Crib for module 3C9 : Fracture Mechanics of Materials and Structures. 2022-23 Part IIA

Q1. (a) Near the fatigue threshold, plasticity induced crack closure is augmented by oxide and roughness induced closure, and this elevates the crack opening stress intensity $K_{op}$.

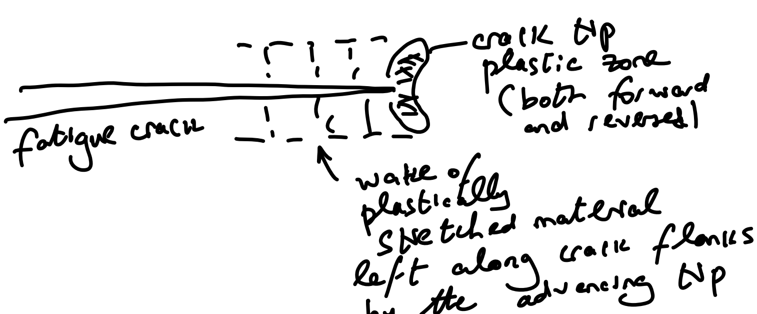
roughness-induced closure

roughness (scale is set by grain size) with a mode II component of K leads to a wedging open of the crack near the tip.

oxide-induced closure

oxide debris on crack flanks is generated by fretting in a moist oxidising environment, such as moist air.

plasticity induced crack closure

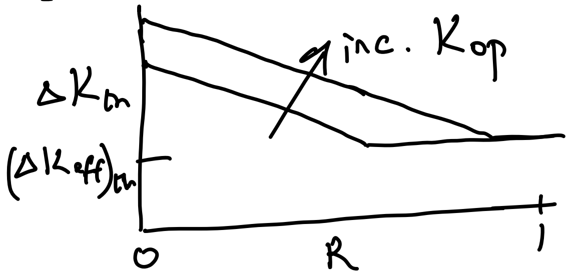



$\Delta K_{eff} = K_{max} - K_{op}$ , $\Delta K = K_{max} - K_{min}$, $R = \frac{K_{min}}{K_{max}}$

At fatigue threshold, $\Delta K_{eff} = (\Delta K_{eff})_{th}$ , a material property

So, $(\Delta K_{eff})_{th} = K_{max} - K_{op} = \frac{\Delta K_{th}}{1-R} - K_{op}$

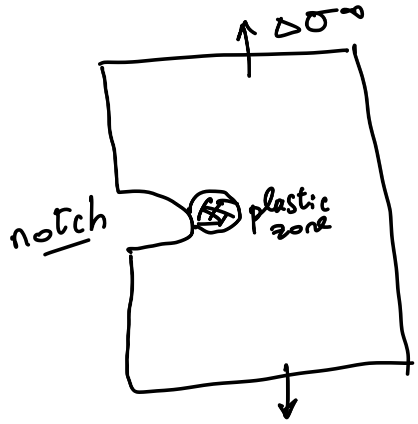
So $\Delta K_{th} = (1-R)\left[(\Delta K_{eff})_{th} + K_{op}\right]$




If $K_{op}$ is constant, then $\Delta K_{th}$ varies with R as shown.

oxide & roughness closure increase $K_{op}$. [20%]

1(b)




Neuber assumed that $\Delta\sigma^{\infty}$ is sufficiently large for cyclic plasticity to exist at the notch root.

Under monotonic loading by $\sigma^{\infty}$, the tensile stress $\sigma_n$ at the notch root is given by $\sigma_n = K_\sigma \sigma^{\infty}$. $K_\sigma$ is less than the elastic value of stress concentration factor $K_T$.

