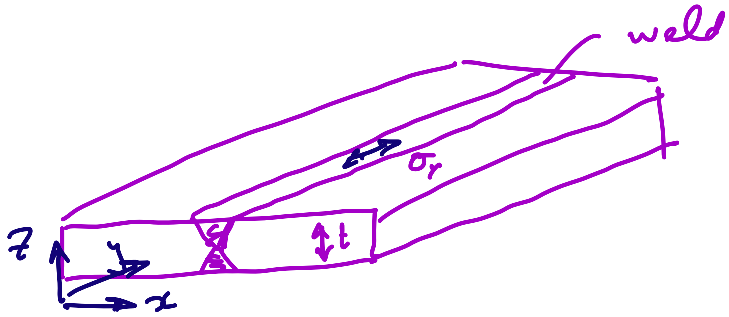
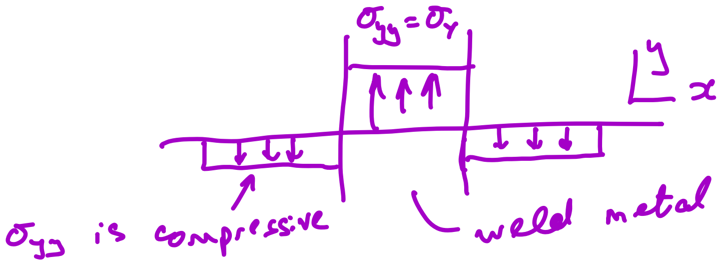
2024/5 Crib for 3C9 : Fracture Mechanics of Materials and Structures

A1. (a)



Assume the weld is along the y direction. Upon solidification of ~~the~~ molten weld pool shrinkage occurs. Shrinkage along the longitudinal y-direction is restrained by adjacent material and the stress component σ_{yy} attains the yield strength σ_Y.



Now consider the transverse stress σ_{xx}.

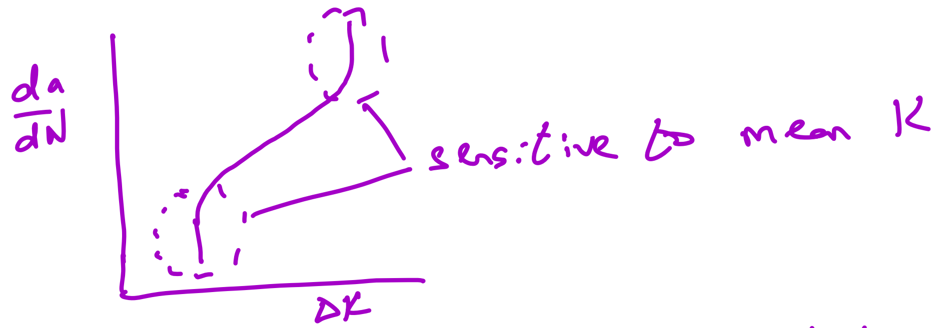
$\frac{1}{t}\int_0^t \sigma_{xx}\, dz = \bar{\sigma}_{xx}$, average membrane stress in x-direction. The value of $\bar{\sigma}_{xx}$ depends upon the remote constraint in ~~the~~ x-direction.

There may be some variation of $\sigma_{xx}(z)$ that depends upon whether the weld is a single pass or multi-pass.

1. (b) The stress intensity factor K on a crack is due to the sum of K_R due to residual stress and K_a due to active, live loads due to traffic and dead loads.

Fracture occurs when $K_R + K_a = K_{IC}$, ~~the~~ the fracture toughness.

Fatigue crack growth is influenced by the mean value of $(K_R + K_a)$ when ΔK_a is near threshold or K_{max} is near K_{IC}.



It is possible for $K_a < 0$ but $K_a + K_R > 0$ due to the fact ~~that~~ $K_R \approx \sigma_Y \sqrt{\pi a}$ for a flaw of length a embedded in a region of tensile residual stress of yield strength magnitude.

1. (c) Most of the fatigue life of a welded structure is spent in ~~the~~ early stages of crack growth.

