Version NAF/2

EGT2
ENGINEERING TRIPOS PART IIA

Friday 2 May 2014 2 to 3.30

Module 3C9

FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C9 Fracture mechanics of materials and structures data sheet (8 pages).
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) A plate contains an edge crack of length a and is subjected to a tensile load P. Show that the energy release rate G is related to the compliance C by

$$G = \frac{P^2}{2B}\frac{\partial C}{\partial a}$$

where B is the thickness of the plate. [30%]

(b) An elastic circular cylindrical fibre of radius a is bonded into a foundation and then subjected to a tensile load P , see Fig. 1. The bonding adhesive was not applied around the circumference of the fibre over a length l ($l >> a$) as shown in Fig. 1.

(i) Assuming no friction between the fibre and the foundation over the debonded zone, calculate the critical load P_c for failure of the adhesive. The toughness of the adhesive is G_c. [40%]

(ii) Assuming that the fractured adhesive exerts a constant frictional stress τ_f, calculate the dependence of applied load P on the extension Δl of the debond. [30%]

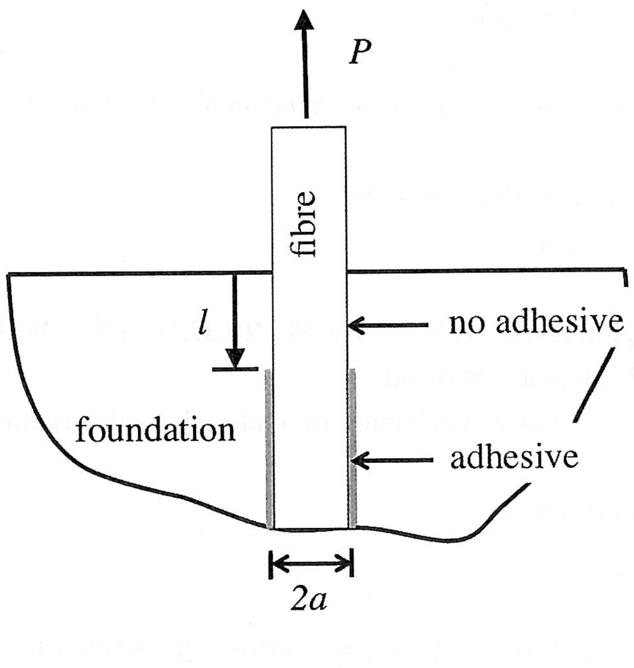


Fig. 1

2 (a) Compare the shape of the R-curve in metals, in ceramic matrix composites and in elastomers, and give the physical basis for the different characteristics. Upon making use of the R-curve for a metal, explain why the tensile strength of a panel containing a central crack of length $2a$ equals that calculated on the basis of the fracture toughness for a sufficiently long crack, but not for a short crack. [40%]

(b) Fatigue failure of the steel transmission shaft of a car engine commonly involves crack growth along a helical path. Explain why this is so. [30%]

(c) Compare the magnitude of the transition flaw size for fatigue crack growth and for ductile fracture. What is their practical significance in design? [30%]

3 (a) Basquin's law states that the fatigue life N_f of a metallic alloy is a function of the stress range $\Delta\sigma_R$ for fully reversed loading, according to $\Delta\sigma_R N_f^{\,a} = C$ where a and C are material parameters. Explain the physical basis of this law with reference to the fatigue failure of a weldment. [20%]

(b) Explain, using a sketch, how Basquin's law is modified for the effect of mean stress, and outline the physical basis for this modification. [20%]

(c) A cantilever beam, of length ℓ, and uniform square cross-section of side t is subjected to a cyclic end load from zero to a maximum value $P_{\max}$. The cantilever is made from a high strength aluminium alloy with fatigue response given by Basquin's law. Obtain an expression for its fatigue life N_f as a function of $P_{\max}$. State any assumptions that you make. [40%]

(d) Explain when the Neuber rule is used in fatigue design in preference to Basquin's law. [20%]

(TURN OVER

4 (a) Explain the physical basis and the practical use of the stress intensity factor K, and of the J-integral. [40%]

(b) A double cantilever beam, of thickness B, is of geometry shown in Fig. 2, and is used for performing a toughness test on ceramics and on steels.

