EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 2 May 2023 9.30 to 11.10

Module 3C9

FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C9 Fracture Mechanics of Materials and Structures (8 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) With reference to the phenomenon of fatigue crack closure in metallic alloys, explain why the threshold value of stress intensity range is sensitive to mean stress. [20%]

(b) Fatigue failure of a notched component involves the initiation and growth of a crack from the notch root under remote cyclic tension. Explain the use of Neuber's rule in predicting the initiation life, and account for the effect of mean stress upon crack initiation from the notch root. If the macroscopic cyclic loading is purely compressive explain why a crack may initiate but not grow from the notch root. [35%]

(c) A paint layer of thickness h is removed from the surface of a ceramic substrate by a frictionless wedge of apex angle α, as shown in Fig. 1. The paint behaves as an elastic solid of Young's modulus E, while the ceramic substrate behaves in a rigid manner. Upon applying an axial force F per unit width of paint layer, the inclined force between the face of the wedge and the lifted-off paint layer is P per unit width, as shown in Fig. 1.

(i) Assume that the paint layer is debonded over a length a. The vertical component of the inclined force P bends the paint layer. By making suitable use of beam theory, obtain an expression for the energy release rate $\mathcal{G}$ at the crack tip as a function of load P and other salient parameters. [30%]

(ii) Assume that delamination proceeds when the energy release rate equals the toughness $\mathcal{G}_c$ of the interface. Determine the required axial force F on the wedge in order to advance the crack tip. Explain whether crack advance is stable or unstable under the prescribed force F. [15%]

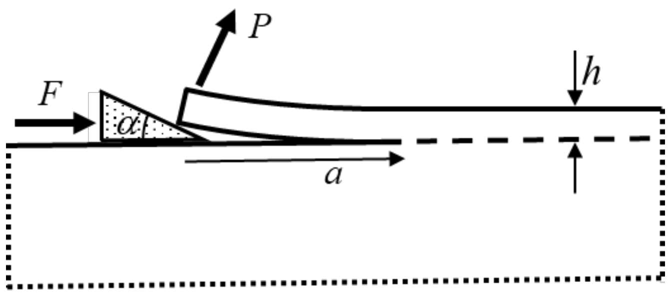

Fig. 1

2 Give a physical explanation for the following observations.

(a) The presence of a circular hole in a steel sheet has no effect upon its fatigue strength when the hole is sufficiently small. [25%]

(b) The presence of inclusions in a steel reduces its fracture toughness. [25%]

(c) The fatigue limit of a welded structure in damp air exceeds that for the same structure in a vacuum. [25%]

(d) Ceramics have a lower toughness than metallic alloys. [25%]

3 (a) Distinguish between the elastic energy release rate $\mathcal{G}$ and the J integral. Under which circumstances should each be used as a fracture parameter? When do both measures fail to provide a suitable failure criterion? [40%]

(b) The wing of an aircraft is manufactured from an aluminium alloy of yield strength σ_Y. In flight, the bottom face of the wing-root experiences a constant tensile stress of magnitude $\sigma_Y/2$ due to aerodynamic lift of the wing (to counterbalance the weight of the aircraft). Additionally, the wing is subjected to gust loading such that the wing root experiences a fully-reversed bending stress of amplitude σ_a that satisfies a probability density function of the form:

$$
\begin{aligned}
p(\sigma_a) &= \frac{A}{(B+\sigma_a)^2}, \quad && 0 < \sigma_a < \sigma_Y/2 \\
&= 0, && \sigma_a > \sigma_Y/2
\end{aligned}
$$

where A and B are constants.

(i) Determine the value of A in terms of B such that the cumulative probability density equals unity. [20%]

(ii) Obtain an integral expression for the number of cycles to initiate a fatigue crack, assuming crack initiation is in the high cycle fatigue regime, and taking into account the effect of mean stress. [40%]

4 Two long ceramic bars have the same rectangular cross-section, of width w and breadth b. The ends of the bars are bonded together by a rubber-based adhesive of thickness h which is negligible compared to w, as shown in Fig. 2. A four-point bend test is used to measure the toughness of the adhesive layer. To do so, the specimen contains a crack of length a that extends across the full breadth of the cross-section. Each of the two central rollers displaces by u under a load F. The deformation of the specimen is idealised by treating the ceramic bars as rigid, and the adhesive layer as a uniform distribution of extensional springs of spring constant k per unit area of adhesive layer. Thus, the adhesive layer behaves as a cohesive zone that carries a tensile traction $T = k\delta$ at any location where the opening displacement equals δ. The springs fail when δ attains the critical tensile value δ_c. The spring stiffness in compression is identical to that in tension, but no failure can occur in compression.