The strain at the notch root $\varepsilon_n$ is related to $\varepsilon^{\infty}$ by $\varepsilon_n = K_\varepsilon \varepsilon^{\infty}$ where $K_\varepsilon > K_T$. Neuber assumed that the local energy density at the notch root $\sigma_n \varepsilon_n$ is the same as for the elastic case such that

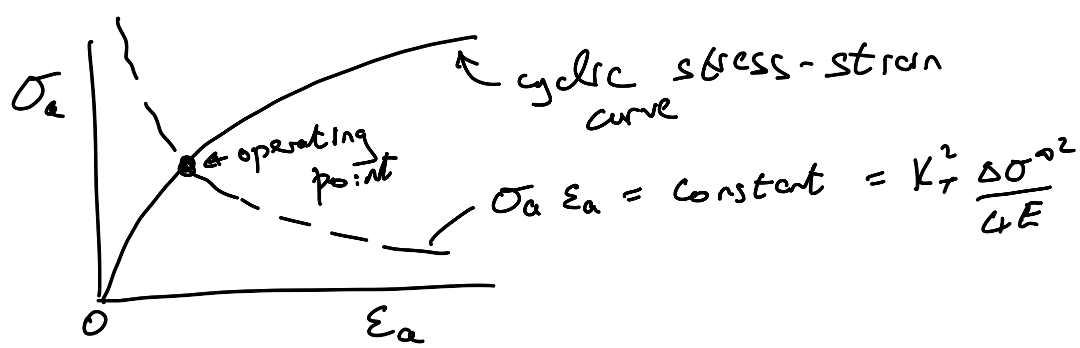
$$K_\sigma K_\varepsilon = K_T^2$$

Hence $$\Delta\sigma \Delta\varepsilon = (K_\sigma \Delta\sigma^{\infty})(K_\varepsilon \Delta\varepsilon^{\infty}) = K_T^2 \frac{\Delta\sigma^{\infty 2}}{E}$$

stress amplitude $\sigma_a = \Delta\sigma/2$

strain amplitude $\varepsilon_a = \Delta\varepsilon/2$

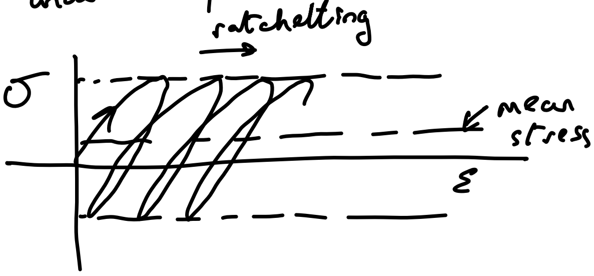
$$\Rightarrow \quad \sigma_a \varepsilon_a = K_T^2 \frac{\Delta\sigma^{\infty 2}}{4E}$$

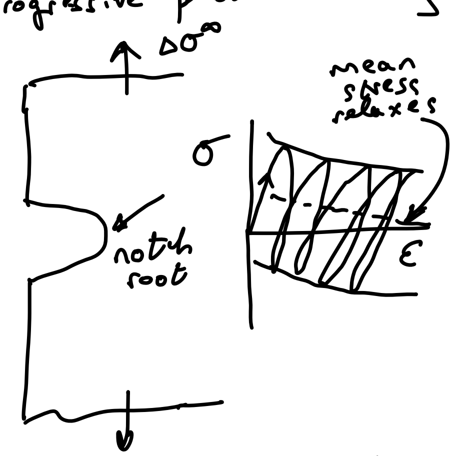



Now substitute into the Coffin–Manson law upon extracting $\Delta\varepsilon^{pl} = 2\left(\varepsilon_a - \frac{\sigma_a}{E}\right)$ such that $\Delta\varepsilon^{pl} N_f^{\beta} = C_2$ to determine the crack initiation life $N_f$.

Q1 (b) cont'd.

Since cyclic plasticity occurs at the notch root, mean stress relaxation can occur by ratchetting : progressive plastic straining occurs under the finite mean stress.




If macroscopic loading is purely compressive, then crack initiation may still occur but the crack will not grow due to the fact that the crack tip will stay closed over the load cycle. [35%]

Q1. (c) (i)


$P' = P\cos\alpha \approx P$ since $\alpha \ll 1$

$u = \dfrac{P'}{3EI}a^3 \qquad I = \frac{1}{12}bh^3 \quad (b=1)$

Compliance $C = u/P' \qquad \mathcal{G} = \frac{1}{2}P'^2 \dfrac{\partial C}{\partial a} \qquad C = \dfrac{a^2}{EI}$

So $\mathcal{G} = \dfrac{\frac{1}{2}P'^2 a^2}{EI} = \dfrac{6P'^2a^2}{Et^3}$ [30%]