(i) Assume that linear elastic fracture mechanics prevails for a ceramic specimen. Calculate the energy release rate and hence the mode I stress intensity factor for the cantilever beam under an end load P. [30%]

(ii) Consider instead a tough steel. Calculate the collapse load P_L as a function of crack length, assuming that the steel behaves in a rigid, ideally plastic manner with a yield strength σ_Y. For a fixed end displacement $u = u_0$ applied to each arm of the beam, obtain an expression for the value of the J-integral. [30%]

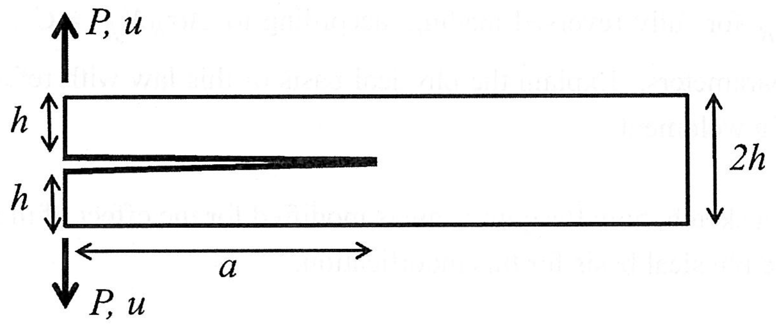


Fig. 2

END OF PAPER

ENGINEERING TRIPOS PART IIA

Module 3C9 – FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

DATASHEET

Crack tip plastic zone sizes

$$\text{diameter, } d_p = \begin{cases} \dfrac{1}{\pi}\left(\dfrac{K_I}{\sigma_y}\right)^2 & \text{Plane stress} \\[2ex] \dfrac{1}{3\pi}\left(\dfrac{K_I}{\sigma_y}\right)^2 & \text{Plane strain} \end{cases}$$

Crack opening displacement

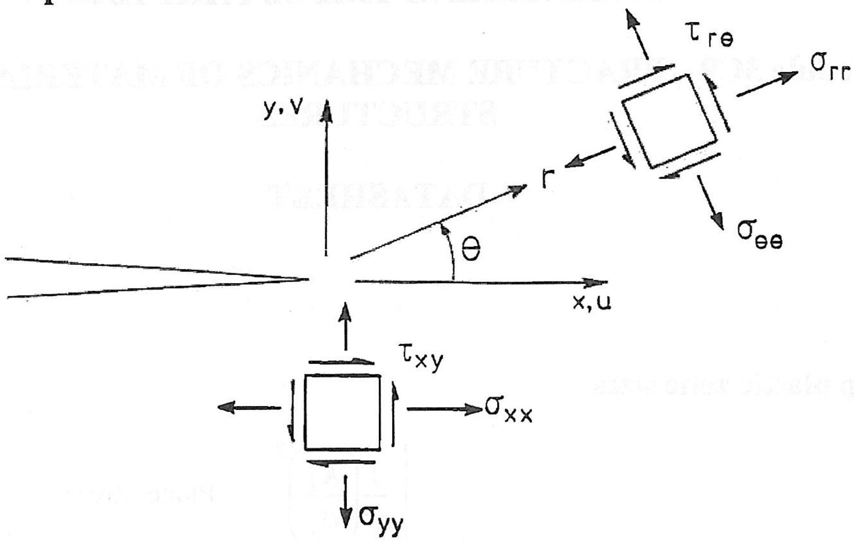
$$\delta = \begin{cases} \dfrac{K_I^2}{\sigma_y E} & \text{Plane stress} \\[2ex] \dfrac{1}{2}\dfrac{K_I^2}{\sigma_y E} & \text{Plane strain} \end{cases}$$

Energy release rate

$$G = \begin{cases} \dfrac{1}{E}K_I^2 & \text{Plane stress} \\[2ex] \dfrac{1-\nu^2}{E}K_I^2 & \text{Plane strain} \end{cases}$$