(a) Relate the roller displacement u to the relative rotation θ of the two ceramic bars, for a fixed crack length a. Likewise, relate the load F on each roller to the moment M on the uncracked ligament of the specimen, that is on the uncracked portion of the adhesive layer of length $(w - a)$. [20%]

(b) Determine the critical value of joint rotation θ_c such that the outermost springs in the cohesive zone fail at the opening displacement δ_c and, by consideration of the traction distribution in the springs, find the corresponding failure moment M_c on the uncracked ligament of the specimen. [20%]

(c) Obtain an expression for the stored energy P of the specimen under a prescribed displacement u, and thereby calculate the energy release rate $\mathcal{G}$. [30%]

(d) Determine the toughness of the joint in terms of k and δ_c, and explain why the toughness of the adhesive joint may differ substantially from that of the bulk adhesive. [30%]

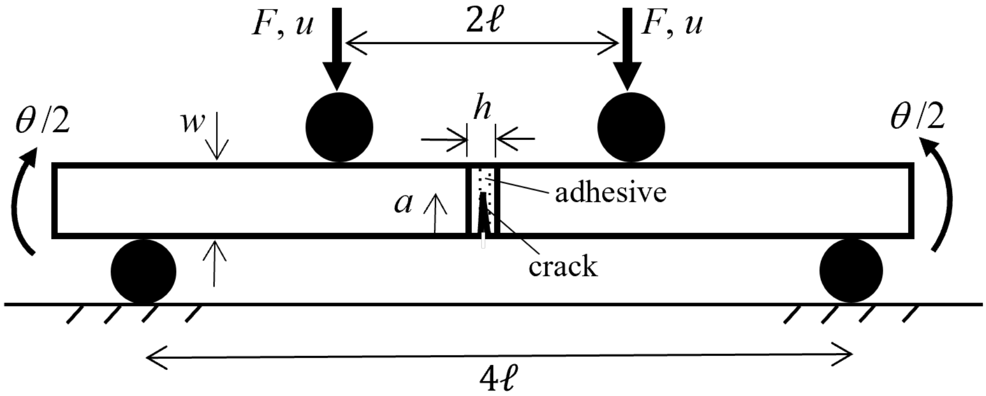

Fig. 2

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ENGINEERING TRIPOS PART IIA

Module 3C9 – FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

DATASHEET

Crack tip plastic zone sizes

$$\text{diameter, } d_p = \begin{cases} \dfrac{1}{\pi}\left(\dfrac{K_I}{\sigma_y}\right)^2 & \text{Plane stress} \\ \dfrac{1}{3\pi}\left(\dfrac{K_I}{\sigma_y}\right)^2 & \text{Plane strain} \end{cases}$$

Crack opening displacement

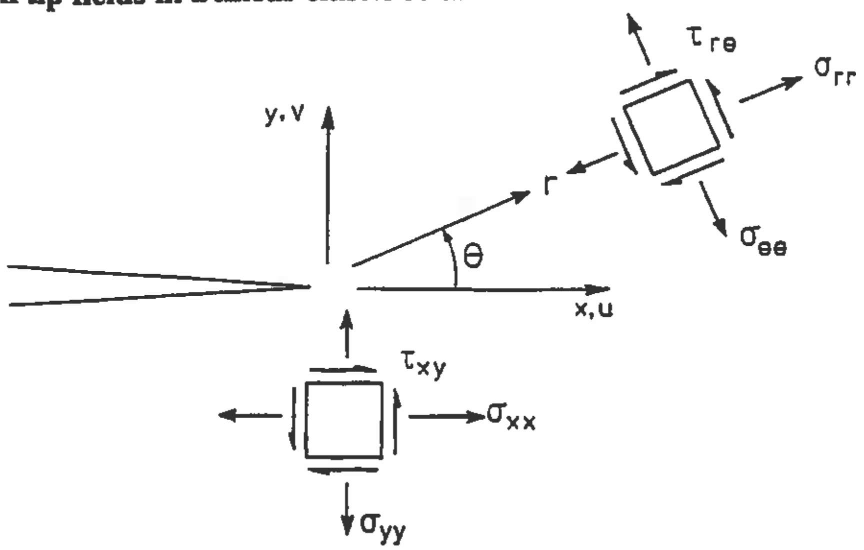
$$\delta = \begin{cases} \dfrac{K_I^2}{\sigma_y E} & \text{Plane stress} \\ \dfrac{1}{2}\dfrac{K_I^2}{\sigma_y E} & \text{Plane strain} \end{cases}$$

