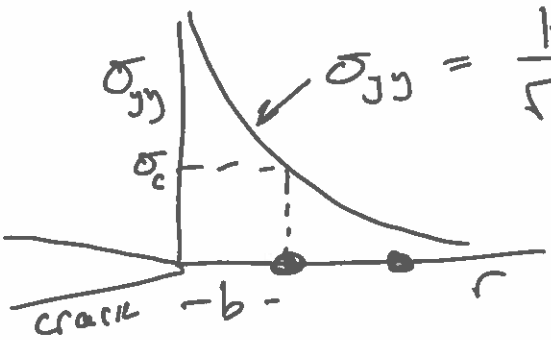
Part IIA 2023-4 crib for paper 3C9

Fracture mechanics of materials and structures

1. (a) Brittle ceramic : fails by cleavage.

Toughness $G_c \approx 2\gamma_s$ where γ_s = surface energy by Griffith



Assume that $K = K_{IC}$ when $\sigma_{yy}(r=b) = \sigma_c$

atomic spacing

ideal strength

$$\Rightarrow \frac{K_{IC}}{\sqrt{2\pi b}} = \sigma_c$$

Ideal strength σ_c is on the order of $E/10$

$$\Rightarrow K_{IC} \approx \frac{E}{10}\sqrt{2\pi b}$$

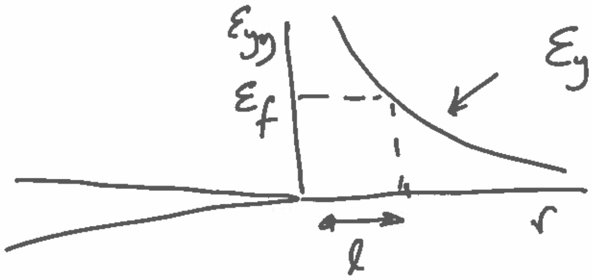
$G_{IC} = \frac{K_{IC}^2}{E}$ by Irwin relation

$$\Rightarrow G_{IC} \approx \frac{2\pi}{100} Eb \approx 2\gamma_s$$

consistent with $\left(\gamma_s \approx \frac{\pi}{100} Eb\right)$ ✓

steel with inclusions

The toughness is governed by the nucleation and growth of voids directly ahead of the crack tip.



$$\varepsilon_{yy} \approx \frac{J}{\sigma_Y r} \quad \text{(HRR singularity)}$$

inclusion spacing $= l$

Assume $\varepsilon_{yy} > \varepsilon_f$ over $0 < r < l$

ε_f ← strain to failure

$$\varepsilon_f \approx f^{-1/2} \exp\left(-\frac{3\sigma_h}{2\sigma_Y}\right) \quad \text{by Rice – Tracy}$$

f ← volume fraction of inclusions

$\sigma_h \approx 2.5\,\sigma_Y$

Thus: $$\varepsilon_{yy} = \frac{J_{IC}}{\sigma_Y l} = \varepsilon_f = f^{-1/2} \exp\left(-\frac{3\sigma_h}{2\sigma_Y}\right)$$

So $J_{IC} \propto \sigma_Y\, l\, f^{-1/2}$

Also, inclusion spacing l scales with $f^{-1/2}$

So a high volume fraction of inclusions implies a small l and a low toughness J_{IC}. Under L.E.F.M., $G_{IC} \equiv J_{IC}$.

1 (b) (i) $\phi(\theta_0) = \frac{1}{2} M\theta_0 = \frac{1}{2}\frac{\theta_0^2}{C}$ where $\theta = CM$

potential energy $P(M_0) = \phi - M_0\theta$
where $-M_0\theta$ is the potential energy of the loading system

$$\Rightarrow P = \tfrac{1}{2} M\theta - M_0\theta = -\tfrac{1}{2} M_0\theta = -\tfrac{1}{2} C M_0^2$$

(ii) $bG = -\left.\dfrac{\partial\phi}{\partial a}\right|_{\theta_0}$ and $EG = K^2$ (Irwin)

$$\Rightarrow bG = -\frac{1}{2}\theta_0^2 \frac{\partial}{\partial a}\left(\frac{1}{C}\right) = \frac{1}{2}\theta_0^2 \frac{1}{C^2}\frac{\partial C}{\partial a}$$

where $C = \dfrac{l}{EI} + \dfrac{24\,ab}{EI(b-a)}$

$$\frac{\partial C}{\partial a} = ? \quad \frac{\partial}{\partial a}\left(\frac{a}{b-a}\right) = \frac{b-a+a}{(b-a)^2} = \frac{b}{(b-a)^2}$$