Related to compliance C: $G = \dfrac{1}{2}\dfrac{P^2}{B}\dfrac{dC}{da}$

Asymptotic crack tip fields in a linear elastic solid



Mode I

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2}-\frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2}+\frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2}+\frac{1}{4}\sin\frac{3\theta}{2}\right)$$

$$u = \begin{cases} \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{1-\nu}{1+\nu}+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(1-2\nu+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{2}{1+\nu}-\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(2-2\nu-\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Crack tip stress fields (cont'd)

Mode II

$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \sin\frac{\theta}{2} \cos\frac{3\theta}{2}$$

$$\sigma_{xx} = -\frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2} \left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{rr} = \frac{K_{II}}{\sqrt{2\pi r}} \left(-\frac{5}{4}\sin\frac{\theta}{2}+\frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{\theta\theta} = -\frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{3}{4}\sin\frac{\theta}{2}+\frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\tau_{r\theta} = \frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{1}{4}\cos\frac{\theta}{2}+\frac{3}{4}\cos\frac{3\theta}{2}\right)$$

$$u = \begin{cases} \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{2}{1+\nu}+\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(2-2\nu+\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{\nu-1}{1+\nu}+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(-1+2\nu+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Mode III

$$\tau_{zx} = -\frac{K_{III}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}$$

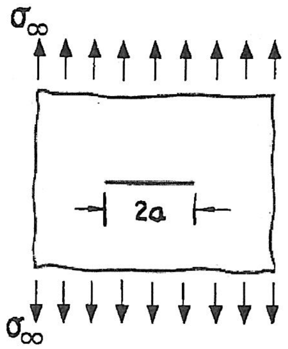
$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \tau_{xy} = 0$$

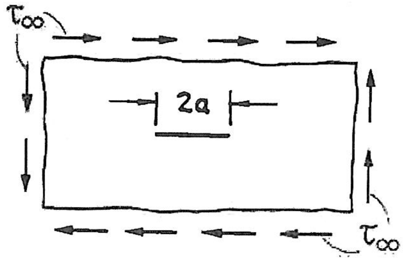
$$w = \frac{K_{III}}{G}\sqrt{\frac{2r}{\pi}} \sin\frac{\theta}{2}$$