(ii) $F = P\sin\alpha$ ~~…~~ where $P'^2 = \dfrac{Et^3}{6a^2}\mathcal{G}_{Ic}$

$\Rightarrow F = \left(\dfrac{Et^3}{6a^2}\mathcal{G}_{Ic}\right)^{1/2}\tan\alpha$

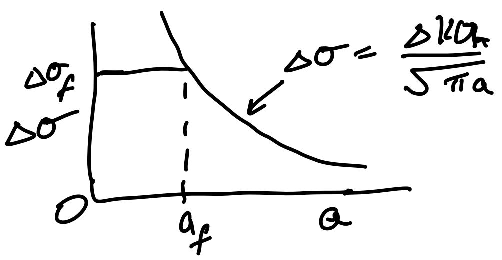
As 'a' increases, at fixed F, $\mathcal{G}$ increases. Hence unstable crack growth.

[15%]

2\. (a) A circular hole is less damaging than a crack of length $2a = D$, where $D$ is the hole diameter.
Recall that a transition flaw size exists for a material $a_T$ such that

$$\Delta K_{th} = \Delta\sigma_f \sqrt{\pi a_f} \Rightarrow a_f = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_f}\right)^2$$

where the stress range at the fatigue limit $\Delta\sigma_f$ is twice the amplitude at the fatigue limit, $\sigma_f$.
So: $a_T = \frac{1}{4\pi}\left(\frac{\Delta K_{th}}{\sigma_f}\right)^2$. Typically $\sigma_f \approx \sigma_Y$, the yield strength.
Thus $a_T$ is often on the order of microns for a steel.

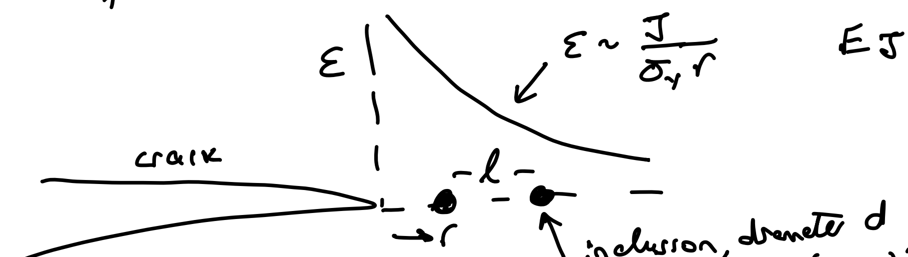



For $a < a_f$ the presence of the crack has no effect upon the fatigue strength.

[25%]

$EJ = K^2$

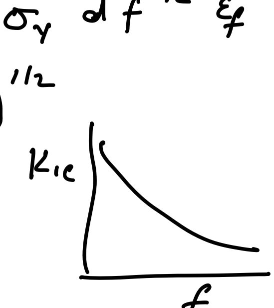
2\. (b)




Assume a volume fraction of inclusions $f \approx (d/l)^3$
or $f \sim (d/l)^2$ if failure occurs on the weakest plane.
void growth: $\frac{\dot{a}}{a} \approx \frac{\sigma_h}{\sigma_Y} \dot{\varepsilon}^\infty \Rightarrow \ln(a/a_0) \approx \frac{\sigma_h}{\sigma_Y} \varepsilon$
Put $\sigma_h/\sigma_Y \approx 3$ and $a \sim l/2$, $a_0 \sim d/2$ at crack tip.
Hence $\varepsilon_f \sim \frac{1}{3} \ln\left(\frac{l}{d}\right) \approx -\frac{1}{6} \ln f$
Put $r \approx l$ to give $J \sim \sigma_Y \varepsilon_f l = \sigma_Y d f^{-1/2} \varepsilon_f$

$$K_{IC} \sim E^{1/2} J^{1/2} \sim \left(\sigma_Y E d f^{-1/2} \frac{\ln f}{-6}\right)^{1/2}$$

So $K_{IC}$ drops as $f$ increases.




[25%]

2. (c) Welded structures contain sharp defects due to the welding process. These defects behave as pre-existing cracks, of length $a_0$. Then, the stress range $\Delta\sigma$ at the fatigue limit is given by $\Delta\sigma\sqrt{\pi a_0} \approx \Delta K_{th}$, the fatigue threshold.