So $bG = \dfrac{1}{2}\dfrac{\theta_0^2}{C^2}\dfrac{24\,b^2}{EI(b-a)^2} \Rightarrow G = \dfrac{12\,M^2}{EI}\dfrac{b}{(b-a)^2}$

$$K^2 = EG = \frac{12\,M^2}{I}\frac{b}{(b-a)^2} \qquad I = \tfrac{1}{12}b^4$$

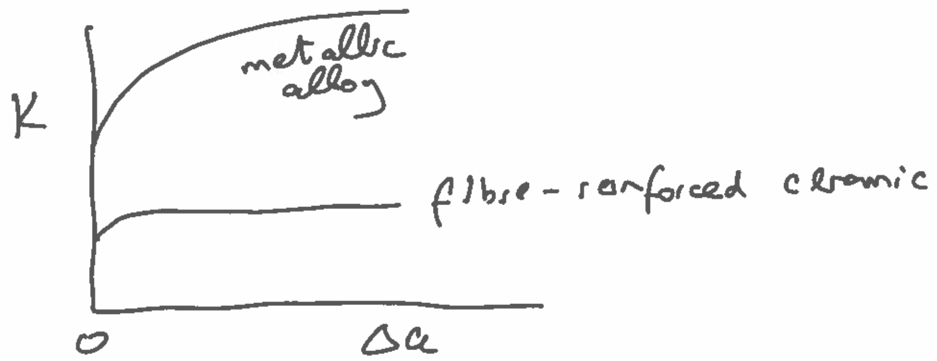
$$\Rightarrow K = \frac{12\,M}{b^2}\frac{\sqrt{b}}{(b-a)}$$

(iii) $bG = -\dfrac{\partial P}{\partial a} = M_0^2\dfrac{\partial C}{\partial a} = \dfrac{1}{2}\left(\dfrac{\theta_0}{C}\right)^2\dfrac{\partial C}{\partial a}$, as above

(iv) $K = \dfrac{12M}{b^2}\dfrac{\sqrt{b}}{(b-a)}$ So $\left.\dfrac{\partial K}{\partial a}\right|_M = \dfrac{12M}{b^2}\sqrt{b}\,\dfrac{1}{(b-a)^2} > 0$

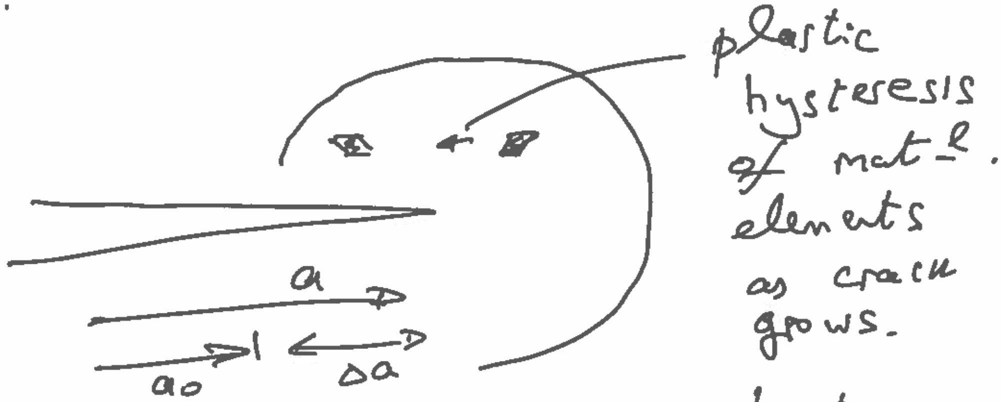
Hence unstable.