$$u = v = 0$$

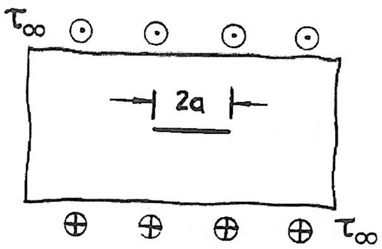
Tables of stress intensity factors



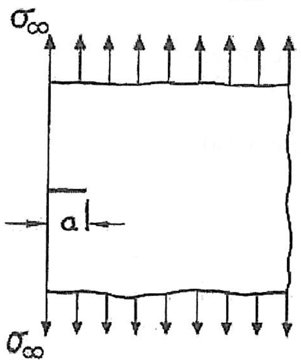
$$K_I = \sigma_\infty \sqrt{\pi a}$$



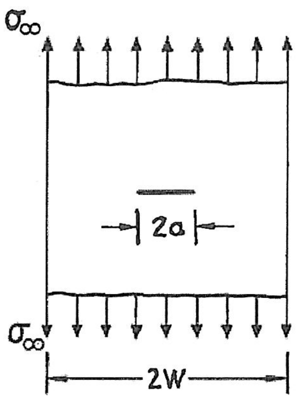
$$K_{II} = \tau_\infty \sqrt{\pi a}$$



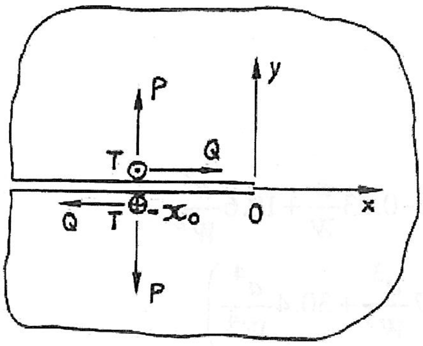
$$K_{III} = \tau_\infty \sqrt{\pi a}$$



$$K_I = 1.12\,\sigma_\infty \sqrt{\pi a}$$



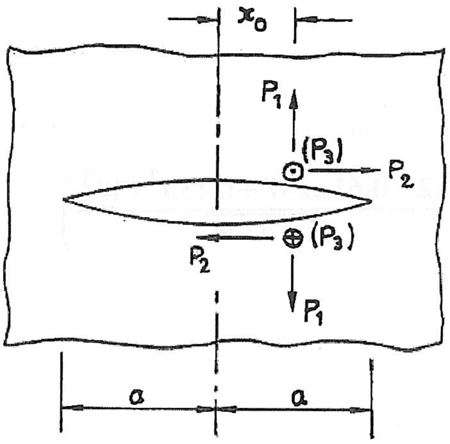
$$K_I = \sigma_\infty \sqrt{\pi a}\left(\frac{1 - a/2W + 0.326a^2/W^2}{\sqrt{1 - a/W}}\right)$$



$$K_I = \frac{2P}{\sqrt{2\pi x_o}}$$

$$K_{II} = \frac{2Q}{\sqrt{2\pi x_o}}$$

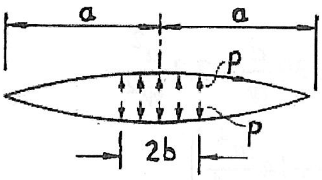
$$K_{III} = \frac{2T}{\sqrt{2\pi x_o}}$$



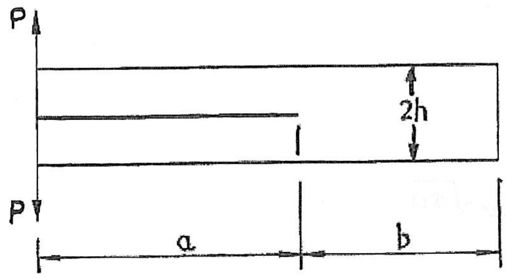
$$K_I = \frac{P_1}{\sqrt{\pi a}}\sqrt{\frac{a+x_o}{a-x_o}}$$

$$K_{II} = \frac{P_2}{\sqrt{\pi a}}\sqrt{\frac{a+x_o}{a-x_o}}$$

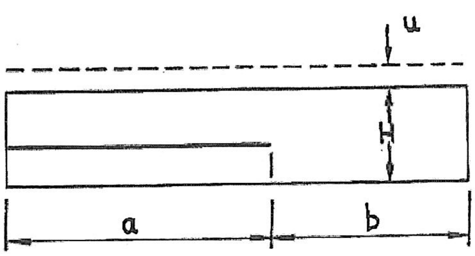
$$K_{III} = \frac{P_3}{\sqrt{\pi a}}\sqrt{\frac{a+x_o}{a-x_o}}$$



$$K_I = \frac{2pb}{\sqrt{\pi a}}\frac{a}{b}\arcsin\frac{b}{a}$$

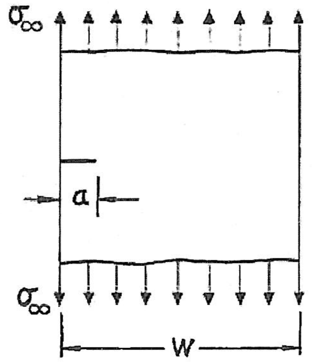


$$K_I = \frac{2\sqrt{3}\,Pa}{h\sqrt{h}\;B} \qquad h \ll a \text{ and } h \ll b$$



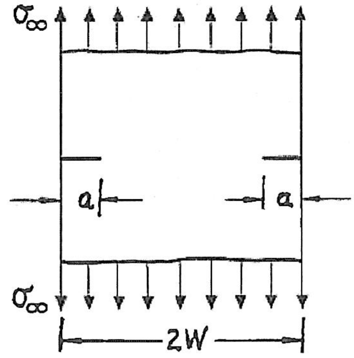
$$K_I = \sqrt{\frac{1}{2\alpha H}}\,Eu \qquad H \ll a \text{ and } H \ll b$$

$$\alpha = \begin{cases} 1-\nu^2 & \text{Plane stress} \\ 1-3\nu^2-2\nu^3 & \text{Plane strain} \end{cases}$$