Assume $\frac{da}{dN} = 0$ for $\Delta K < \Delta K_{th}$

$$\frac{da}{dN} = A \Delta K^n \quad \text{for } \Delta K_{th} < \Delta K < K_{Ic}(1-R)$$

where ~~the~~ load ratio $R = K_{min}/K_{max}$

Assume an initial flaw size a_0 and a final flaw size a_f.

Apply $\Delta\sigma$ and a K-calibration of $K = \sigma\sqrt{\pi a}$.

$$N_f = \int_0^{N_f} dN = \int_{a_0}^{a_f} \left(\frac{da}{dN}\right)^{-1} da = \int_{a_0}^{a_f} \frac{1}{A(\Delta\sigma\sqrt{\pi a})^n}\, da$$

$$= \frac{1}{A\Delta\sigma^n}\pi^{-n/2}\int_{a_0}^{a_f} a^{-n/2}\, da$$

$$= \frac{\pi^{-n/2}}{A\,\Delta\sigma^n}\left[\frac{2a^{\frac{2-n}{2}}}{(2-n)}\right]_{a_0}^{a_f}$$

$$= \frac{2\pi^{-n/2}}{(n-2)A\,\Delta\sigma^n}\left[\left(\frac{1}{a_0}\right)^{\frac{n-2}{2}} - \left(\frac{1}{a_f}\right)^{\frac{n-2}{2}}\right]$$

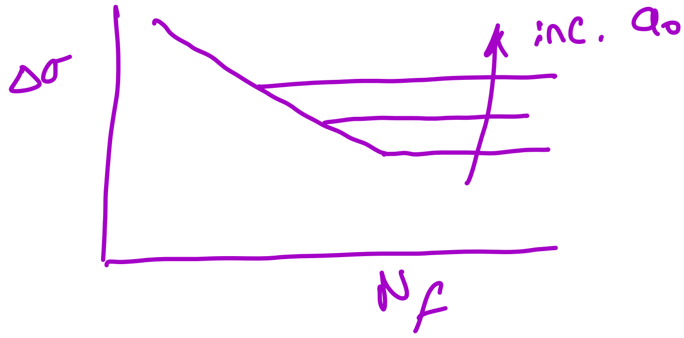
We can drop ~~the~~ term $\left(\frac{1}{a_f}\right)^{\frac{n-2}{2}}$ for $n > 2$

eg. $n = 4$, since $a_f \gg a_0$.

$$\frac{\Delta\sigma}{(1-R)}\sqrt{\pi a_f} = K_{Ic} \Rightarrow a_f = \frac{(1-R)^2 K_{Ic}^2}{\pi\,\Delta\sigma^2}$$

$$\frac{da}{dN} = A\,\Delta\sigma^n\,\pi^{n/2}\,a^{n/2}$$

Note if $\Delta\sigma\sqrt{\pi a_0} < \Delta K_{th}$
then ~~the~~ fatigue life is infinite.



1. (d) $a_f = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_Y}\right)^2$

So choose a microstructure that gives a high fracture toughness K_{IC} and a low yield strength σ_Y.

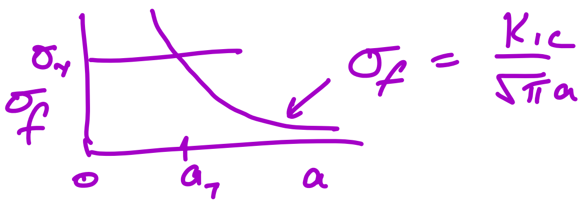
σ_Y is decreased by a larger grain size, and a wider precipitate spacing and dislocation spacing.

K_{IC} is increased by increasing ~~the~~ the inclusion spacing — make the steel cleaner.

If the steel is annealed then work hardening is removed and dislocation spacing increases. Precipitates coarsen and grain size increases. Consequently, σ_Y drops.