2. (a)

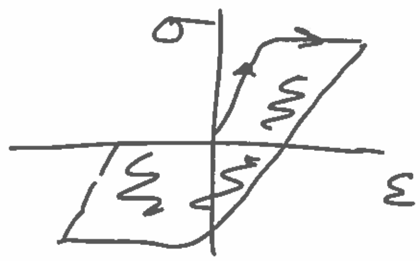


Alloy

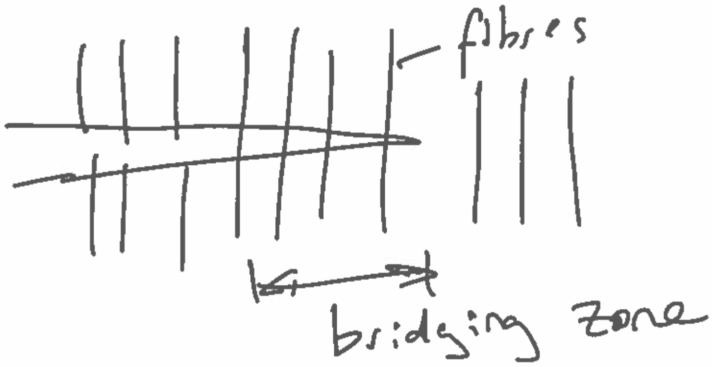
Crack growth initiates at $K = K_{Ic}$ and $K - \Delta a$ curve rises to a steady state value of approx. $2K_{Ic}$ due to irreversible plastic dissipation.



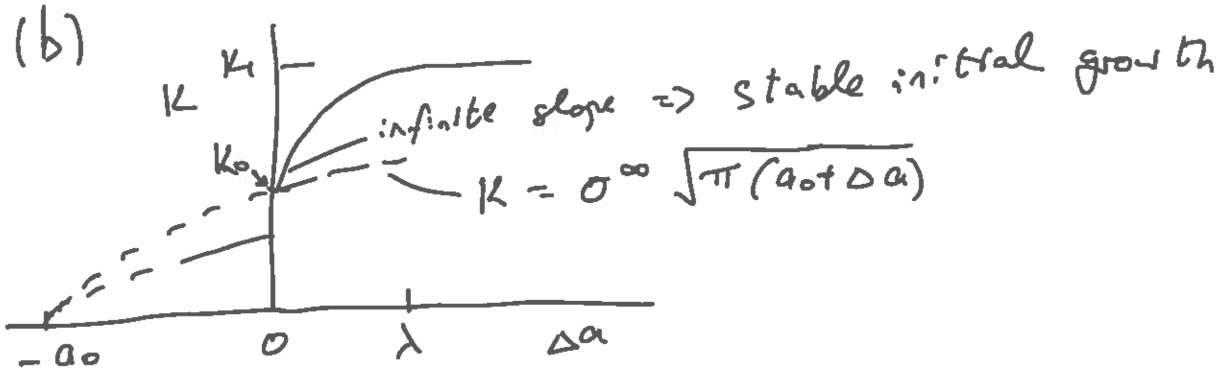
Near crack tip, the $\Delta\sigma - \Delta\varepsilon$ history is non-proportional.



Fibre-reinforced ceramic



(b)



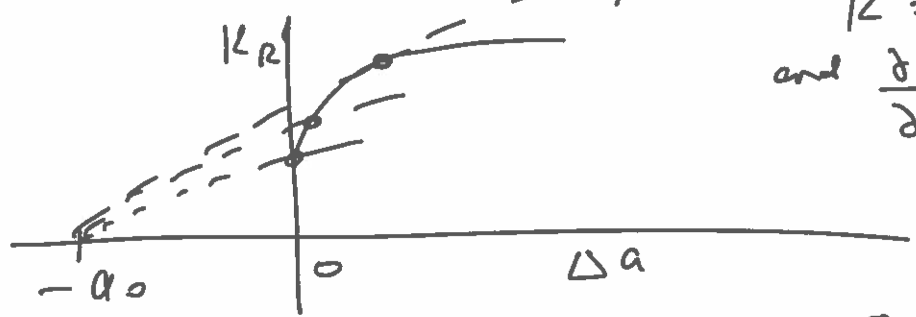
(i) $\sigma^\infty \sqrt{\pi a_0} = K_0$

