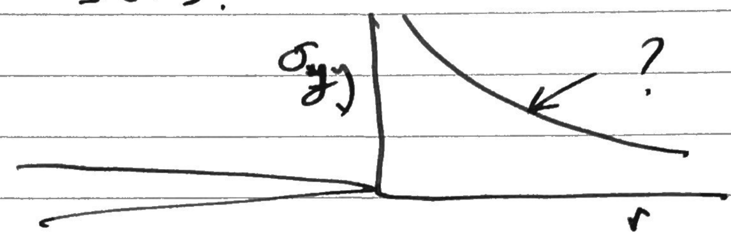
Crib for Part IIA paper 3C9

'Fracture Mechanics of Materials & Structures'

Year 2020 – 21

Q1 (a) Williams analysis shows that the crack tip stress field can be expressed by an infinite series.



In mode I we have:

$$\sigma_{yy} = \underbrace{\frac{K_I}{\sqrt{2\pi r}}}_{\text{leading term.}} + \alpha_1 \sqrt{r} + \alpha_2 r + \alpha_3 r^{3/2} + \dots$$

Note that more singular terms must vanish in order for ~~the~~ elastic energy to be ~~the~~ finite in ~~the~~ structure, and for ~~the~~ crack opening to go to zero, as $r \to 0$.

A small inelastic zone may exist at ~~the~~ crack tip (process zone where cleavage or void growth occurs, and a crack tip plastic zone) but ~~this~~ zone is fully embedded within ~~the~~ crack tip K-field. Hence K is a valid loading parameter.

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Q1. (b) If a component has a crack of length a, and ligament dimension $(W-a)$, then the crack tip may be surrounded by a J-field and/or a K-field.

Note that $\ell = \left(\frac{K_{IC}}{\sigma_Y}\right)^2$ is an intrinsic material length scale.

(i) If $a, (W-a) > 2.5\,\ell$ then an outer K-field exists and K_{IC} can be used as a fracture property. Crack growth occurs at $K = K_{IC}$.

(ii) If $a, (W-a) > 25\left(\frac{J_{IC}}{\sigma_Y}\right)$ then a J-field exists around the crack tip.

Now $E J_{IC} \sim K_{IC}^2$ and so

$$25\,\frac{J_{IC}}{\sigma_Y} = 25\,\frac{K_{IC}^2}{\sigma_Y^2}\,\frac{\sigma_Y}{E} = 25\,\frac{\sigma_Y}{E}\,\ell$$

So, if

$$25\,\frac{\sigma_Y}{E}\,\ell < a, (W-a) < 2.5\,\ell$$

then an outer J-field exists ahead of the crack tip but an outer K-field does not exist.

(iii) If $a, (W-a) < 25\,\frac{\sigma_Y}{E}\,\ell$ then there is no crack tip J-field. Also, there is no crack

Q1. (b) contd.

tip K-field. Plastic collapse occurs and (K_{IC}, J_{IC}) are not relevant parameters.

~~Q1. (c) The cyclic crack opening displacement δ drives fatigue crack growth where~~ $\Delta\delta = \dfrac{\Delta K^2}{2\sigma_Y E}$

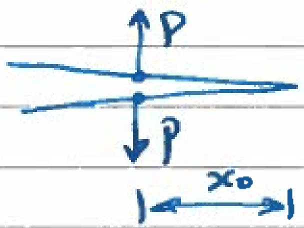
~~Now~~ $G_{max} = \dfrac{K_{max}^2}{E}$, $G_{min} = \dfrac{K_{min}^2}{E}$

~~So,~~ $\Delta G = G_{max} - G_{min} = \dfrac{1}{E}(K_{max} + K_{min})\Delta K$

$\Rightarrow \Delta G = \dfrac{1}{E}(K_{min} + \Delta K)\Delta K$.

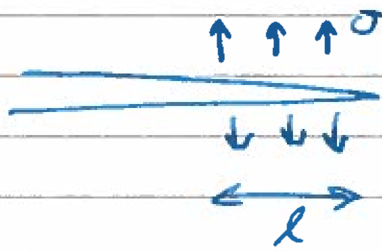
~~Note that~~ $E\Delta G \neq \Delta K^2$.