The value of $\Delta K_{th}$ is increased in a moist air environment due to the increase in crack opening stress intensity factor $K_{op}$, as a result of the build up of oxide on the fracture surface. This is due to fretting.

[25%]

(d) Ceramics fail by cleavage with little crack tip blunting. The fracture toughness $K_{IC}$ is of the order of $K_{IC} \sim \sigma_c\sqrt{\pi b}$ where $\sigma_c$ = cleavage strength and $b$ = atomic spacing.

Metallic alloys fail by void coalescence at the crack tip. Thus, $K_{IC} \approx \sigma_Y\sqrt{\pi l}$ where $\sigma_Y$ is yield strength and $l$ is the void spacing. Dislocations are easily emitted from the crack tip and thereby blunt the crack so that cleavage does not occur.

[25%]

3. (a) $\mathcal{G}$ is defined for a linear elastic solid, $\sigma = E\varepsilon$

$J$ is defined for a non-linear elastic solid, $\sigma = \sigma(\varepsilon)$

strain energy $w(\varepsilon) = \int_0^{\varepsilon} \sigma(\varepsilon')\, d\varepsilon'$ in both cases.

Potential energy, $PE = \int_V w(\varepsilon)\, dV - P_0 u$

$$\mathcal{G} = -\frac{\partial (PE)}{\partial A}$$ where $A$ = crack area and $w(\varepsilon)$ is for a linear elastic solid

$$J = -\frac{\partial (PE)}{\partial A}$$ where $w(\varepsilon)$ is for a non-linear elastic solid.

Use $\mathcal{G}$ when a crack tip $K$-field exists. This is the case for small scale yielding (ssy), such that (a, ligament size) $< 2.5 \dfrac{K^2}{\sigma_Y^2}$

Use $J$ provided $a > 25\,\delta$ ← crack tip opening

i.e. $a > 25 \dfrac{K^2}{\sigma_Y E}$, such that a crack tip $J$-field exists.

For $a < 25 \dfrac{K^2}{\sigma_Y E}$ there is no $J$-field and no crack tip $K$-field. So use a different criterion such as level of applied strain.

[40%]

3(b)(i) $$\int_0^{\sigma_Y/2} A\,(B+\sigma_a)^{-2}\, d\sigma_a = 1$$

$$\Rightarrow \left[A\,(B+\sigma_a)^{-1}\right]_{\sigma_a=\sigma_Y/2}^{\sigma_a=0} = 1$$

$$\Rightarrow \frac{A}{B} - \frac{A}{B+(\sigma_Y/2)} = 1$$

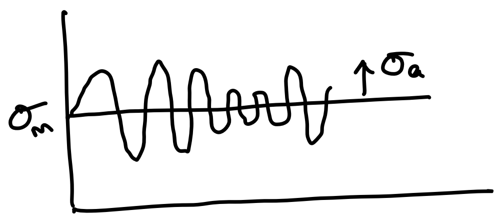
$$\Rightarrow A\,[\,B+(\sigma_Y/2) - B\,] = B\,[\,B+(\sigma_Y/2)\,]$$

$$\Rightarrow A = \frac{2B}{\sigma_Y}\left[B + \frac{\sigma_Y}{2}\right]$$

[20%]

3(b)(ii)

$$\sigma_m = \frac{\sigma_Y}{2}$$




Assume that the structure fails after $N_s$ cycles. Mines law states $\sum_i \frac{N_i}{N_{fi}} = 1$

Discrete → Continuous

At $\sigma_a$ : $N_{fi} \rightarrow N_f(\sigma_a)$

$N_i \rightarrow N_s\, p(\sigma_a)\, d\sigma_a$

So : $$N_s \int_0^{\infty} \frac{p(\sigma_a)}{N_f(\sigma_a)}\, d\sigma_a = 1 \qquad (*)$$

Now, $\sigma_a = \sigma_a^0 \left(1 - \frac{\sigma_m}{\sigma_{UTS}}\right)$ by Goodman's rule

Hence $\sigma_a^0 = \sigma_a \left(1 - \frac{\sigma_Y}{2\sigma_{UTS}}\right)^{-1}$