2. (a)

$a_T = \frac{1}{\pi}\frac{K_{IC}^2}{\sigma_Y^2}$



Failure at $\sigma_f \approx \sigma_Y$ for $a < a_T$

and at $\sigma_f \approx \frac{K_{IC}}{\sqrt{\pi a}}$ for $a > a_T$

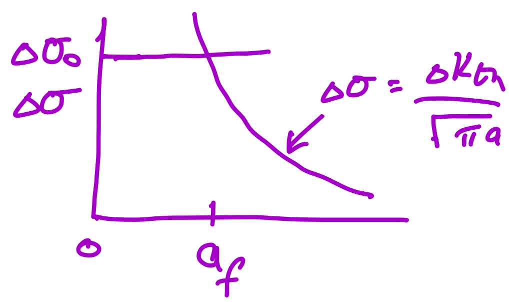
For cyclic loading define a cyclic transition flaw size a_f by $a_f = \frac{1}{\pi}\frac{\Delta K_{th}^2}{\Delta\sigma_0^2}$

where ΔK_{th} = fatigue threshold

and $\Delta\sigma_0$ = stress range at ~~the~~ the fatigue limit

Typically, $\Delta K_{th} \approx \frac{1}{10} K_{IC}$ and $\Delta\sigma_0 \approx \sigma_Y$ for metallic alloys.

Hence $a_f \approx \frac{a_T}{100}$



Consequently, alloys are much more flaw sensitive to cyclic loading than to mo0

2. (b)(i) Basquin's Law from materials data book is

$$\Delta\sigma \, N_f^{\alpha} = C_1$$

$\Delta\sigma = 2\sigma_0$ for fully reversed loading and $N_f = 10^7$ cycles

$$\Rightarrow \quad 2\sigma_0 \, 10^{7\alpha} = C_1 \qquad -(1)$$

Also, $N_f = 10^4$ cycles at $\Delta\sigma = 3\sigma_0$

$$\Rightarrow \quad 3\sigma_0 \, 10^{4\alpha} = C_1 \qquad -(2)$$

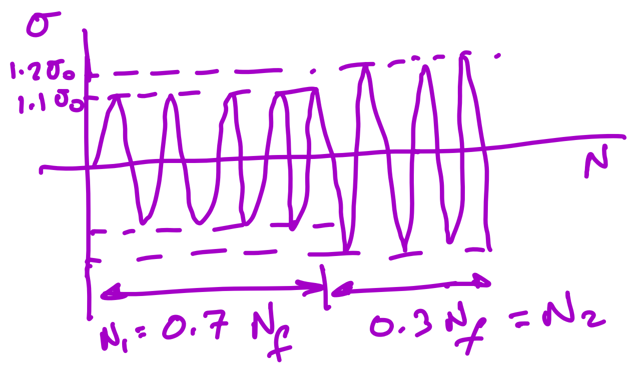
Take ratio of (1) and (2) $\Rightarrow \frac{3}{2} 10^{-3\alpha} = 1$

$$\Rightarrow \quad 10^{3\alpha} = \frac{3}{2} \qquad \Rightarrow 3\alpha = \log_{10} 3/2$$

$$\Rightarrow \alpha = 0.05870$$

$$(2) \Rightarrow \quad C_1 = 3\sigma_0 \, 10^{4\alpha}$$

$$= 3\sigma_0 \left(\frac{3}{2}\right)^{4/3} \sigma_0 = \underline{5.151\,\sigma_0}$$



Miner's rule states $\sum_i \frac{N_i}{N_{f_i}} = 1$

$\Delta\sigma_1 = 2.2\sigma_0$, $N_1 = 10^7 N_f$

$N_{f1} = ?$ by Basquin

$$\Delta\sigma \, N_f^{\alpha} = C_1 \quad \Rightarrow \underbrace{2\sigma_0 \, 10^{7\alpha}}_{\text{at fatigue limit}} = 2.2\sigma_0 \, N_{f1}^{\alpha}$$

$$\text{so} \quad N_{f1} = 10^7 \left(\frac{2}{2.2}\right)^{1/\alpha}$$

$\Delta\sigma_2 = 2.4\sigma_0$, $N_{f2} = ?$ $\qquad 2\sigma_0 \, 10^{7\alpha} = 2\sigma_0 \, N_{f2}^{\alpha}$

So $N_{f2} = 10^7 \left(\frac{2}{2.4}\right)^{1/\alpha}$ $\qquad$ Here, $N_2 = 0.3 N_f$

2.(b) contd.