(ii)

point of instability at max^m. σ^∞

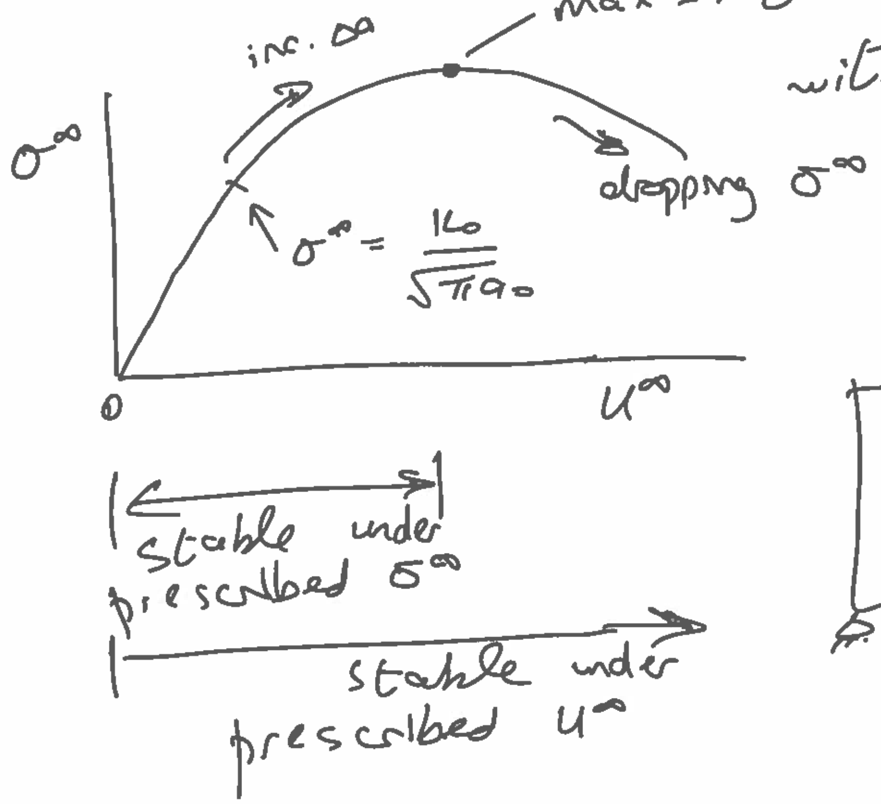
$K = K_R$

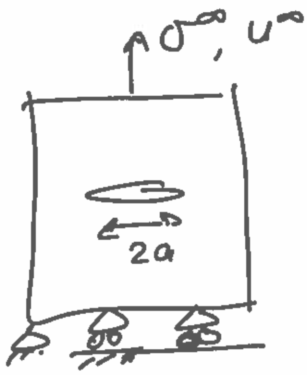
and $\dfrac{\partial K}{\partial a} = \dfrac{\partial K_R}{\partial a}$



max^m. σ^∞ at $\dfrac{\partial K}{\partial a} = \dfrac{\partial K_R}{\partial a}$ with $K = K_R$

(iii)



