Part IIA

Crib for paper 3C9 :

'Fracture Mechanics of Materials and Structures'

1

(a)

$$C = \frac{u}{P}$$

$\Psi \equiv$ Potential energy of body + loading system

$$\Psi = W - Pu$$

$$= \frac{1}{2} Pu - Pu$$

$$= -\frac{1}{2} C P^2$$

$$G = -\frac{1}{B} \frac{\partial \Psi}{\partial a}$$

$$= -\frac{1}{B} \frac{\partial}{\partial a} \left(-\frac{1}{2} P^2 C \right)$$

$$= \frac{1}{2B} P^2 \frac{\partial C}{\partial a}$$

1 (b)

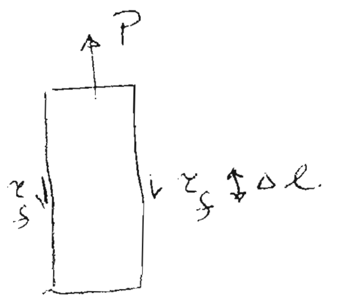
(i) Let crack advance by δl, then energy release

$$-\delta E = \frac{1}{2}\frac{\sigma^2}{E}\pi a^2\,\delta l = 2\pi a\,\delta l\,G$$

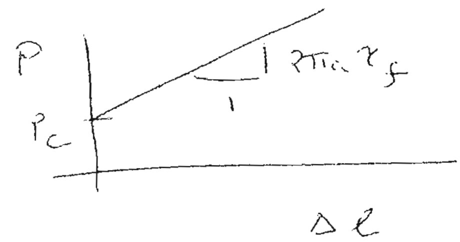
$$\Rightarrow G = \frac{\sigma^2 a}{4E}$$

$$P_c = 2\pi a^{3/2}\sqrt{EG_c}$$

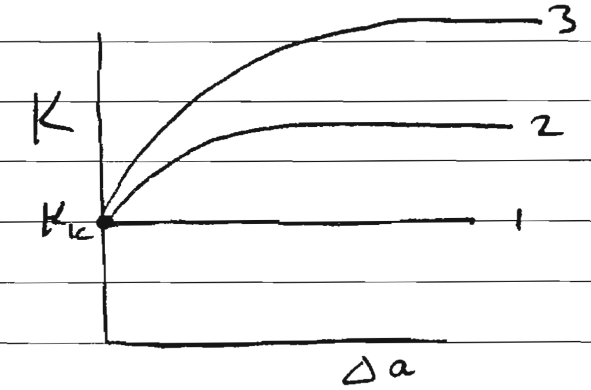
(ii)



$$P = P_c + 2\pi a\,\Delta l\,\tau_f$$



2. (a)



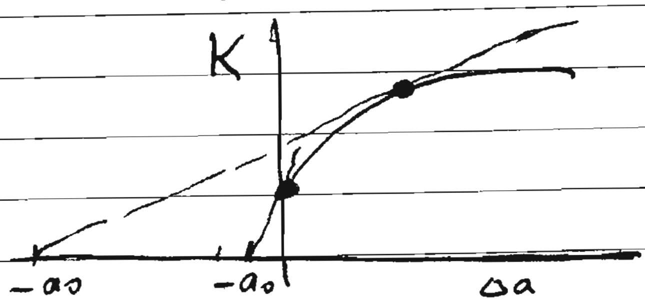
R - curves

Curve ① : Elastomer. A reversible, ~~elastic~~ solid with the fracture toughness dictated by fracture surface energy and by blunting of the crack tip.

② : Ceramic matrix composite. As the crack grows it is bridged by fibres or particles to give a rising R-curve

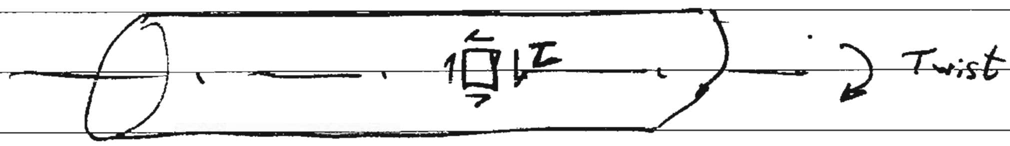
③ : Metals. Crack growth at the tip is dictated by microvoid coalescence or by cleavage.

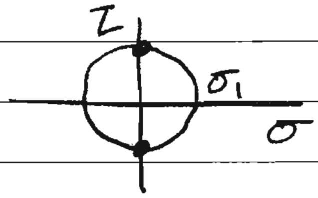
For a centre cracked panel, $K = \sigma^{\infty}\sqrt{\pi(a_o+\Delta a)}$ where a_o is the initial flaw size.



- critical K value is sensitive to the magnitude of a_o.