Q1. (d) Recall the relevant K-calibration from the data sheet:



$$K = \frac{2P}{\sqrt{2\pi x_0}}$$

Hence:



$$K = \int_0^{\ell} \frac{2\sigma_Y}{\sqrt{2\pi x_0}}\, dx_0$$

$$\Rightarrow K = \frac{4\sigma_Y\sqrt{\ell}}{\sqrt{2\pi}}$$

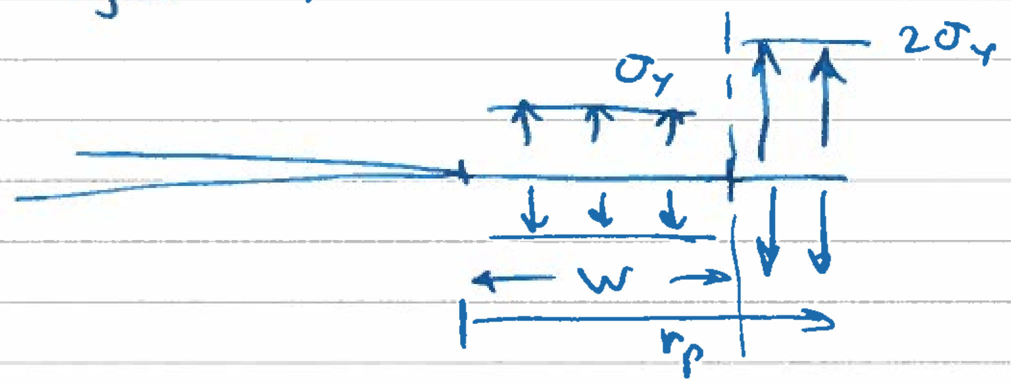
Q1 k) contd.

At a small value of K_I, the plastic zone r_p does not reach the interface.

In this case, $r_p = \frac{2\pi}{16}\frac{K_I^2}{\sigma_Y^2}$

At $r_p = w$ we have $K_I^* = \frac{4\sigma_Y}{\sqrt{2\pi}}\sqrt{w}$

At $K_I > K_I^*$, the plastic zone extends into the right-hand plate of yield strength $2\sigma_Y$.



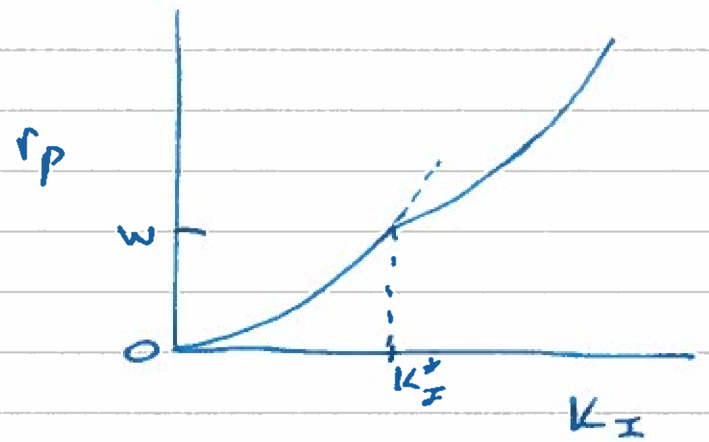
Use superposition & idealise the crack tip loading by σ_Y over a length r_p, added to a loading of σ_Y over $(r_p - w)$.

Thus $K_I = \frac{4\sigma_Y\sqrt{r_p}}{\sqrt{2\pi}} + \frac{4\sigma_Y}{\sqrt{2\pi}}\sqrt{(r_p - w)}$

This could be inverted to read

$$\left(\sqrt{r_p} + \sqrt{(r_p - w)}\right)^2 = \frac{2\pi}{16}\left(\frac{K_I}{\sigma_Y}\right)^2$$

Q1 (c) contd.