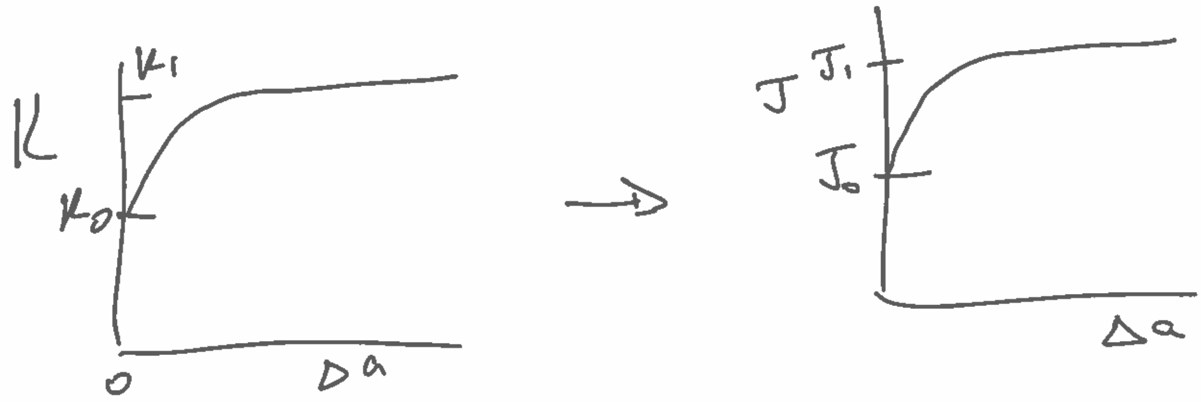


(iv) $EJ = K^2$ so

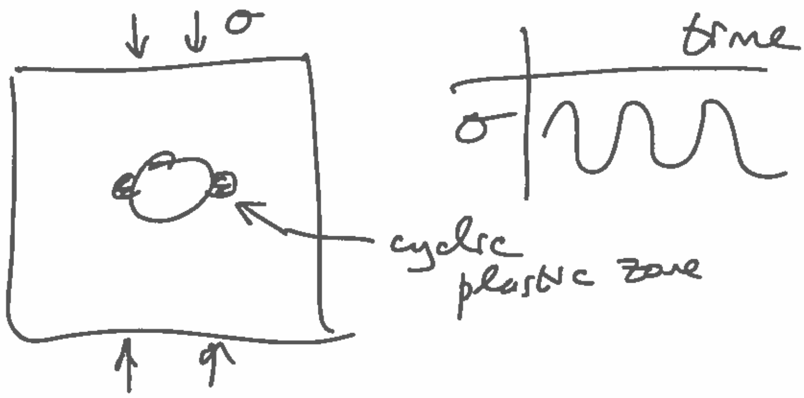
convert $K \to J$ for the

case of small specimens

where J is valid but not K.



3. (a)



A fully reversed plastic zone exists adjacent to the hole, leading to the initiation and growth of a fatigue crack. The number of cycles to initiate the crack N_i depends upon $\Delta\varepsilon$ at the edge of the hole, and $\Delta\varepsilon$ is determined from $\Delta\sigma^\infty$ by Neuber's rule for example $(K_\sigma K_\varepsilon = K_T^2)$.

After the crack has reached the periphery of the cyclic plastic zone, it continues to grow in a cyclic compressive stress field until the point where $\Delta K < \Delta K_{th}$.

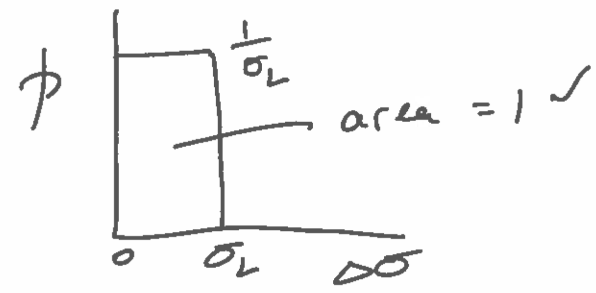
(b) (i) For a thumbnail-shaped edge crack, the 3C9 datasheet gives:

$$K = \frac{1.12}{\Phi}\sigma\sqrt{\pi a} \qquad \Phi = \Phi(a/c)$$

For $a/c = 1 \Rightarrow \Phi = 1.56$

$$\Rightarrow K = 0.718\,\sigma\sqrt{\pi a}$$

(ii)



Consider n cycles of random loading.
The number of cycles of stress with a range of $\Delta\sigma$ to $(\Delta\sigma + \delta\Delta\sigma)$ is $n\,p(\Delta\sigma)\,.\,\delta\Delta\sigma$.
The crack growth due to these cycles is

$$\frac{da}{dN}\, n\, p(\Delta\sigma)\, \delta\Delta\sigma$$

Total growth for n cycles is

$$\int_0^\infty \frac{da}{dN}\, n\, p(\Delta\sigma)\, d\Delta\sigma$$

So, total growth for $n = 1$ is

$$\left\langle \frac{da}{dN} \right\rangle = \int_0^\infty \frac{da}{dN}(\Delta K)\, p(\Delta\sigma)\, d\Delta\sigma$$

where $\Delta K = 0.718\, \Delta\sigma \sqrt{\pi a}$

(iii) N_T cycles required to grow a crack from length a_0 to $2a_0$. $N_T = ?$

$$\left\langle \frac{da}{dN} \right\rangle = \int_0^{\sigma_L} A \left(0.718\, \Delta\sigma \sqrt{\pi a}\right)^n \frac{1}{\sigma_L}\, d\Delta\sigma$$

$$\Rightarrow \left\langle \frac{da}{dN} \right\rangle = A \quad 0.718^{n} (\pi a)^{n/2} \quad \frac{\sigma_L^{n}}{n+1}$$

