EGT2
ENGINEERING TRIPOS PART IIA

Monday 10 May 2021 1.30 to 3.10

Module 3C9

FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet and at the top of each answer sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 3C9 Fracture mechanics of materials and structures data sheet (8 pages).
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 15 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 (a) With reference to the Williams solution for the asymptotic fields near a crack tip, explain why the stress intensity factor K is an adequate loading parameter for the prediction of brittle fracture. [20%]

(b) Explain the circumstances under which the yield strength σ_y, the fracture toughness K_{IC} and the critical value of J-integral J_{IC} are appropriate fracture parameters. [30%]

(c) A semi-infinite plate of yield strength σ_y contains a semi-infinite crack as shown in Fig. 1. The plate is perfectly bonded along its rightmost edge to an uncracked semi-infinite plate of yield strength $2\sigma_y$, but identical isotropic elastic properties. The crack tip is at a distance w from the interface, and the cracked assembly is subjected to remote mode I loading, K_I. By making use of the Dugdale strip-yield model, obtain the dependence of the plastic zone size r_p ahead of the crack tip upon K_I. Sketch the form of this relation.

[50%]

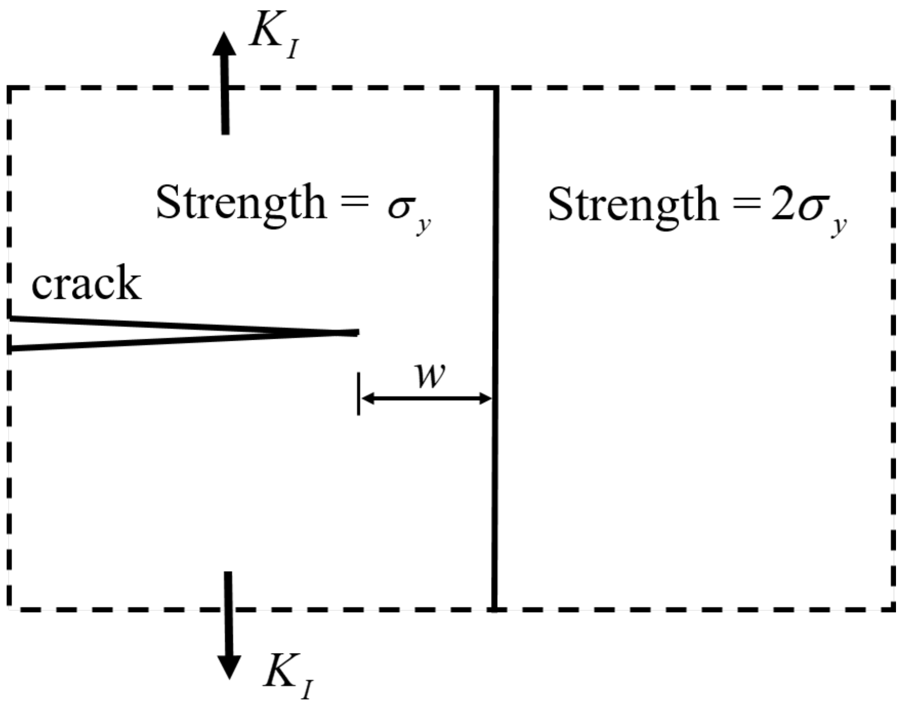

Fig. 1

2 (a) Explain why stress-relief of a welded joint increases its fatigue life. [25%]

(b) Explain why the toughness of metallic alloys is sensitive to inclusion content. [25%]

(c) Two steel bars, each of square cross-section $b \times b$, are adhesively bonded together in a lap-joint configuration by an elastomeric adhesive of thickness h to give a contact area of $b \times w$, as shown in Fig. 2. An edge crack of length a is present within the adhesive. The elastic shear modulus of the adhesive G is much less than that of the steel bars, such that the bars can be treated as rigid. The bars are end-loaded by an axial force P and a moment M.