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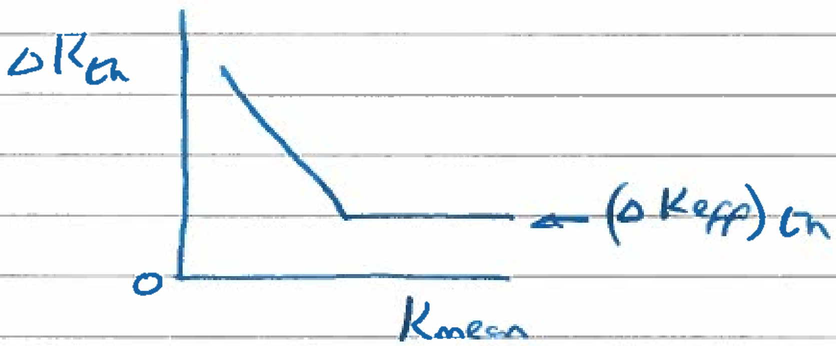
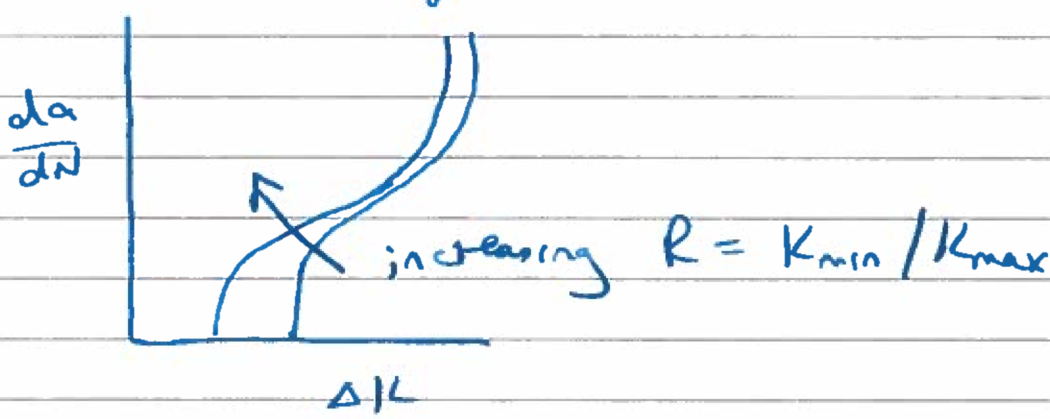
Q2. (a) Fusion welding leads to thermal stresses of yield strength σ_Y magnitude. Welds contain flaws which act as pre-cracks.

A tensile mean stress reduces the initiation life and a raised K_{mean} for a crack increases the crack growth rate, particularly in the near threshold regime.



Near threshold, crack closure mechanisms, such as 'oxide' and 'roughness induced crack' closure, are active and reduce the cyclic opening of the crack tip. Upon increasing K_{mean} the crack faces remain open and the crack growth rate is increased.



Q 2. (b)

Consider inclusions of diameter d and volume fraction f. To a good approx^n, the inclusion spacing ℓ satisfies the relation $f \approx \left(\frac{d}{\ell}\right)^2$, so $\ell \approx d f^{-1/2}$

The tensile strain to failure ε_f is sensitive to mean stress σ_h such that

$$f^{-1/2} \approx \frac{\ell}{d} \approx \varepsilon_f \exp\left(\frac{3\sigma_h}{2\sigma_y}\right)$$

↖ yield strength

Now the tensile strain ahead of a crack tip scales as

$$\varepsilon \sim \frac{J_{Ic}}{r} \sim \frac{K_{Ic}^2}{Er}$$

Assume $\varepsilon \approx \varepsilon_f$ at $r = \ell$

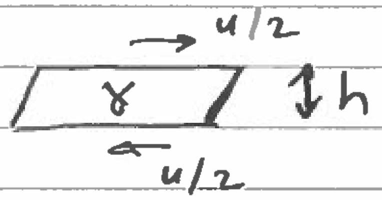
Then $J_{Ic} \approx \varepsilon_f \ell$

$$\approx \ell f^{-1/2} \exp\left(-\frac{3\sigma_h}{2\sigma_y}\right)$$

$$\Rightarrow J_{Ic} \approx \frac{d}{f} \exp \frac{-3\sigma_h}{2\sigma_y}$$

Q2 (c) (i) $M = \frac{P(b+h)}{2}$ by taking moments.