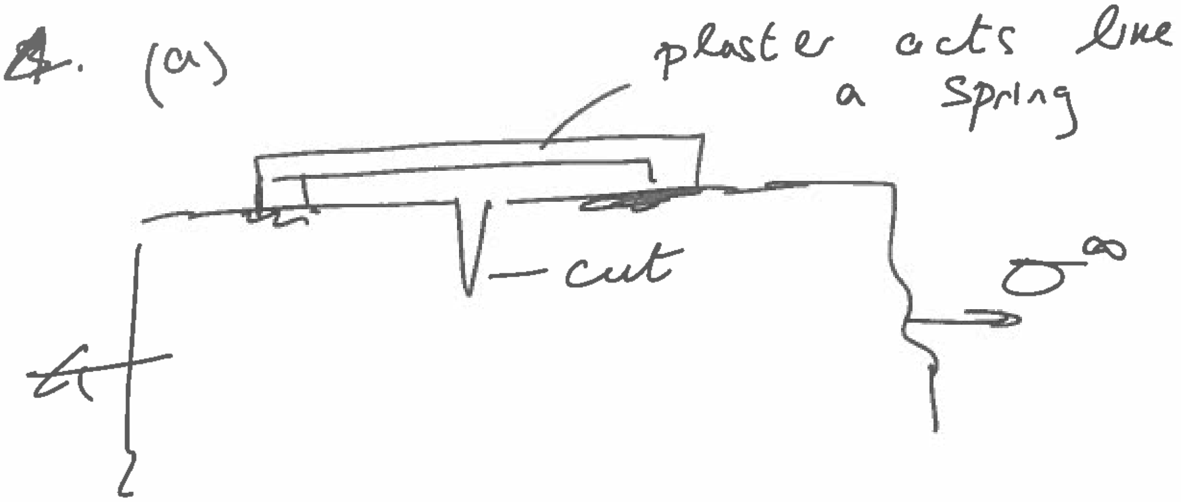
$$\Rightarrow N_T = \int_{a_0}^{2a_0} \left\langle \frac{da}{dN} \right\rangle^{-1} da$$

$$= \int_{a_0}^{2a_0} \frac{(n+1)}{A\,\sigma_L^{n}} \frac{1}{(0.718)^{n}\,\pi^{n/2}} a^{-\frac{n}{2}}\, da$$

$$\Rightarrow N_T = \frac{(n+1)}{A\,(0.718)^{n}\,\pi^{n/2}} \frac{1}{\sigma_L^{n}} \left[a^{\frac{2-n}{2}} \right]_{a_0}^{2a_0} \frac{2}{2-n}$$

(iv) Single peak overloads can lead to a severe retardation in growth rate and so the actual fatigue life is much greater than that predicted by a linear summation of damage (i.e. by Miner's law).

4. (a)

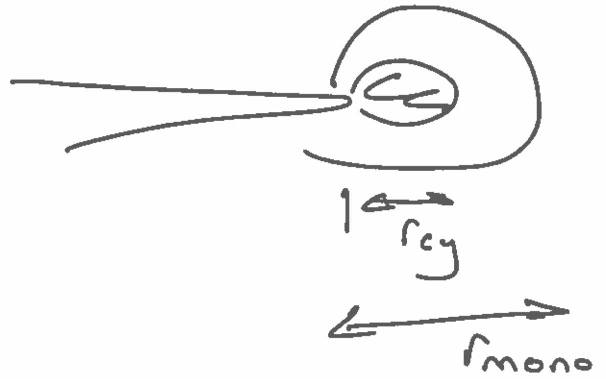


- plaster bridges the crack and develops tension to reduce the K at the crack tip.