Energy release rate

$$G = \begin{cases} \dfrac{1}{E}K_I^2 & \text{Plane stress} \\ \dfrac{1-\nu^2}{E}K_I^2 & \text{Plane strain} \end{cases}$$

Related to compliance C: $G = \dfrac{1}{2}\dfrac{P^2}{B}\dfrac{dC}{da}$

Asymptotic crack tip fields in a linear elastic solid



Mode I

$$\sigma_{yy}=\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{xx}=\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy}=\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_{rr}=\frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2}-\frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$\sigma_{\theta\theta}=\frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2}+\frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$\tau_{r\theta}=\frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2}+\frac{1}{4}\sin\frac{3\theta}{2}\right)$$

$$u=\begin{cases}\dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{1-\nu}{1+\nu}+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane stress}\\[2ex] \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(1-2\nu+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane strain}\end{cases}$$

$$v=\begin{cases}\dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{2}{1+\nu}-\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane stress}\\[2ex] \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(2-2\nu-\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane strain}\end{cases}$$

$$w=0$$

Crack tip stress fields (cont'd)

Mode II

$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \sin\frac{\theta}{2} \cos\frac{3\theta}{2}$$

$$\sigma_{xx} = -\frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{rr} = \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{5}{4}\sin\frac{\theta}{2}+\frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{\theta\theta} = -\frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{3}{4}\sin\frac{\theta}{2}+\frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\tau_{r\theta} = \frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{1}{4}\cos\frac{\theta}{2}+\frac{3}{4}\cos\frac{3\theta}{2}\right)$$

$$u = \begin{cases} \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{2}{1+\nu}+\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(2-2\nu+\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{\nu-1}{1+\nu}+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(-1+2\nu+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Mode III

$$\tau_{zx} = -\frac{K_{III}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}$$

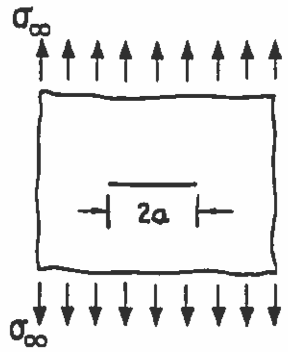
$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \tau_{xy} = 0$$

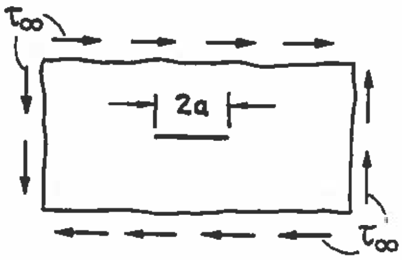
$$w = \frac{K_{III}}{G}\sqrt{\frac{2r}{\pi}} \sin\frac{\theta}{2}$$