(ii)



$$u = \gamma h = \frac{\tau}{G} h$$

$$\Rightarrow \quad u = \frac{P h}{b(w-a)G}$$

(iii) $$C = \frac{u}{P} = \frac{h}{Gb(w-a)}$$

$$\Rightarrow \quad \frac{\partial C}{\partial a} = \frac{h}{Gb(w-a)^2}$$

$$g = \frac{1}{2}\frac{P^2}{b}\frac{\partial C}{\partial a} = \frac{P^2 h}{2Gb^2(w-a)^2}$$

At fracture, $g = G_{Ic}$

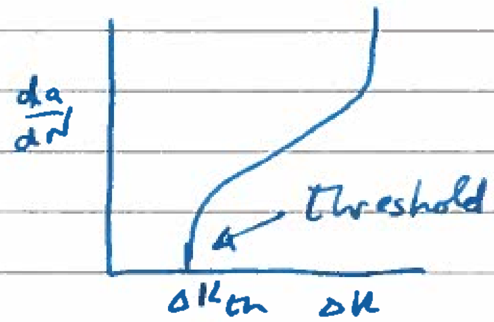
$$\Rightarrow \quad P^2 = \frac{2Gb^2(w-a)^2}{h} G_{Ic}$$

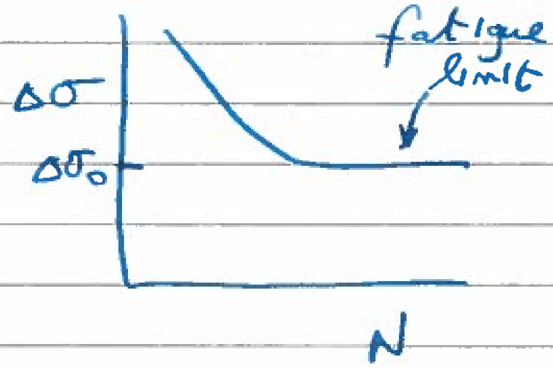
3.(a) For monotonic loading $\delta = \dfrac{K^2}{\sigma_Y E}$

Now unload the crack tip by reducing K by ΔK. Then, the stresses ahead of the crack tip reverse from $+\sigma_Y$ to $-\sigma_Y$ and we deduce that $\Delta\delta = (\Delta K)^2/(2\sigma_Y E)$.

Fatigue crack growth is driven by reversed plasticity at the crack tip. Consequently, striations of width $\Delta\delta/2$ form at the crack tip and can give rise to the crack growth increment Δa per cycle.

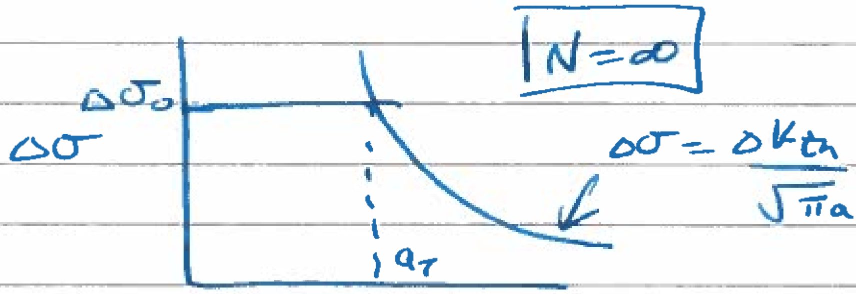
3.(b)





For a long crack, $\frac{da}{dN} \to 0$ as $\Delta K \to \Delta K_{th}$

For crack initiation, $N \to \infty$ as $\Delta\sigma \to \Delta\sigma_0$

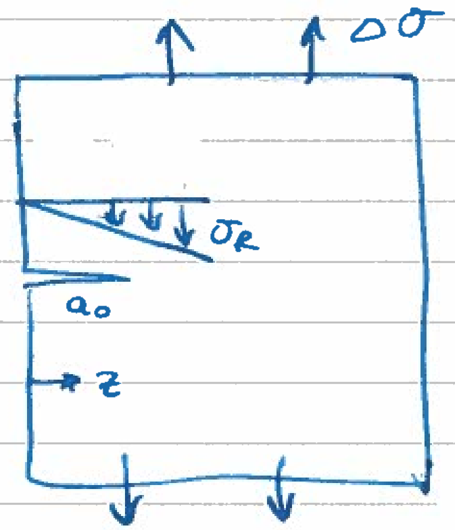


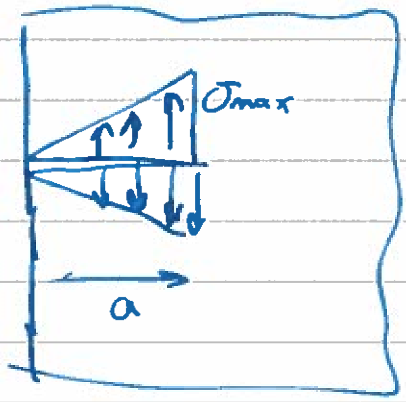
$$a_T = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2$$

3 (b) contd.