2. (b) Torsional loading induces shear stressing in the shaft.

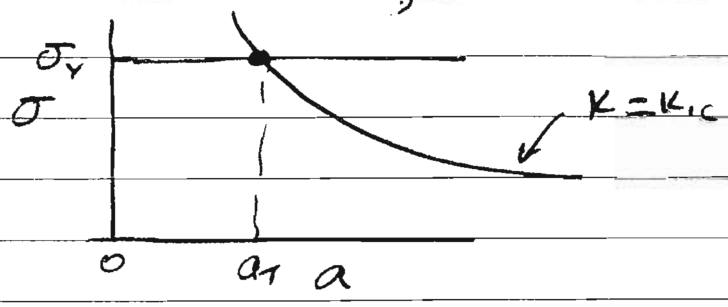




So maximum tension $\sigma_1 = \tau$ is at 45° to the axis, along a helical path.

Fatigue cracks grow in local mode I, hence the helical path.

(c) Ductile fracture occurs when $\sigma = \sigma_Y$ or when $K = K_{IC}$, where $K = \sigma\sqrt{\pi a}$



$$a_T = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_Y}\right)^2$$

For $a < a_T$ failure is at $\sigma = \sigma_Y$

$a > a_T$ failure is at $\sigma = \dfrac{K_{IC}}{\sqrt{\pi a}}$

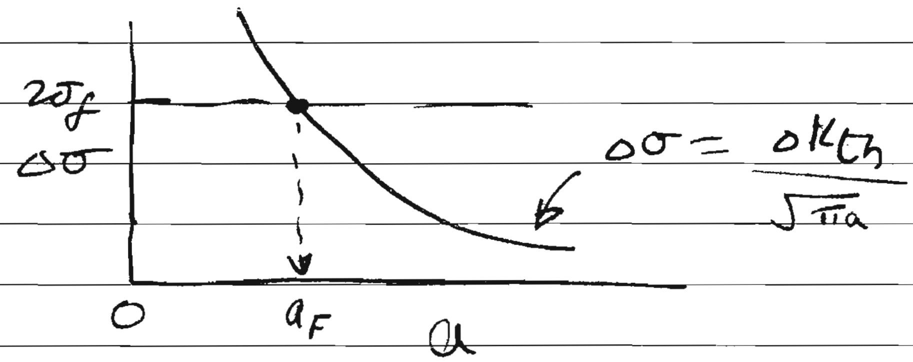
Similarly in fatigue: fatigue crack initiation occurs when $\Delta\sigma \leq 2\,\sigma_f$ where σ_f is the fatigue limit in the absence of a large flaw.

When a large flaw (of length a_o) is

2(c) contd.

present, fatigue failure occurs when

$$\Delta\sigma = \frac{\Delta K_{th}}{\sqrt{\pi a_0}}$$



The transition flaw size in fatigue is given by

$$a_F = \frac{1}{\pi}\left(\frac{\Delta K_{TH}}{2\sigma_F}\right)^2$$

Now, $\Delta K_{TH} << K_{IC}$ for metals, hence $a_F << a_T$.

3. (a) Basquin's law is obtained by integration of the Paris law, for fatigue crack growth:

$$da/dN = C \Delta K^m$$

For a crack at a weld root:

$$\Delta K \simeq \Delta\sigma \sqrt{\pi a}$$

$$\Rightarrow \frac{da}{dN} = C \Delta\sigma^m \pi^{m/2} a^{m/2}$$

$$\Rightarrow \int_{a_0}^{a_F} \frac{1}{a^{m/2}} da = C \Delta\sigma^m \pi^{m/2} N_f$$

$$\Rightarrow \left(1-\frac{m}{2}\right)^{-1} \left[a^{1-\frac{m}{2}}\right]_{a_0}^{a_F} = C \Delta\sigma^m \pi^{m/2} N_f$$

Now, $a_F \gg a_0 \Rightarrow$

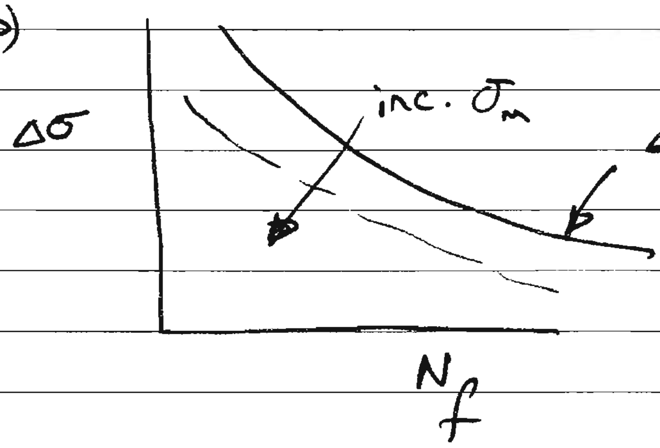
$$\left(\frac{m-2}{2}\right) C \pi^{m/2} \Delta\sigma^m N_f = a_0^{1-\frac{m}{2}}$$

$$\Rightarrow \Delta\sigma^m N_f = \text{constant}$$

This is the same as the Basquin Law.