$$K_{\substack{\text{plaster}\\\text{present}}} = \alpha \cdot K_{\text{no-plaster}}$$

$$\alpha < 1$$

(b)



$$r_{cy} \sim \frac{1}{4\pi} \frac{\Delta K^2}{\sigma_Y^2}$$

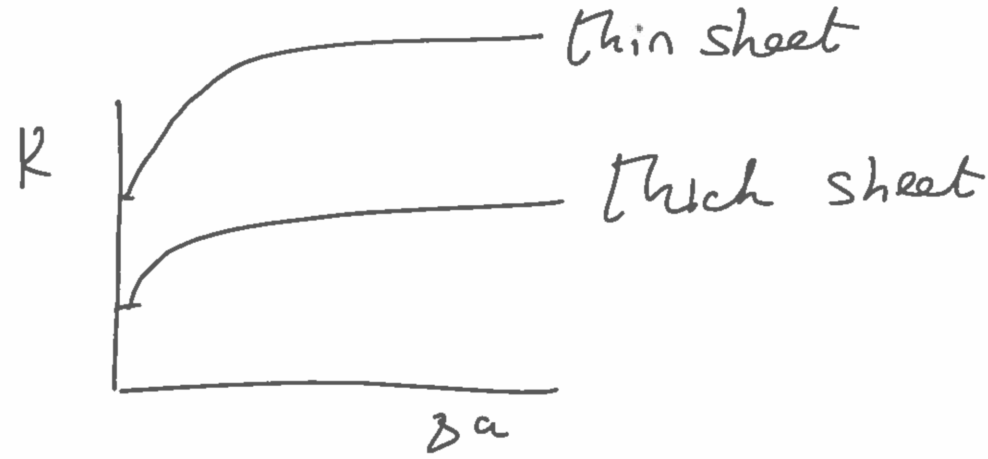
$$r_{mono} \sim \frac{1}{\pi} \frac{K_{max}^2}{\sigma_Y^2}$$

Paris Law : $\frac{da}{dN} = A \, \Delta K^n$

$$= A \left(4\pi r_{cy} \sigma_Y^2\right)^{n/2}$$

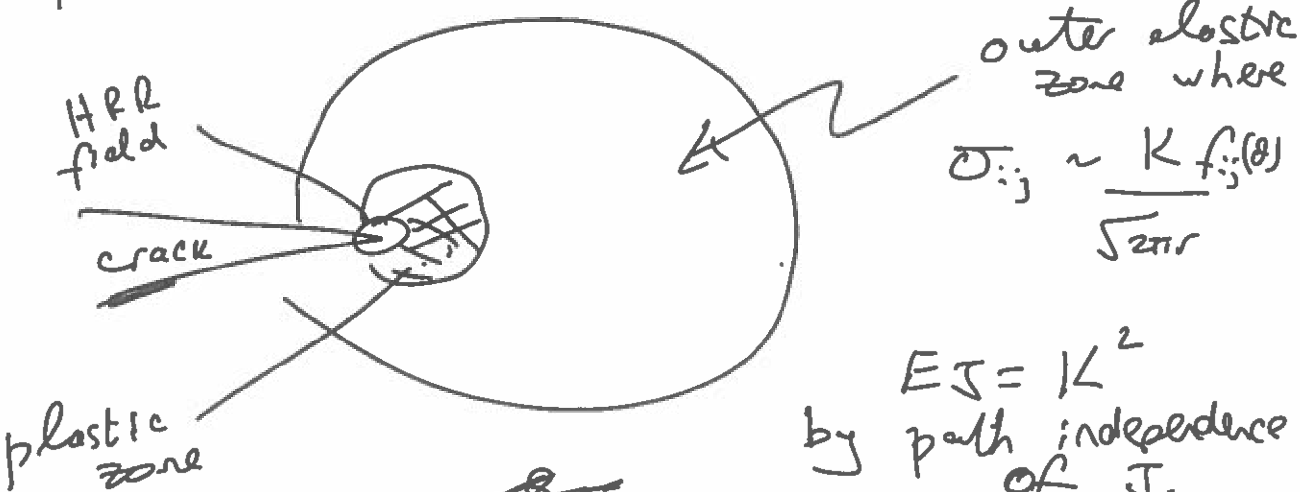
$$\propto r_{cy}^{n/2}$$

(c)



Assume that crack advance is by void growth.
A thick sheet has a high tensile hydrostatic stress at the crack tip, thereby reducing ductility (Rice - Tracy) and consequently reducing the toughness.
So a stack of thin sheets behave as a material of high toughness compared to a single thick sheet.

(d)



$EJ = K^2$
by path independence of J.

END