An aggressive environment can lead to stress corrosion cracking: chemical attack at the crack tip. Also, the crack faces can corrode, leading to wedging of the crack flanks and to an increased value of K for crack tip opening, K_{op}.

3 (c)





From data book,
$K = 0.683\,\sigma_{max}\sqrt{\pi a}$

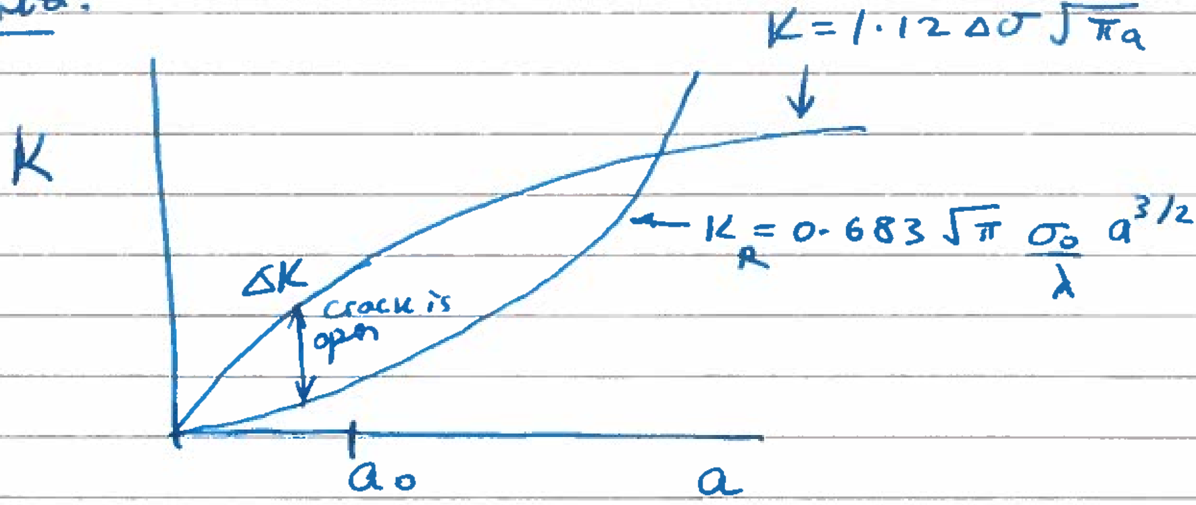
Hence, the residual K induced from σ_R is

$$K_R = -\left(\frac{\sigma_0 a}{\lambda}\right) 0.683\sqrt{\pi a}$$

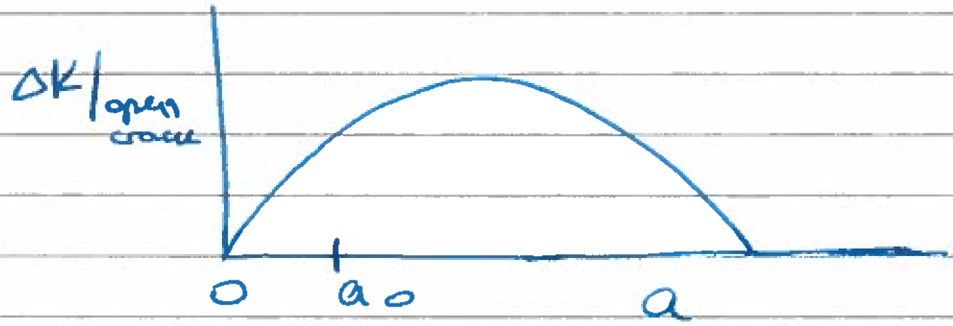
$$\Rightarrow \quad K_R = -0.683\,\frac{\sigma_0}{\lambda}\sqrt{\pi}\;a^{3/2}$$

Also $K = 1.12\,\sigma^{\infty}\sqrt{\pi a}$ where σ^{∞} varies from 0 to $\Delta\sigma$.