Now substitute into Miner's Rule to get

$$\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} = 1 \qquad \text{Recall } \alpha = 0.05870$$

$$\Rightarrow \frac{0.7\, N_f}{10^7 (2/2.2)^{1/\alpha}} + \frac{0.3\, N_f}{10^7 (2/2.4)^{1/\alpha}} = 1$$

$$\Rightarrow N_f = 10^7 \left[0.7 \left(\frac{2.2}{2}\right)^{1/\alpha} + 0.3 \left(\frac{2.4}{2}\right)^{1/\alpha} \right]^{-1}$$

$$= 10^7 [\, 3.550 + 6.699 \,]^{-1} \text{ cycles}$$

$$= 9.76 \times 10^5 \text{ cycles}$$

2.(b)(ii) Goodman's rule from materials data book is

$$\Delta\sigma = \Delta\sigma_0 \left(1 - \frac{\sigma_m}{\sigma_{ts}} \right) \qquad \text{Here, } \frac{\sigma_m}{\sigma_{ts}} = 0.2$$

$$\Rightarrow \Delta\sigma_0 = 1.25\, \Delta\sigma$$

mean stress absent

mean stress present

So $\Delta\sigma_1 = 2.2\, \sigma_0$ becomes $\Delta\sigma_1 = 1.25 \times 2.2\, \sigma_0 = 2.75\, \sigma_0$

$\Delta\sigma_2 = 2.4\, \sigma_0$ becomes $\Delta\sigma_2 = 1.25 \times 2.4\, \sigma_0 = 3\, \sigma_0$

Consequently, N_{f1} becomes $N_{f1} = 10^7 \left(\frac{2}{2.75} \right)^{1/\alpha}$

and N_{f2} becomes $N_{f2} = 10^7 \left(\frac{2}{3} \right)^{1/\alpha}$

Miners rule is again: $\frac{0.7\, N_f}{N_{f1}} + \frac{0.3\, N_f}{N_{f2}} = 1$

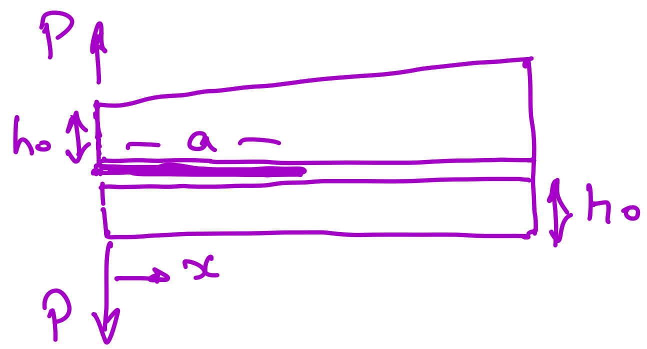
$$\Rightarrow \quad \frac{0.7\,N_f}{10^7}\left(\frac{2.75}{2}\right)^{1/\alpha} + \frac{0.3\,N_f}{10^7}\left(\frac{3}{2}\right)^{1/\alpha} = 1$$

So $$N_f = 10^7\left[0.7\left(\frac{2.75}{2}\right)^{1/\alpha} + 0.3\left(\frac{3}{2}\right)^{1/\alpha}\right]^{-1}$$

$$= 10^7\left[158.9 + 299.9\right]^{-1}$$

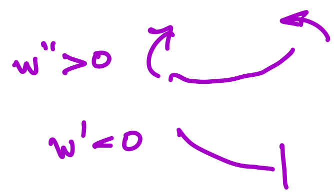
$$= \underline{2.18 \times 10^4 \text{ cycles}}$$

3. (a)



Assume that the upper beam deflects by $w(x)$ over the crack length $0 < x < a$.

$$EI\,w'' = M = Px$$



where $I = \frac{1}{12} b h^3 = \frac{1}{12} b (h_0 + \alpha x)^3$

So $$w'' = \frac{Px}{EI} = \frac{12P}{Eb} \frac{x}{(h_0 + \alpha x)^3}$$

At $x = a$, $w = w' = 0$

$$\Rightarrow \quad w'(x) = \frac{12P}{Eb} \int_x^a \frac{x}{(h_0+\alpha x)^3}\, dx$$