$$u = v = 0$$

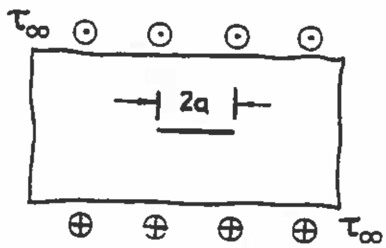
Tables of stress intensity factors



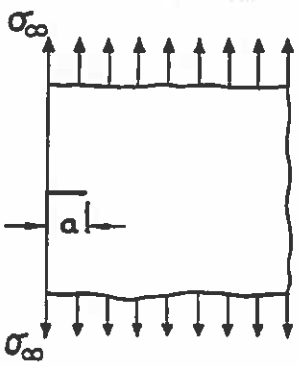
$$K_I = \sigma_\infty \sqrt{\pi a}$$



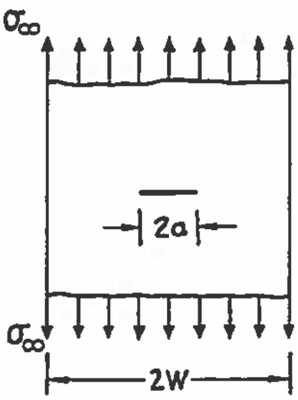
$$K_{II} = \tau_\infty \sqrt{\pi a}$$



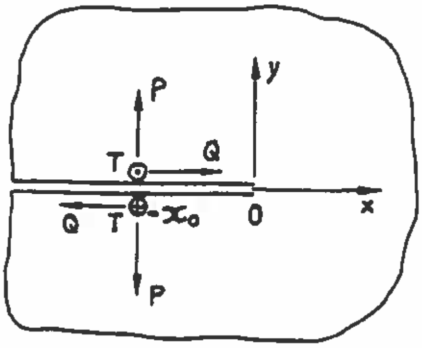
$$K_{III} = \tau_\infty \sqrt{\pi a}$$



$$K_I = 1.12\,\sigma_\infty \sqrt{\pi a}$$



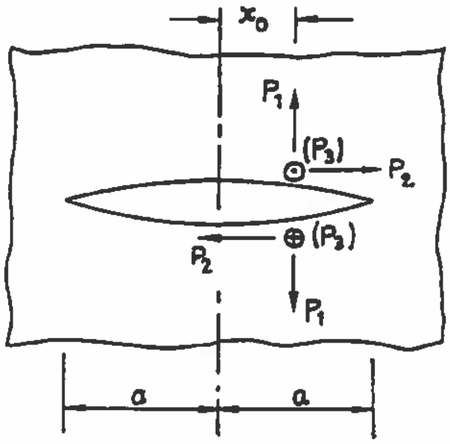
$$K_I = \sigma_\infty \sqrt{\pi a}\left(\frac{1 - a/2W + 0.326a^2/W^2}{\sqrt{1 - a/W}}\right)$$



$$K_I = \frac{2P}{\sqrt{2\pi x_o}}$$

$$K_{II} = \frac{2Q}{\sqrt{2\pi x_o}}$$

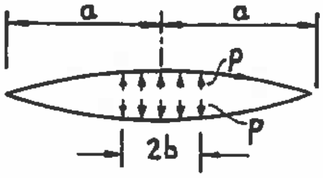
$$K_{III} = \frac{2T}{\sqrt{2\pi x_o}}$$



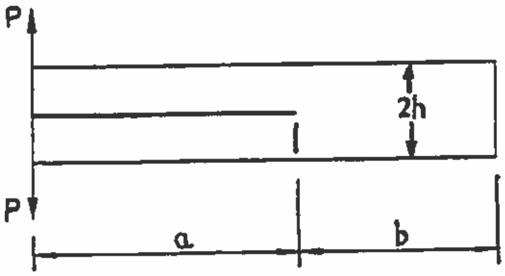
$$K_I = \frac{P_1}{\sqrt{\pi a}}\sqrt{\frac{a+x_o}{a-x_o}}$$

$$K_{II} = \frac{P_2}{\sqrt{\pi a}}\sqrt{\frac{a+x_o}{a-x_o}}$$

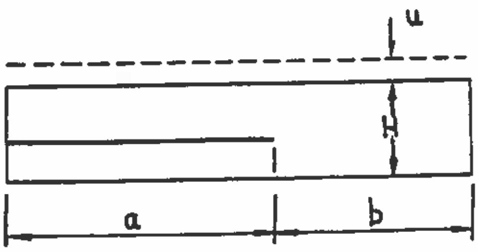
$$K_{III} = \frac{P_3}{\sqrt{\pi a}}\sqrt{\frac{a+x_o}{a-x_o}}$$



$$K_I = \frac{2pb}{\sqrt{\pi a}}\frac{a}{b}\arcsin\frac{b}{a}$$

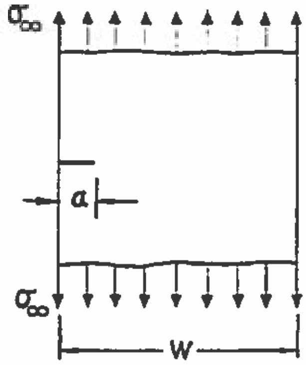


$$K_I = \frac{2\sqrt{3}}{h\sqrt{h}}\frac{Pa}{B} \qquad h \ll a \text{ and } h \ll b$$