3. (c) contd.



$$\Delta K\Big|_{\substack{\text{open}\\\text{crack}}} = 1.12\,\Delta\sigma\sqrt{\pi a} - 0.683\sqrt{\pi}\,\frac{\sigma_o}{\lambda}\,a^{3/2}$$



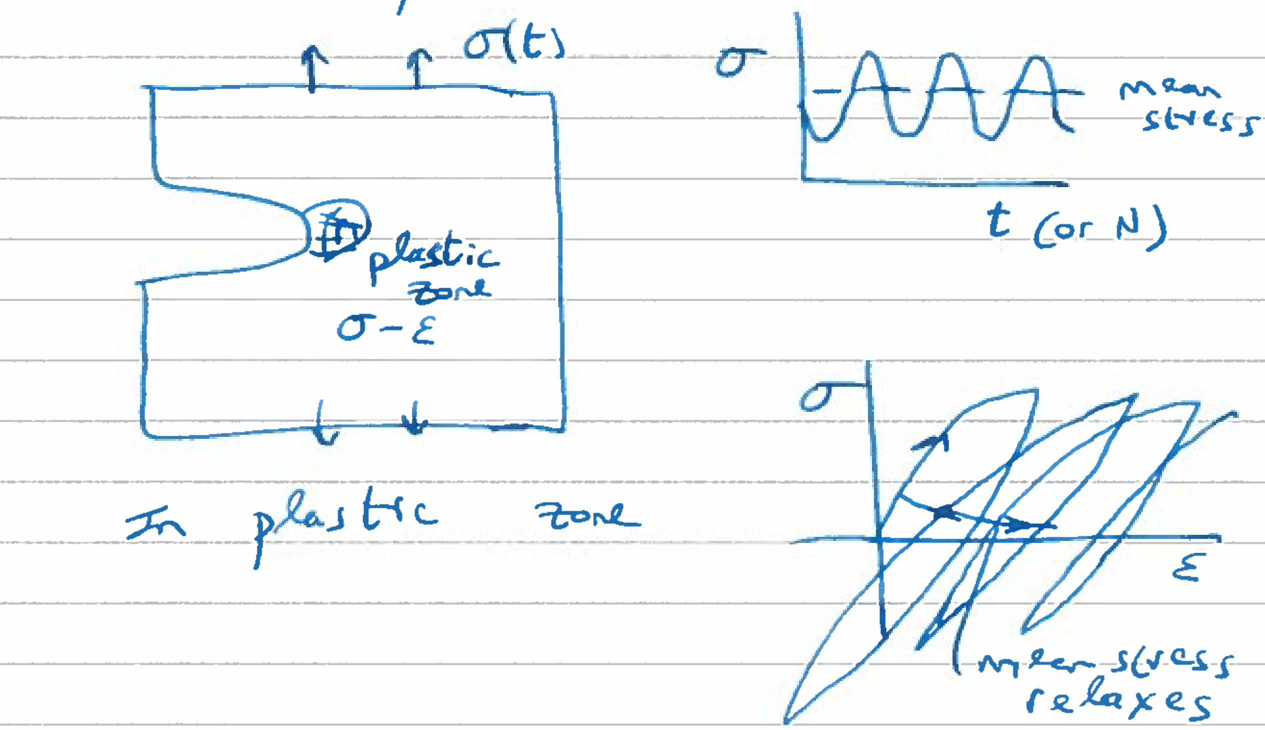
$$N = \int_{a_o}^{2a_o} \frac{dN}{da}\,da$$

where $\frac{da}{dN} = C\,\Delta K^m_{\substack{\text{open}\\\text{crack}}}$ Paris law

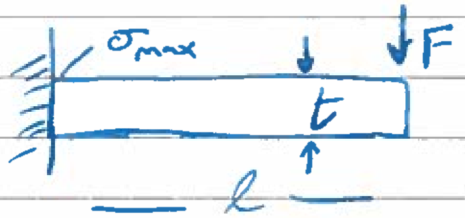
So, $N = \int_{a_o}^{2a_o} \left(C\,\Delta K^m_{\substack{\text{open}\\\text{crack}}}\right)^{-1} da$

4. (a) When a metal is cyclically loaded into the plastic range with a non-zero mean stress, the material will progressively strain in the direction of the mean stress. The phenomenon resembles creep, hence the term 'cyclic creep' or ratchetting.

