Version MPFS/4

EGT2
ENGINEERING TRIPOS PART IIA

Monday 28 April 2014 2 to 3.30

Module 3C8

MACHINE DESIGN

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The **approximate** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number **not** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Module 3C8 data sheet (9 pages)
Supplementary page: one extra copy of Fig 1 (Question 1)
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Figure 1 shows the performance of an internal combustion engine that is to be used to power a vehicle of mass 1000 kg with wheel radius of 0.36 m. Apart from starting, the engine must only operate between 1000 and 6000 rpm. The total road resistance F (in Newtons) on a level road is given by

$$F = 0.3V^2$$

where V is the speed in m s^{-1}.

(a) What is the maximum speed the vehicle could achieve on a level road and what value of the gear ratio G (engine speed : road wheel speed) would be needed? [25%]

(b) For a gear ratio $G = 6$ plot the load characteristic on the supplementary copy of Fig. 1 and hence show that the maximum gradient that the vehicle can climb in this gear is about 9°. What is the vehicle speed under these conditions? [25%]

(c) The vehicle is to be provided with a continuously variable transmission such that G can vary from 3 to 15.

(i) If the highest gear ratio were to be selected, estimate the maximum acceleration that could be achieved on a level road, and [25%]

(ii) estimate the fuel consumption in litres per 100 km when running at maximum economy on a level road at a steady speed of 35 m s^{-1}. The density of the fuel is 0.75 kg per litre. [25%]

A copy of Fig. 1 is provided and should be handed in with your script.

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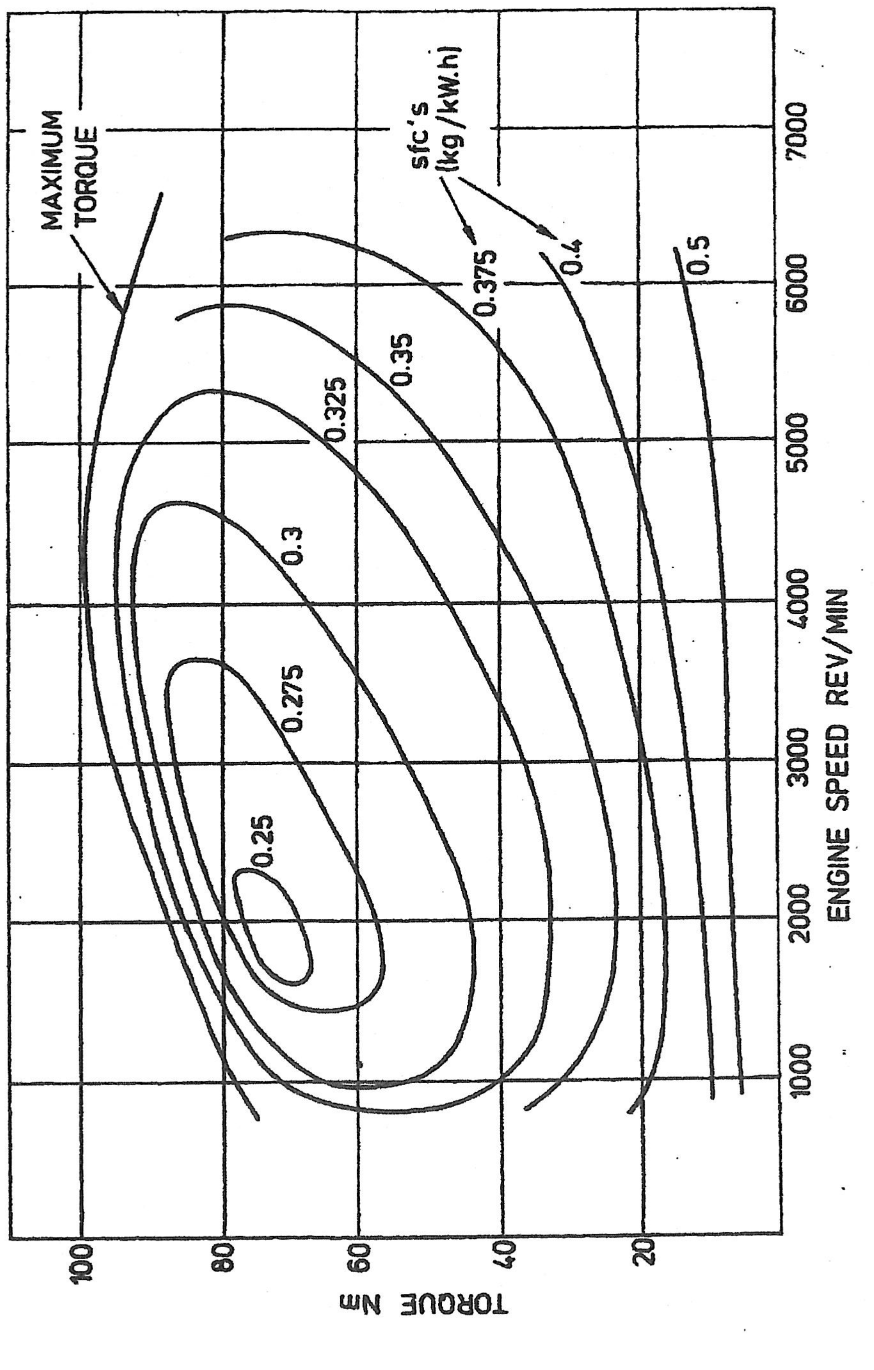