Now use hint by taking $A = h_0$ and $B = \alpha$, to give

$$w'(x) = \frac{12P}{\alpha^2 Eb} \left[\frac{h_0}{2} (h_0 + \alpha x)^{-2} - (h_0 + \alpha x)^{-1} \right]_x^a < 0$$

$$\Rightarrow \quad w'(x) = \frac{12P}{\alpha^2 Eb} \left[\frac{h_0}{2} (h_0 + \alpha a)^{-2} - (h_0 + \alpha a)^{-1} - \frac{h_0}{2} (h_0 + \alpha x)^{-2} + (h_0 + \alpha x)^{-1} \right]$$

Write $$D \equiv \frac{h_0}{2} (h_0 + \alpha a)^{-2} - (h_0 + \alpha a)^{-1}$$

$$\frac{\partial D}{\partial a} = -h_0 \alpha (h_0 + \alpha a)^{-3} + \alpha (h_0 + \alpha a)^{-2}$$

$$\Rightarrow \quad w(x) = \frac{12P}{\alpha^2 Eb}\left[Dx + \frac{h_o}{2}(h_o+\alpha x)^{-1} + \ln(h_o+\alpha x)\right]_x^a$$

$$\Rightarrow \quad W(x) = \frac{12P}{\alpha^2 Eb}\left[D(a-x) + \frac{h_o}{2}(h_o+\alpha a)^{-1} - \frac{h_o}{2}(h_o+\alpha x)^{-1} + \ln\left(\frac{h_o+\alpha a}{h_o+\alpha x}\right)\right]$$

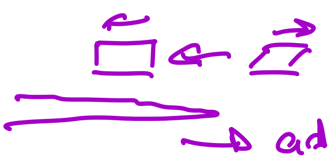
$$\Rightarrow \quad w(x=0) = \frac{12P}{\alpha^2 Eb}\left[Da + \frac{h_o}{2}(h_o+\alpha a)^{-1} - \frac{1}{2} + \ln\left(1+\frac{\alpha a}{h_o}\right)\right]$$

(b) $C = \dfrac{2w(0)}{P} = C(a)$

$$\frac{\partial C}{\partial a} = \frac{24}{\alpha^2 Eb}\left[D + a\frac{\partial D}{\partial a} - \frac{h_o}{2}\alpha(h_o+\alpha a)^{-2} + \frac{\alpha}{h_o}\left(1+\frac{\alpha a}{h_o}\right)^{-1}\right]$$

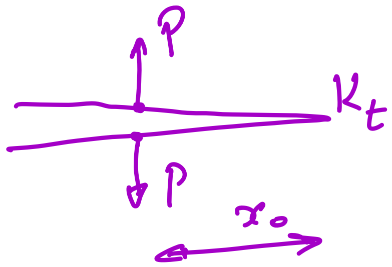
$Gb = \frac{1}{2}P^2\dfrac{\partial C}{\partial a}$ $\qquad$ Pot. Energy $= -\frac{1}{2}P\,w(0)$

(c) Non-proportional straining of material elements occur as the crack advances. This leads to additional plastic dissipation for a metallic alloy whereas stored elastic energy is recoverable in an elastic solid.



in metal as the crack advances.

4. (a)

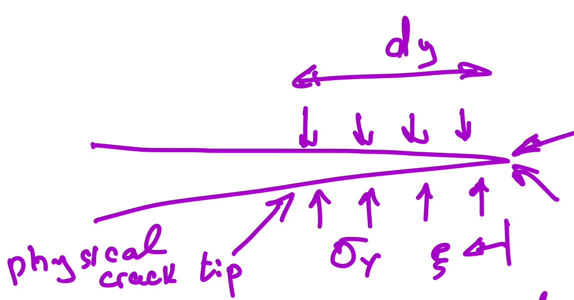


$$K_t = \frac{2P}{\sqrt{2\pi x_0}} + K^\infty$$

Can use linear superposition of K because we are considering linear elasticity theory whereby ~~the~~ equations of equilibrium, strain-displacement and ~~the~~ constitutive law are all linear. stress fields add and consequently K fields add.