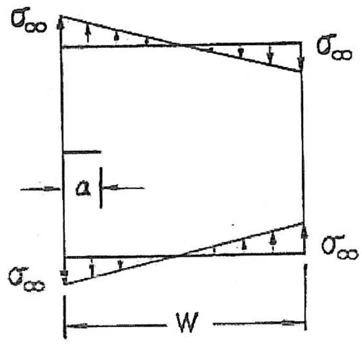


$a/W < 0.7$

$$K_I = \sigma_\infty \sqrt{\pi a}\left(1.12 - 0.23\frac{a}{W} + 10.6\frac{a^2}{W^2} - 21.7\frac{a^3}{W^3} + 30.4\frac{a^4}{W^4}\right)$$

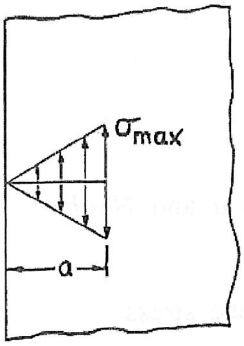


$$K_I = \sigma_\infty \sqrt{\pi a}\left(\frac{1.12 - 0.61a/W + 0.13a^3/W^3}{\sqrt{1 - a/W}}\right)$$

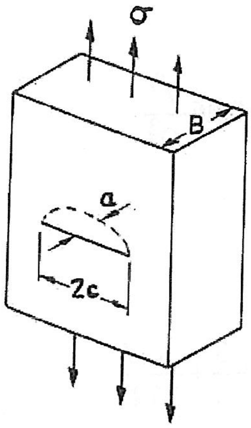


$a/W < 0.7$

$$K_I = \sigma_\infty \sqrt{\pi a}\left(1.12 - 1.39\frac{a}{W} + 7.3\frac{a^2}{W^2} - 13\frac{a^3}{W^3} + 14\frac{a^4}{W^4}\right)$$

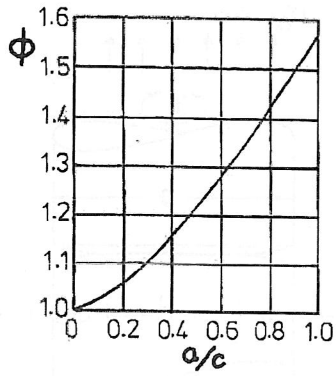


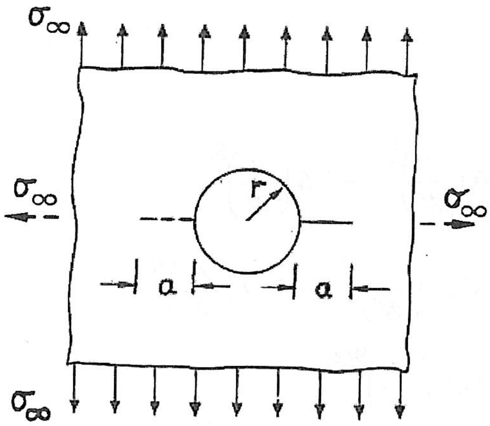
$$K_I = 0.683\,\sigma_{max}\sqrt{\pi a}$$



$$K_I = \frac{1.12}{\Phi}\sigma\sqrt{\pi a}$$

$$\Phi = \int_0^{\pi/2}\left(1-\frac{c^2-a^2}{c^2}\sin^2\theta\right)^{\frac{1}{2}}d\theta$$