(i) Obtain the value of M as required for global equilibrium. [10%]

(ii) Estimate the relative axial displacement of the two bars, assuming that the adhesive layer is in a state of simple shear. [20%]

(iii) Hence determine the energy release rate at the crack tip, and determine the critical value of force P for fracture of the joint in terms of the toughness of the adhesive G_{IC} . [20%]

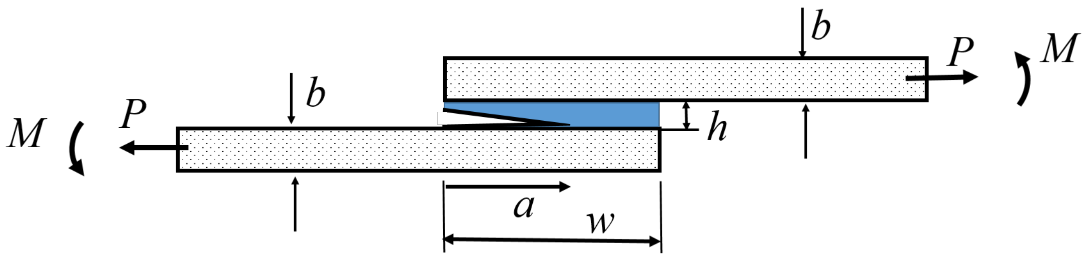

Fig. 2

3 (a) Derive the dependence of the cyclic crack tip opening displacement $\Delta\delta$ of a fatigue crack upon the stress intensity factor, and explain the relevance of $\Delta\delta$ to fatigue crack growth rate. [20%]

(b) Distinguish between the threshold stress intensity factor and the fatigue limit in design. What role does environment play in influencing the threshold stress intensity factor for metallic alloys? [40%]

(c) The surface layers at the edge of a large steel plate are put into a state of residual compressive stress of magnitude σ_R parallel to the edge, as shown in Fig. 3. This stress varies with depth z according to $\sigma_R = \sigma_0 z/\lambda$ where σ_0 and λ are constants. The plate is also subjected to a remote tensile cyclic stress σ^∞ that ranges from zero to a maximum value of $\Delta\sigma^\infty$. Assume that the plate has an edge crack of length a_0 that grows in accordance with the Paris law, provided the stress intensity factor is positive. Obtain an expression for the number of cycles required to double the crack length, without conducting any integration. Suitable use may be made of the calibrations for stress intensity factor in the 3C9 datasheet. [40%]

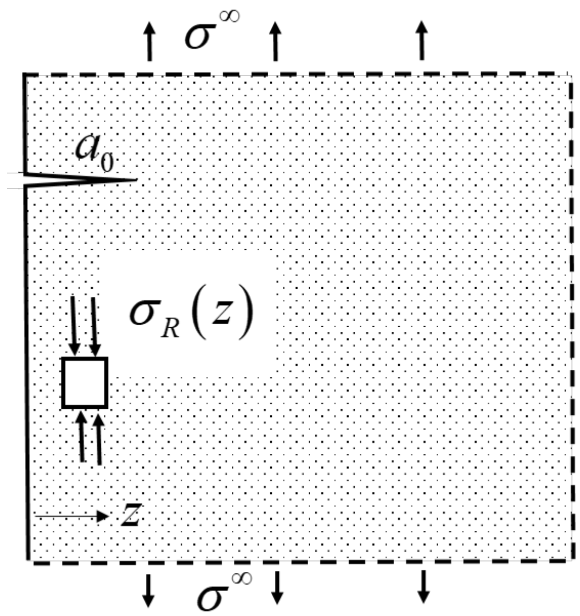

Fig. 3

4 (a) A metallic component, containing a machined edge notch, is loaded in cyclic tension with a superimposed tensile mean stress. The alloy from which the component is made undergoes ratchetting in a low cycle fatigue test. Explain how the presence of ratchetting affects the fatigue life of the notched component. [20%]

(b) The steel nib of a fountain pen has a finite fatigue life. The nib can be idealised as an end-loaded cantilever beam of length l, thickness t and width b, such that its free end is subjected to a transverse force F that varies from zero to a value F_m in each load cycle. The loading is random, and satisfies the exponential probability density function $p(F_m) = A\exp(-F_m/F_0)$ where A and F_0 are constants.