(b)

K^∞



fictitious crack tip, a distance d_y ahead of physical crack tip

$K_t = 0$

Total stress intensity factor at ~~the~~ crack tip K_t is

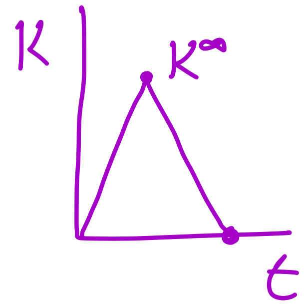
$$K_t = K^\infty - \int_0^{d_y} \frac{2\sigma_Y}{\sqrt{2\pi\xi}}\, d\xi = 0$$

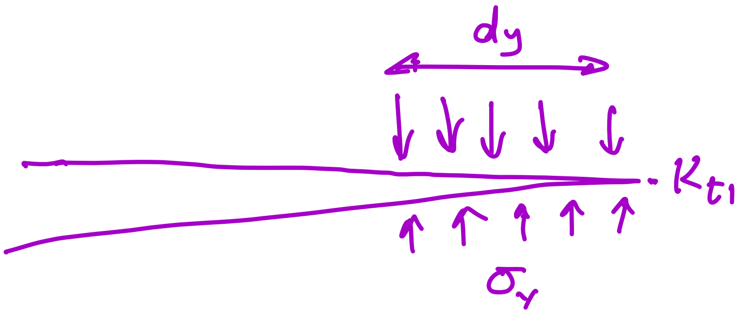
Now $\displaystyle\int_0^{d_y} \sqrt{\frac{2}{\pi}}\, \sigma_Y\, \xi^{-1/2}\, d\xi = 2\sqrt{\frac{2}{\pi}}\, \sigma_Y\, d_y^{1/2}$

Hence $\displaystyle d_y = \frac{\pi}{8}\left(\frac{K^\infty}{\sigma_Y}\right)^2$

(c)

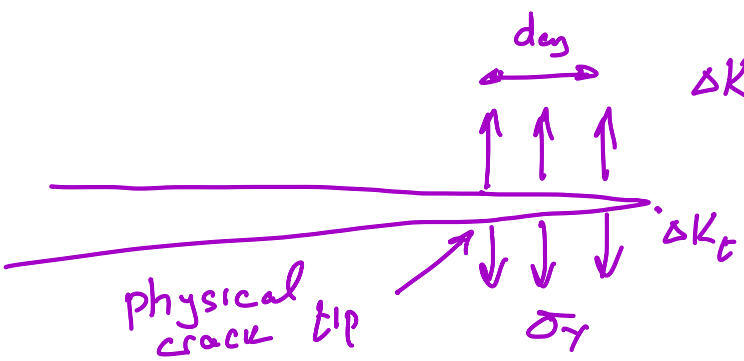
At K^∞ we have :





$$K_{t1} = 0 = K^\infty - 2\sqrt{\frac{2}{\pi}}\,\sigma_Y d_y^{1/2}$$

After unloading by ΔK^∞ $(= -K^\infty)$ we have :



$$\Delta K_t = 0 = \Delta K^\infty + \int_0^{d_{cy}} \sqrt{\frac{2}{\pi}}\,\frac{2\sigma_Y}{\sqrt{\xi}}\,d\xi$$

Hence $$d_{cy} = \frac{\pi}{8}\left(\frac{K^\infty}{2\sigma_Y}\right)^2$$

Note : over d_{cy} the stress bridging the crack jumps by $2\sigma_Y$ from $-\sigma_Y$ to σ_Y as a result of the unloading by ΔK^∞.

(d) Small scale yielding assumes that the plastic zone at the crack tip is embedded within an outer K-field. The domain of the K-field scales with the crack length a.

$$a > 2.5 \frac{K^2}{\sigma_Y^2} = 2.5\pi \underbrace{\frac{1}{\pi} \frac{K^2}{\sigma_Y^2}}_{= r_p}$$

So $a > 2.5\pi\, r_p$