Fig. 1

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2 (a) Summarise briefly the idealizations that are made in the Hertz theory of elastic contact. [10%]

(b) By direct appeal to the data sheet, show that in an elastic contact between a smooth sphere of radius R and a smooth, plane surface carrying a normal load P, as illustrated in Fig. 2(a), that the peak Hertzian pressure p_0, and the true area of contact A can be written as

$$p_0 = K P^{1/3} \quad \text{and} \quad A = K_1 P^{2/3}$$

where K and K_1 are appropriate constants which should be defined.

In such a contact it can be assumed that the pressure $p(r)$ within the contact area is distributed according to the relation

$$p(r) = p_0\sqrt{1 - r^2/a^2}$$

where a is the radius of the circular contact spot. On this basis, write down the contribution δP made to P by the annular ring within the contact area of radius r and width δr. [30%]

(c) The surface of the sphere is in fact covered by a population of small spherical protuberances, or asperities, each of radius R_1 $(\ll R)$. There are m of these per unit area as illustrated in Fig. 2(b). Show that within the chosen annulus, the average load per asperity w is

$$w = \frac{K P^{1/3}}{m}\sqrt{1 - r^2/a^2} \; .$$ [25%]

(d) The true contact area A_1 between the rough sphere and the flat surface can now be established by first considering the asperities within the chosen annulus and then integrating for all such annuli between the limits $r = 0$ and $r = a$. By carrying out these steps show that

$$A_1 \propto P^n$$

where the value of n should be determined. There is no need to evaluate explicitly the constant of proportionality. [25%]

(e) Comment on the relevance of this analysis to the contact between real machine components with rough surfaces. [10%]

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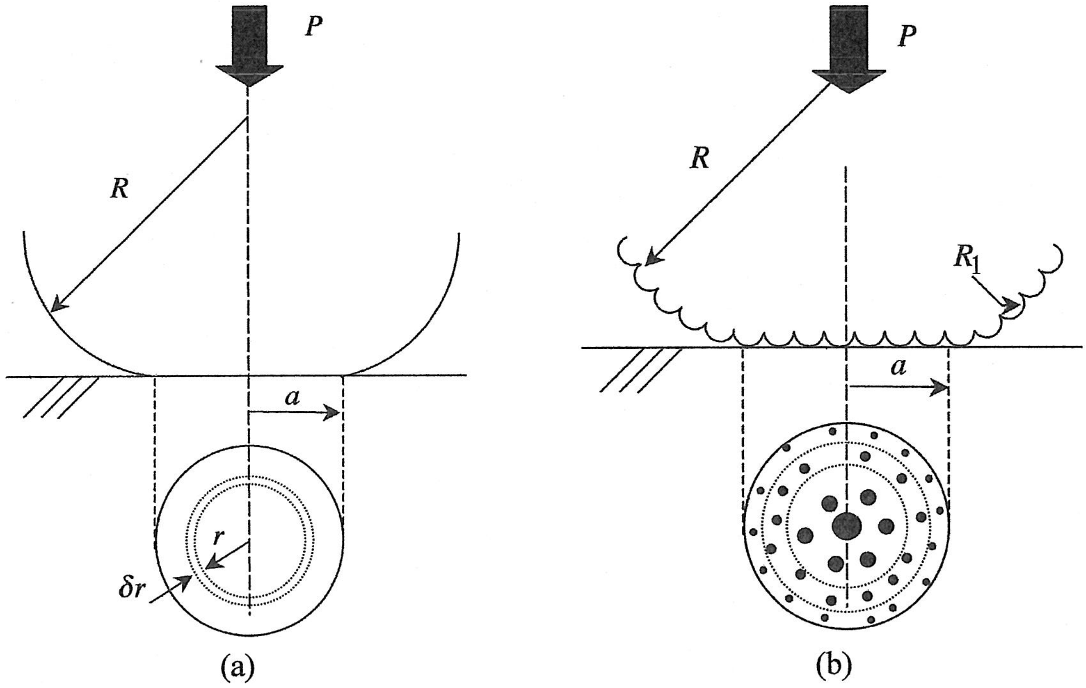