Basquin's law : $2\sigma_a^0 N_f^{\alpha} = C_1 \Rightarrow N_f = \left(\frac{C_1}{2\sigma_a^0}\right)^{1/\alpha}$

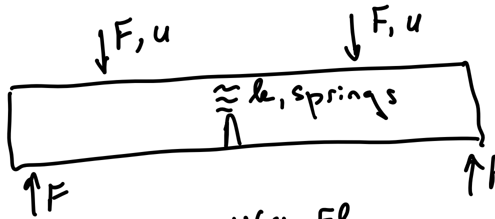
So $N_f(\sigma_a) \rightarrow \left(\frac{C_1}{2\sigma_a^0}\right)^{1/\alpha}$

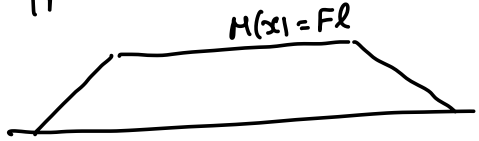
Q 3 (b) (ii) contd.

So: $$\frac{1}{N_S} = \int_0^{\sigma_Y/2} \frac{A}{(B+\sigma_a)^2} \left(\frac{2\sigma_a^{0}}{C_1}\right)^{1/\alpha} d\sigma_a$$

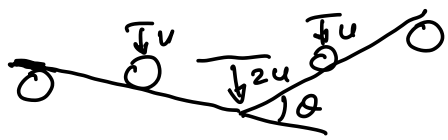
where $$\sigma_a^{0} = \sigma_a \left(1 - \frac{\sigma_Y}{2\,\sigma_{UTS}}\right)^{-1}$$

Q4 (a)




moment distribution in beam.




deflections.

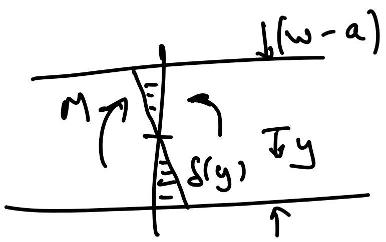
Kinematics: $\theta = \dfrac{2u}{\ell}$

Statics $M = F\ell$

$\delta W = 2F\,\delta u = M\,\delta\theta$

[20%]

4(b)




$\delta = \theta y$ , $T = k\delta$

$$M = 2\int_0^{\frac{(w-a)}{2}} T\,y\,dy$$

$$= 2k\theta \int_0^{\frac{(w-a)}{2}} y^2\,dy$$

$$\Rightarrow \quad M = \frac{2k\theta}{3}\left(\frac{w-a}{2}\right)^3$$

$F = \dfrac{M}{\ell}$ and $\theta = \dfrac{2u}{\ell}$ $\Rightarrow$ $F = \dfrac{4k}{3}\dfrac{u}{\ell^2}\left(\dfrac{w-a}{2}\right)^3$

Note: $\delta = \theta y$

$\Rightarrow \quad \delta_c = \theta_c \left(\dfrac{w-a}{2}\right)$ at $y = \dfrac{w-a}{2}$ [20%]

(c) At onset of crack growth, $\theta_c = \dfrac{2\delta_c}{(w-a)}$

and $u_c = \dfrac{l}{2}\theta_c = \dfrac{l\delta_c}{(w-a)}$

$$\Rightarrow \quad F_c = \frac{1}{6} k (w-a)^3 \frac{u_c}{l^2} = \frac{1}{6} k (w-a)^2 \frac{\delta_c}{l}$$

Stored energy $P = \frac{1}{2}(2Fu) = Fu = \frac{1}{6} k (w-a)^3 \dfrac{u^2}{l^2}$

$$G = -\frac{\partial P}{\partial a} = \frac{1}{2} k (w-a)^2 \frac{u^2}{l^2}$$

[30%]

(d) $G = G_c$ and $u_c = \dfrac{l\delta_c}{(w-a)}$

Hence $G_c = \frac{1}{2} k \delta_c^2$

The adhesive layer is constrained within the stiff substrates and this can lead to higher hydrostatic stress than in the bulk. The crack tip plastic zone may be restricted in size due to this plastic constraint. Additionally, interfacial failure may occur. Consequently the toughness is reduced.

[30%]