(i) What is the value of A in order for the cumulative probability of loading to equal unity? [20%]

(ii) Calculate the tensile stress at the most heavily loaded location of the nib in terms of the tip force F and the nib geometry. [20%]

(iii) By suitable consideration of fatigue crack initiation, obtain an integral expression for the fatigue life of the nib with F_m as the independent variable for the integral. [40%]

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ENGINEERING TRIPOS PART IIA

Module 3C9 – FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

DATASHEET

Crack tip plastic zone sizes

$$\text{diameter, } d_p = \begin{cases} \dfrac{1}{\pi}\left(\dfrac{K_I}{\sigma_y}\right)^2 & \text{Plane stress} \\ \dfrac{1}{3\pi}\left(\dfrac{K_I}{\sigma_y}\right)^2 & \text{Plane strain} \end{cases}$$

Crack opening displacement

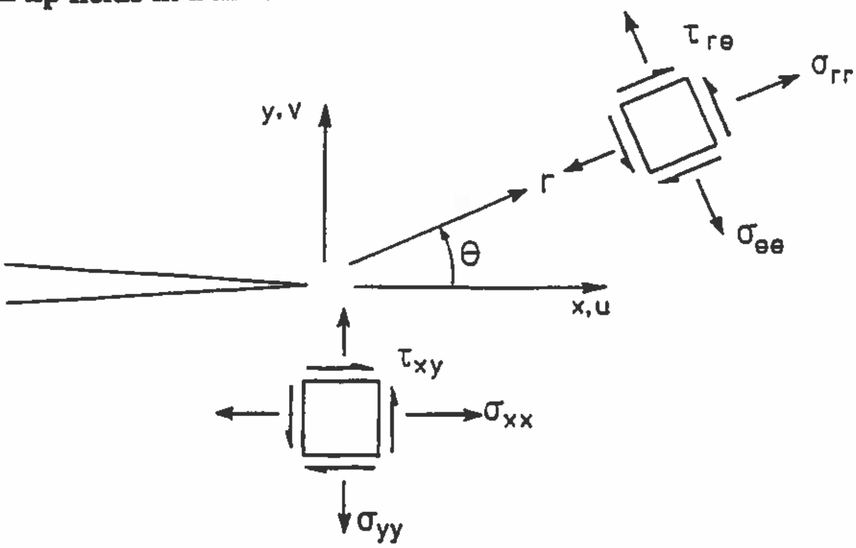
$$\delta = \begin{cases} \dfrac{K_I^2}{\sigma_y E} & \text{Plane stress} \\ \dfrac{1}{2}\dfrac{K_I^2}{\sigma_y E} & \text{Plane strain} \end{cases}$$

Energy release rate

$$G = \begin{cases} \dfrac{1}{E}K_I^2 & \text{Plane stress} \\ \dfrac{1-\nu^2}{E}K_I^2 & \text{Plane strain} \end{cases}$$

Related to compliance C: $G = \dfrac{1}{2}\dfrac{P^2}{B}\dfrac{dC}{da}$

Asymptotic crack tip fields in a linear elastic solid



Mode I

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\,\sin\frac{\theta}{2}\,\cos\frac{3\theta}{2}$$

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right)$$

$$u = \begin{cases} \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{1-\nu}{1+\nu} + \sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(1 - 2\nu + \sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{2}{1+\nu} - \cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(2 - 2\nu - \cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Crack tip stress fields (cont'd)

Mode II

$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \sin\frac{\theta}{2} \cos\frac{3\theta}{2}$$

$$\sigma_{xx} = -\frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2} \left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{rr} = \frac{K_{II}}{\sqrt{2\pi r}} \left(-\frac{5}{4}\sin\frac{\theta}{2}+\frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{\theta\theta} = -\frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{3}{4}\sin\frac{\theta}{2}+\frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\tau_{r\theta} = \frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{1}{4}\cos\frac{\theta}{2}+\frac{3}{4}\cos\frac{3\theta}{2}\right)$$

$$u = \begin{cases} \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{2}{1+\nu}+\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(2-2\nu+\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{\nu-1}{1+\nu}+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(-1+2\nu+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Mode III

$$\tau_{zx} = -\frac{K_{III}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}$$