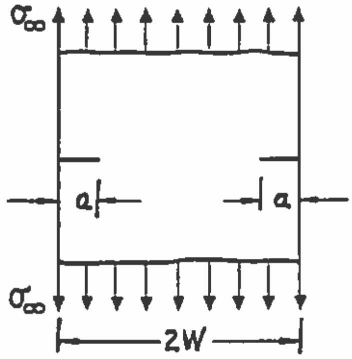
$$K_I = \sqrt{\frac{1}{2\alpha H}}\,Eu \qquad H \ll a \text{ and } H \ll b$$

$$\alpha = \begin{cases} 1-\nu^2 & \text{Plane stress} \\ 1-3\nu^2-2\nu^3 & \text{Plane strain} \end{cases}$$

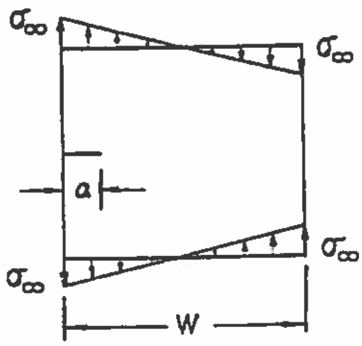


$a/W < 0.7$

$$K_I = \sigma_\infty \sqrt{\pi a}\left(1.12 - 0.23\frac{a}{W} + 10.6\frac{a^2}{W^2} - 21.7\frac{a^3}{W^3} + 30.4\frac{a^4}{W^4}\right)$$

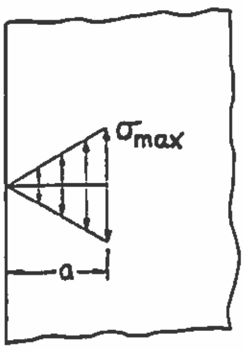


$$K_I = \sigma_\infty \sqrt{\pi a}\left(\frac{1.12 - 0.61a/W + 0.13a^3/W^3}{\sqrt{1 - a/W}}\right)$$

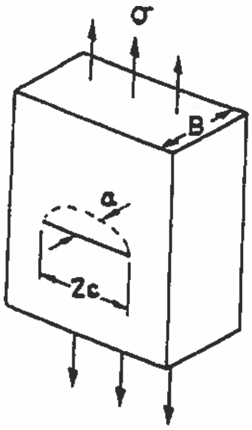


$a/W < 0.7$

$$K_I = \sigma_\infty \sqrt{\pi a}\left(1.12 - 1.39\frac{a}{W} + 7.3\frac{a^2}{W^2} - 13\frac{a^3}{W^3} + 14\frac{a^4}{W^4}\right)$$

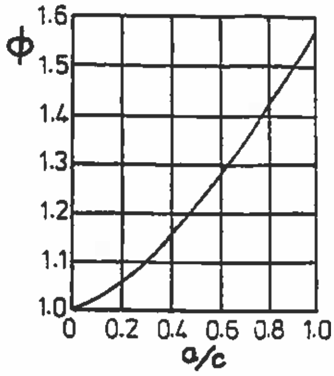


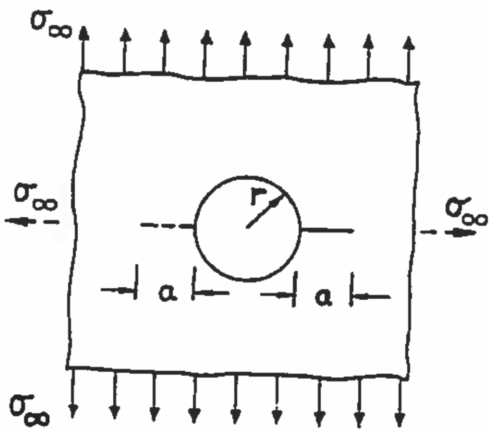
$$K_I = 0.683\ \sigma_{max}\sqrt{\pi a}$$



$$K_I = \frac{1.12}{\Phi}\sigma\sqrt{\pi a}$$

$$\Phi = \int_0^{\pi/2}\left(1 - \frac{c^2 - a^2}{c^2}\sin^2\theta\right)^{\frac{1}{2}} d\theta$$