<u>Note</u>: Basquins law is also useful for crack growth initiation in smooth specimens at low stress levels (i.e. at long lives).

3. (b)



$\Delta\sigma_R N_f^a = C$ at zero mean stress, $\sigma_m = 0$.

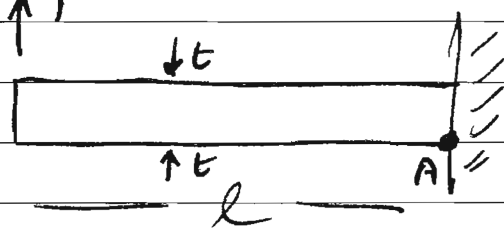
Goodman's rule: the drop in stress range $\Delta\sigma$, at fixed fatigue life, is linear in the mean stress σ_m, and $\Delta\sigma$ drops to zero at $\sigma_m = \sigma_{UTS}$.

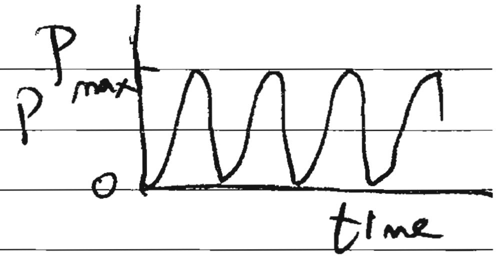
$$\Delta\sigma = \Delta\sigma_R \left(1 - \frac{\sigma_m}{\sigma_{UTS}} \right) \quad \text{at fixed } N_f.$$

So, choose a value for N_f. Then, Basquin's law gives $\Delta\sigma_R$. Now apply $\Delta\sigma$ at a mean stress σ_m. Goodman's rule gives the magnitude of $\Delta\sigma$.

Physical basis: the fatigue threshold decreases with increasing mean stress (the crack stays open for a larger fraction of the load cycle)!

3. (c)





Maximum stress is at location A.
Neglect the presence of any stress raiser at this site!

$$M = P.l$$

Beam theory gives '$\frac{\sigma}{y} = \frac{M}{I}$'

$$y = t/2 \qquad I = \tfrac{1}{12} t^4$$

$$\Rightarrow \qquad \sigma = \frac{6Pl}{t^3}$$

$$\sigma_{max} = \frac{6l}{t^3} P_{max} \qquad \sigma_m = \tfrac{1}{2}\sigma_{max}$$

Basquin's law $\Rightarrow$ $\sigma_{max} = \Delta\sigma_R \left(1 - \frac{\sigma_{max}}{2\sigma_{UTS}}\right)$

where $\Delta\sigma_R = \frac{C}{N_f^a}$

write $\bar{\sigma} \equiv \frac{\sigma_{max}}{\sigma_{UTS}} \Rightarrow \bar{\sigma} = \frac{C}{\sigma_{UTS} N_f^a} \cdot \left(1 - \frac{\bar{\sigma}}{2}\right)$

write $\bar{C} = \frac{C}{\sigma_{UTS} N_f^a}$

Then $\bar{\sigma} = \bar{C} . \left(1 - \tfrac{1}{2}\bar{\sigma}\right) \Rightarrow \bar{\sigma}\left(1 + \tfrac{1}{2}\bar{C}\right) = \bar{C}$

$$\Rightarrow \qquad \underline{\bar{\sigma} = \bar{C} . \left(1 + \tfrac{1}{2}\bar{C}\right)^{-1}}$$

3. (c) contd.