Fig. 2

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3 Figure 3 illustrates part of a proposed continuously variable transmission for an automobile. The input is via a conical disc which rotates at speed Ω about a horizontal axis. The cone face is inclined at an angle $\alpha = 20°$ to the vertical. The disc contacts a sphere at a distance $R = 90$ mm from the axis of rotation of the disc. The sphere of radius $r = 15$ mm is supported via bearings, so that it rotates about a vertical axis at angular velocity ω with its centre O remaining fixed.

(a) Assuming no slip conditions at the contact point between the disc and sphere, derive expressions for the angular velocity of the sphere and the spin velocity at the contact in terms of Ω, α, R and r. [15%]

(b) A force N normal to the contact is applied to the sphere to keep it in contact with the disc. Assume that Hertzian conditions apply at the contact between the sphere and disc and that the contact can be approximated as a sphere of radius r in contact with a flat, with a contact modulus $E^* = 115$ GPa. Find the value of N which gives a maximum Hertzian pressure of 1.2 GPa. What is the radius of the corresponding contact patch? [15%]

(c) Explain how the Hertzian contact frictional loss curves in the data sheet imply a limiting input torque Q that can be transmitted. What are the sliding conditions in the contact corresponding to this limiting torque? Write down an expression for the limiting value of Q in terms of N, the local friction coefficient μ and the geometric parameters of the drive. [15%]

(d) Use the frictional loss curves in the data sheet to calculate the efficiency of the drive when the torque input Q equals one third of its limiting value and with N equal to the value found in part (b). [35%]

(e) For the same conditions as part (d), what are the minimum and maximum relative sliding velocities in the contact, for an input angular velocity $\Omega = 100$ rpm? [10%]

(f) Sketch an appropriate bearing arrangement for supporting the input disc shaft, explaining why you have chosen this solution. [10%]

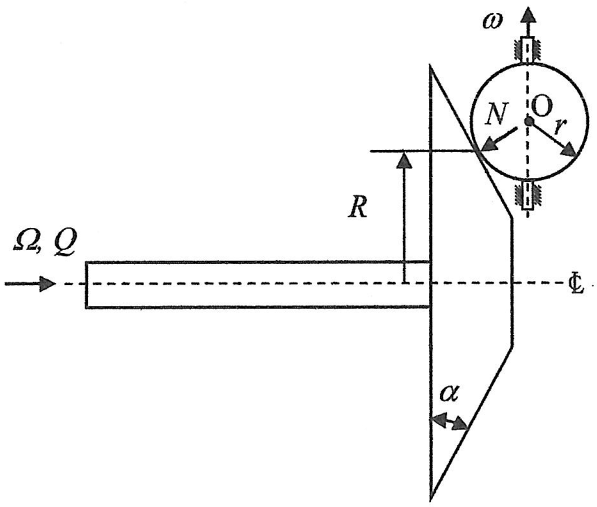

Fig. 3.

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4 (a) The final drive of a machine transmits a power of 15 MW. It is proposed to use a single pinion and wheel for the drive, with tooth numbers of 50 and 200 for the pinion and wheel, respectively. The speed of the wheel is 200 rpm. Spur gears with a standard involute tooth profile are to be used, with the addendum equal to the module and a pressure angle equal to 20°. A square pinion is proposed, with a face width equal to its pitch circle diameter. Select an appropriate module for precision gears made of steel with bending and surface fatigue strengths of 480 MPa and 1000 MPa, respectively. [50%]

(b) An alternative solution for the final drive is proposed, using four square pinions to drive the wheel, again with the same material, pinion and wheel speeds and power as in part (a). What module is now required to avoid surface failure? Do not consider tooth bending failure. State any assumptions you make. [40%]

(c) What are the advantages and disadvantages of the solutions proposed in parts (a) and (b)? [10%]

END OF PAPER