\omega_2}\frac{\omega_2}{\Omega}\frac{F}{\mu N a}}{\frac{v}{\Omega}\ \frac{F}{\mu N A}}$$

$$= \left|\left(\cos\alpha - \frac{R}{r}\tan\alpha\right)\right|\left(I_m + \frac{e}{a}I_F\right) \Big/ \frac{R}{a} I_F$$

$$= \left|\cos 20 - \frac{90}{15}\tan 20\right|\left(0.54 + 0.29 \times \tfrac{1}{3}\right) \Big/ \cancel{\tfrac{90}{0.4}}\left(\frac{90}{0.246} \times \frac{1}{3}\right)$$

$$= 6.50 \times 10^{-3} \Rightarrow \eta = 0.994$$

(e)

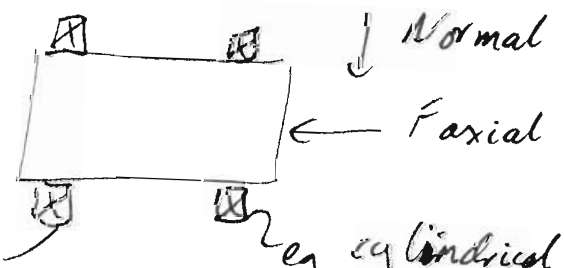


$e+a = 0.317\,\text{mm}$, $e-a = 0.175\,\text{mm}$

$$\omega_2 = \underbrace{10.47}_{\Omega} \times \underbrace{1.244}_{(\cos\alpha - \frac{R}{r}\tan\alpha)}\ \text{rad/s}$$

$\Delta v_{max} = 4.13\,\text{mm/s}$, $\Delta v_{min} = 2.28\,\text{mm/s}$

(f)



→ need significant axial force

→ modest speed

→ low cost

eg deep grove ball bearing

or perhaps taper roller bearing for higher axial loads.

eg cylindrical roller bearing

Q4. w = wheel, p = pinion } subscripts

Power $Q = T_w \omega_w$

Face width $w = 2r_p$ (square pinion)

$r = mN/2$

line load $P' = \dfrac{T}{w\, r \cos\phi} = \dfrac{Q}{\omega_w \cdot \underbrace{2r_p}_{w} \cdot r_w \cos\phi}$

$$= \frac{Q}{\omega_w \cdot 2 \cdot m\frac{N_p}{2} \cdot m\frac{N_w}{2} \cos\phi} = \frac{152.4}{m^2} \text{ N/m}$$

$\omega_w = \dfrac{200 \times 2\pi}{60}$

Bending $\sigma_b = \dfrac{P_T'}{Jm} = \dfrac{P'\cos\phi}{Jm}$

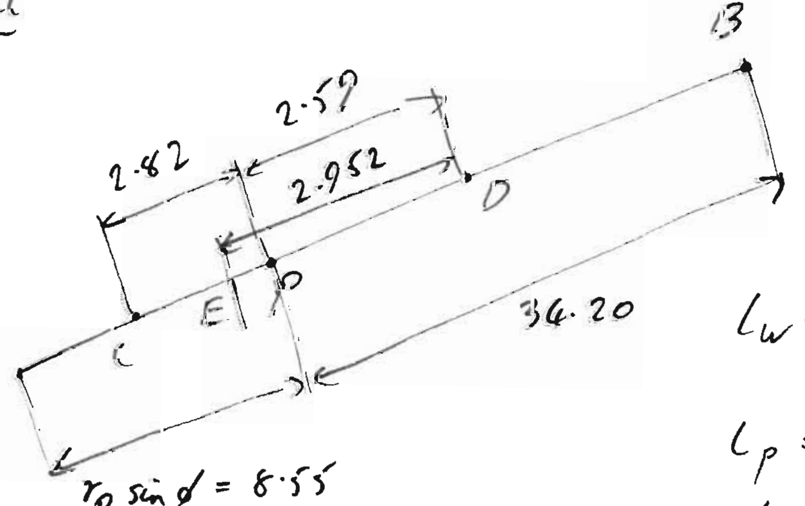
Critical case is pinion with $J = 0.46$, $\sigma_b = 480$ MPa

$$\Rightarrow m = \frac{P'\cos\phi}{\sigma_b J} \Rightarrow m^3 = \frac{152.4 \cdot \cos 20}{480\times10^6 \cdot 0.46} = 6.49\times10^{-7}\ \text{m}^3$$

$m = 8.66$ mm

Contact

Dimensions in terms of m



$l_w = \sqrt{0.02924 N^2 + N + 1} - 0.1710 N = 2.82m$

$l_p = 2.59m$

$p_b = p\cos\phi = \pi m \cos\phi = 2.952m$

4 (a) (cont)

1 pair
$$\frac{1}{R} = \frac{1}{8.55 + 2.59 - 2.952} + \frac{1}{34.20 - 2.59 + 2.952}$$

$$\Rightarrow R = 6.62\,m$$

2 pair
$$\frac{1}{R} = \frac{1}{8.55 - 2.82} + \frac{1}{34.20 + 2.82}$$

$$\Rightarrow R = 4.96\,m$$

One pair critical as $6.62 < 2 \times 4.96$

$$p_0 = \left(\frac{P' E^*}{\pi R}\right)^{\frac{1}{2}} \Rightarrow \left(1000 \times 10^6\right)^2 = \frac{152.4}{m^2} \cdot \frac{115 \times 10^9}{\pi . 6.62 m}$$

$$\Rightarrow m^3 = 8.63 \times 10^{-7}\, m^3$$

$$m = 9.65\,mm$$

Surface contact is critical, choose $m = 10\,mm$

(b) Similar geometry, R scales with m, r scales with m

w scales with m (as square pinions), line load $P' \times \frac{1}{4}$

Assume loads equally divided.

A - part (a), B - part (b)

Power constant $\Rightarrow \frac{Q_A}{Q_B} = 1$

Alternatively plug numbers into appropriate formulae accounting for $\times \frac{1}{4}$ load

number of pinions

Line loads $$\frac{P'_A}{P'_B} = \frac{Q_A}{Q_B}\,\frac{w_B}{w_A}\,\frac{r_B}{r_A}\,\frac{n_B}{n_A} = 4\,\frac{m_B^2}{m_A^2}$$

$$\left(\frac{\sigma_B}{\sigma_A}\right)^2 = 1 = \frac{P'_B}{P'_A}\,\frac{R_A}{R_B} = \frac{m_A^2}{4 m_B^2} \cdot \frac{m_A}{m_B}$$

$$\Rightarrow m_B = m_A \sqrt[3]{1/4} = 9.65 \sqrt[3]{1/4} = 5.95 \rightarrow 6\,mm$$

(c) (b) is significantly smaller than (a) which is likely to lead to a significant reduction in cost of these large gears (eg $r = m\,n/2 = 10 \times 100 = 1000\,mm$)

But (b) is more complex and need to design carefully to ensure ~~and~~ an even load split.