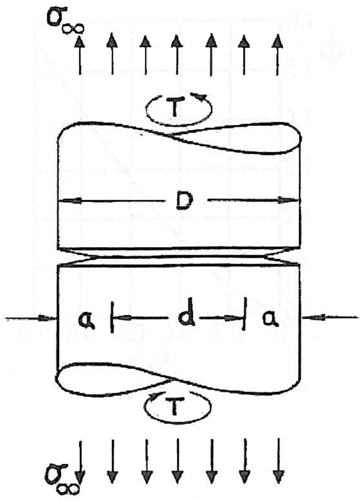


$$K_I = \sigma_\infty\sqrt{\pi a}\ F\left(\frac{a}{r}\right)$$

value of $F(a/r)$†

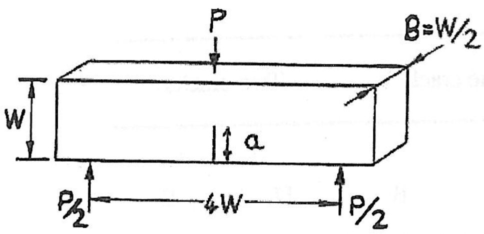
$\frac{a}{r}$	One crack U	One crack B	Two cracks U	Two cracks B
0.00	3.36	2.24	3.36	2.24
0.10	2.73	1.98	2.73	1.98
0.20	2.30	1.82	2.41	1.83
0.30	2.04	1.67	2.15	1.70
0.40	1.86	1.58	1.96	1.61
0.50	1.73	1.49	1.83	1.57
0.60	1.64	1.42	1.71	1.52
0.80	1.47	1.32	1.58	1.43
1.0	1.37	1.22	1.45	1.38
1.5	1.18	1.06	1.29	1.26
2.0	1.06	1.01	1.21	1.20
3.0	0.94	0.93	1.14	1.13
5.0	0.81	0.81	1.07	1.06
10.0	0.75	0.75	1.03	1.03
∞	0.707	0.707	1.00	1.00

†U = uniaxial σ_∞ B = biaxial σ_∞.

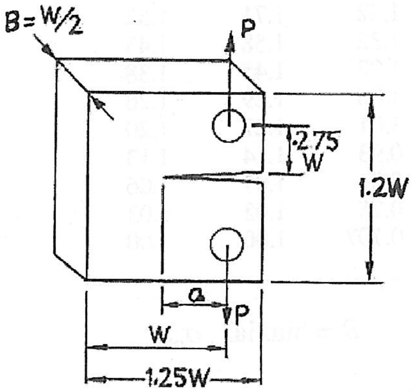


$$K_I = \sigma_\infty \sqrt{\pi a}\left(\frac{D}{d}+\frac{1}{2}+\frac{3}{8}\frac{d}{D}-0.36\frac{d^2}{D^2}+0.73\frac{d^3}{D^3}\right)\frac{1}{2}\sqrt{\frac{D}{d}}$$

$$K_{III} = \frac{16T}{\pi D^3}\sqrt{\pi a}\left(\frac{D^2}{d^2}+\frac{1}{2}\frac{D}{d}+\frac{3}{8}+\frac{5}{16}\frac{d}{D}+\frac{35}{128}\frac{d^2}{D^2}+0.21\frac{d^3}{D^3}\right)\frac{3}{8}\sqrt{\frac{D}{d}}$$



$$K_I = \frac{4P}{B}\sqrt{\frac{\pi}{W}}\left\{1.6\left(\frac{a}{W}\right)^{1/2}-2.6\left(\frac{a}{W}\right)^{3/2}+12.3\left(\frac{a}{W}\right)^{5/2}-21.2\left(\frac{a}{W}\right)^{7/2}+21.8\left(\frac{a}{W}\right)^{9/2}\right\}$$



$$K_I = \frac{P}{B}\sqrt{\frac{\pi}{W}}\left\{16.7\left(\frac{a}{W}\right)^{1/2}-104.7\left(\frac{a}{W}\right)^{3/2}+369.9\left(\frac{a}{W}\right)^{5/2}-573.8\left(\frac{a}{W}\right)^{7/2}+360.5\left(\frac{a}{W}\right)^{9/2}\right\}$$

NAF
March 2010