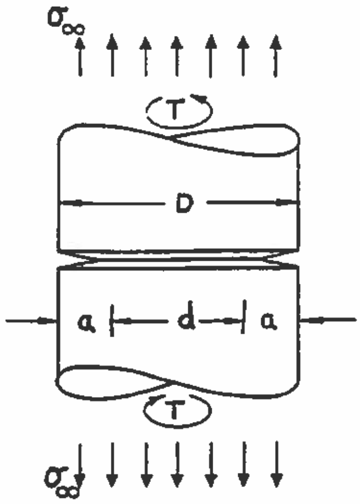


$$K_I = \sigma_\infty\sqrt{\pi a}\ F\left(\frac{a}{r}\right)$$

value of $F(a/r)$†

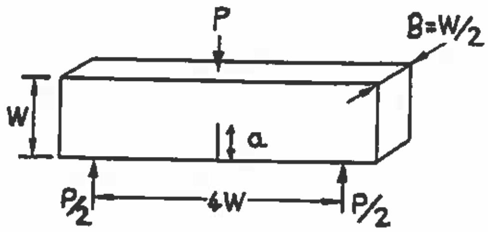
$\frac{a}{r}$	One crack U	One crack B	Two cracks U	Two cracks B
0.00	3.36	2.24	3.36	2.24
0.10	2.73	1.98	2.73	1.98
0.20	2.30	1.82	2.41	1.83
0.30	2.04	1.67	2.15	1.70
0.40	1.86	1.58	1.96	1.61
0.50	1.73	1.49	1.83	1.57
0.60	1.64	1.42	1.71	1.52
0.80	1.47	1.32	1.58	1.43
1.0	1.37	1.22	1.45	1.38
1.5	1.18	1.06	1.29	1.26
2.0	1.06	1.01	1.21	1.20
3.0	0.94	0.93	1.14	1.13
5.0	0.81	0.81	1.07	1.06
10.0	0.75	0.75	1.03	1.03
∞	0.707	0.707	1.00	1.00

†U = uniaxial σ_∞ B = biaxial σ_∞.

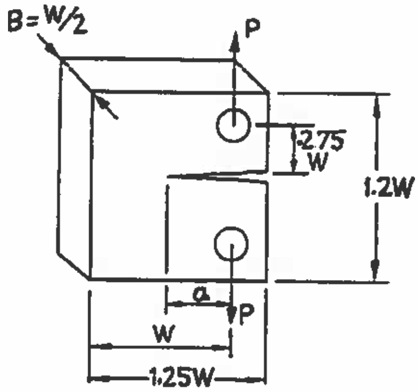


$$K_I = \sigma_\infty \sqrt{\pi a}\left(\frac{D}{d} + \frac{1}{2} + \frac{3}{8}\frac{d}{D} - 0.36\frac{d^2}{D^2} + 0.73\frac{d^3}{D^3}\right)\frac{1}{2}\sqrt{\frac{D}{d}}$$

$$K_{III} = \frac{16T}{\pi D^3}\sqrt{\pi a}\left(\frac{D^2}{d^2} + \frac{1}{2}\frac{D}{d} + \frac{3}{8} + \frac{5}{16}\frac{d}{D} + \frac{35}{128}\frac{d^2}{D^2} + 0.21\frac{d^3}{D^3}\right)\frac{3}{8}\sqrt{\frac{D}{d}}$$



$$K_I = \frac{4P}{B}\sqrt{\frac{\pi}{W}}\left\{1.6\left(\frac{a}{W}\right)^{1/2} - 2.6\left(\frac{a}{W}\right)^{3/2} + 12.3\left(\frac{a}{W}\right)^{5/2} - 21.2\left(\frac{a}{W}\right)^{7/2} + 21.8\left(\frac{a}{W}\right)^{9/2}\right\}$$



$$K_I = \frac{P}{B}\sqrt{\frac{\pi}{W}}\left\{16.7\left(\frac{a}{W}\right)^{1/2} - 104.7\left(\frac{a}{W}\right)^{3/2} + 369.9\left(\frac{a}{W}\right)^{5/2} - 573.8\left(\frac{a}{W}\right)^{7/2} + 360.5\left(\frac{a}{W}\right)^{9/2}\right\}$$

NAF
March 2010