Consider a component with a notch, under sufficient loading that cyclic plasticity occurs at the notch root. If a non-zero mean stress is present at the notch root, then the material progressively strains and this relaxes the mean stress, and increases the crack initiation life.



4 (b)



$M = F.l$ at root of cantilever

$$\frac{\sigma_{max}}{y} = \frac{M}{I} \qquad I = \frac{1}{12} bt^3 \qquad y = t/2$$

$$\Rightarrow \quad \sigma_{max} = \frac{12 F.l}{bt^3} \frac{t}{2} = \frac{6Fl}{bt^2}$$

(i) $$\int_0^\infty p(F)\, dF = 1$$

$$\Rightarrow \int_0^\infty A \exp\left(-\frac{F}{F_0}\right) dF = 1$$

$$\Rightarrow \quad A F_0 = 1 \quad \Rightarrow \quad A = \frac{1}{F_0}$$

(ii) $\sigma_{max} = \dfrac{6Fl}{bt^2}$, as explained above

(iii) Recall the Basquin law : $\Delta\sigma_0 N_f^a = C_1$ for fully reversed loading, of range $\Delta\sigma_0$.

Here, a load cycle ranges from 0 to F_m with mean value $F_m/2$.

Thus $\Delta\sigma = \dfrac{6F_m l}{bt^2}$, mean value $= \sigma_m = \dfrac{3F_m l}{bt^2}$

Goodmans rule : $\Delta\sigma = \Delta\sigma_0 \left(1 - \dfrac{\sigma_m}{\sigma_{ts}}\right)$

Thus, $\Delta\sigma_0 = \frac{6F_m l}{bt^2}\left(1 - \frac{3F_m l}{bt^2\sigma_{ts}}\right)^{-1}$

Now consider Miner's law.
Write dn as the number of cycles of load range between F_m and $(F_m + dF_m)$. Then,

$$\frac{dn}{N_f^T} = p(F_m)\, dF_m$$

where N_f^T is the total number of cycles to failure.

Miner's law : $\int_0^{N_f^T} \frac{dn}{N_f(F_m)} = 1$

Recall that $p(F_m) = \frac{1}{F_0}\exp\left(-\frac{F_m}{F_0}\right)$

Then, by Miner's law :

$$1 = \int_0^{N_f^T} \frac{dn}{N_f(F_m)} = \int_0^{\infty} \frac{N_f^T \cdot p(F_m)\, dF_m}{N_f(F_m)}$$

By Basquin's law : $N_f = \left(\frac{C_1}{\Delta\sigma_0}\right)^{1/a}$

Hence,

$$\frac{1}{N_f^T} = \int_0^{\infty} \frac{(\Delta\sigma_0)^{1/a}}{C_1^{1/a}}\, p(F_m)\, dF_m \Rightarrow N_f^T$$

where $p(F_m) = \frac{1}{F_0}\exp(-F_m/F_0)$

and $\Delta\sigma_0 = \frac{6F_m l}{bt^2} \cdot \left(1 - \frac{3F_m l}{bt^2\sigma_{ts}}\right)^{-1}$

END

Comments by the Principal Assessor on candidate performance

Q1 Dugdale model for plasticity and the Williams stress singularity at a crack tip.

The least popular question of the exam. It was a straightforward question and candidates understood the main ideas, but many answers lacked depth.

Q2 Residual stress effects in fatigue.

This popular question was generally well done. The first easy part (a) was answered in too cursory a manner and students did not talk about the effect of mean stress upon both fatigue crack initiation and fatigue crack growth. The more challenging part (c) was well done.

Q3. Infinite life design against fatigue crack initiation and growth, and on how to factor in residual stress effects into a fatigue crack growth calculation.

Most candidates showed a good understanding of the fatigue crack growth threshold, and of the notion of linear superposition of stress intensity factors. The derivation of the cyclic crack opening was a challenge, and none sketched how Kmin and Kmax varied with crack length in part (c).

Q4. Fatigue crack initiation under random loading.

Most understood the main ideas on ratchetting at a notch root. The analysis by beam theory was well executed. Marks were lost in part (c) by not invoking the Goodman and Basquin laws.