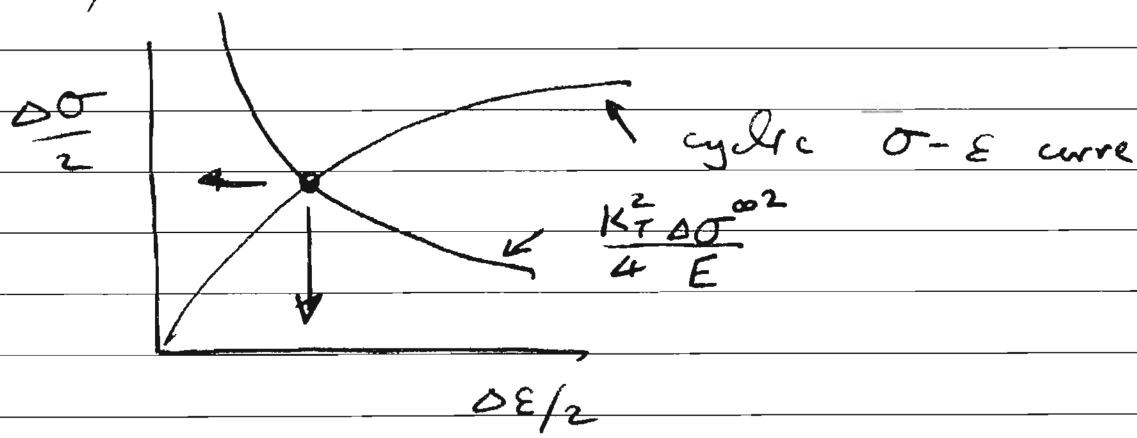
$$\bar{\sigma} = \bar{C} \cdot \left(1 - \tfrac{1}{2}\bar{\sigma}\right) \Rightarrow \bar{C} = \frac{\bar{\sigma}}{1 - \frac{1}{2}\bar{\sigma}}$$

$$\Rightarrow \frac{N_f^a \, \sigma_{UTS}}{C} = \frac{1}{\bar{C}} = \frac{1 - \frac{1}{2}\bar{\sigma}}{\bar{\sigma}} \qquad \text{QED}$$

3 (d) Neuber's rule allows for Notch root plasticity. It states:

$$K_T^2 = K_\sigma K_\varepsilon$$

elastic stress concentration factor; stress concⁿ. factor; strain concentration factor



Calculate $\frac{\Delta\varepsilon}{2}$ at the notch root & determine N_f from a chart of N_f versus $\frac{\Delta\varepsilon}{2}$.

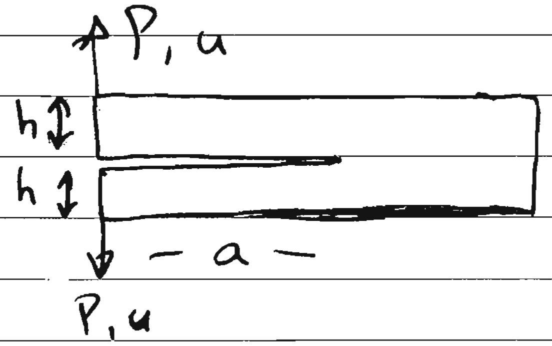
4. (a) K gives the intensity of stress & strain in an elastic solid and also when LEFM prevails.

J gives the intensity of crack tip loading when much of the specimen/structure is yielding (LEFM is violated).

J = energy release rate for crack growth in a non-linear elastic body.

$K^2 = EG$, so under LEFM $G = J$.

(b) (i)



Recall from Structures data book:

$$u = \frac{Pa^3}{3EI} \qquad I = \text{'}\tfrac{1}{12} bt^3\text{'} = \tfrac{1}{12} t^4$$

Assume fixed load. Potential energy Ψ per arm is

$$\Psi = \tfrac{1}{2} Pu - Pu = -\tfrac{1}{2} Pu = -\tfrac{1}{2} \frac{P^2 a^3}{3EI}$$

4. (b) (i) contd.

2 arms!

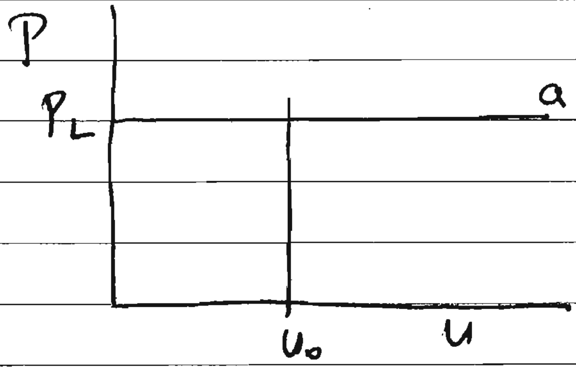
$$G = \frac{-2\,\partial \Psi}{t\,\partial a} = \frac{P^2\, 3a^2}{3EIt} = \frac{P^2 a^2}{EIt}$$

$$K^2 = EG = \frac{P^2 a^2}{It} = \frac{12\, P^2 a^2}{t^5}$$

$$\Rightarrow \quad K = \frac{2\sqrt{3}\, P a}{t^2 \sqrt{E}}$$

(ii) Plastic collapse of each beam arm occurs at $P_L\, a = M_Y = \dfrac{\sigma_Y\, t^3}{4}$

$$\Rightarrow \quad P_L = \frac{\sigma_Y t^3}{4a}$$



$\Psi_p = P_L \cdot u_0$

plastic

$$J = \frac{-1}{t}\frac{\partial}{\partial a} 2\Psi_p$$

$$\Rightarrow \quad J = \frac{\sigma_Y t^2\, u_0}{2a^2}$$