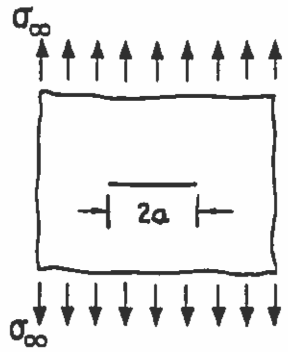
$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \tau_{xy} = 0$$

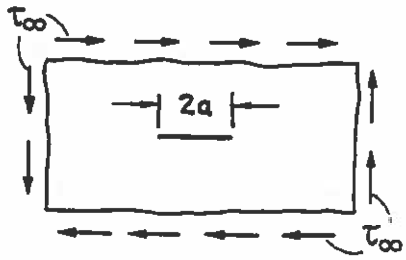
$$w = \frac{K_{III}}{G}\sqrt{\frac{2r}{\pi}} \sin\frac{\theta}{2}$$

$$u = v = 0$$

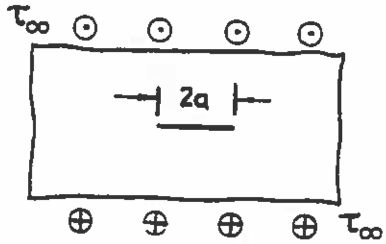
Tables of stress intensity factors



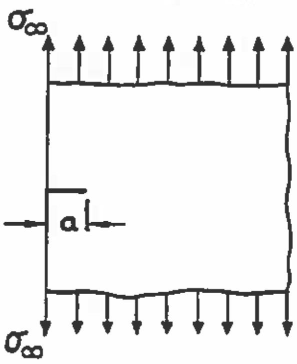
$$K_I = \sigma_\infty \sqrt{\pi a}$$



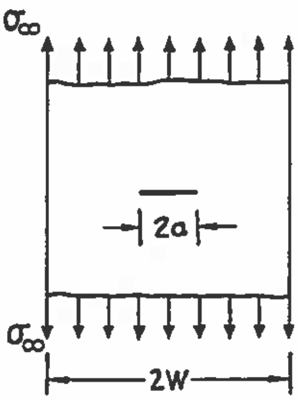
$$K_{II} = \tau_\infty \sqrt{\pi a}$$



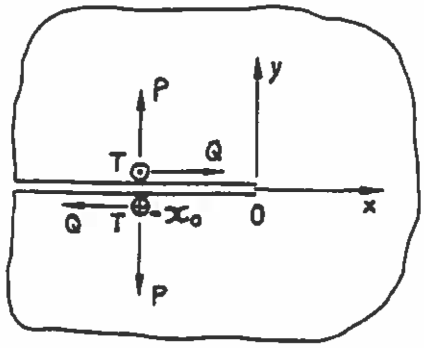
$$K_{III} = \tau_\infty \sqrt{\pi a}$$



$$K_I = 1.12\, \sigma_\infty \sqrt{\pi a}$$



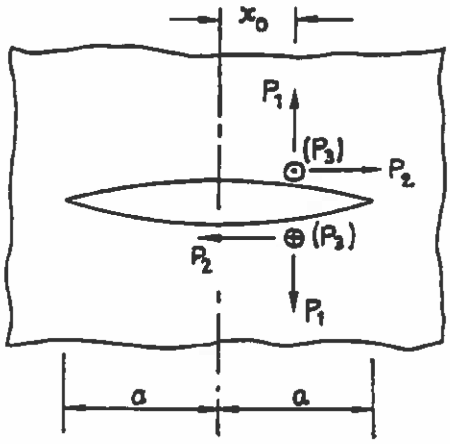
$$K_I = \sigma_\infty \sqrt{\pi a} \left(\frac{1 - a/2W + 0.326 a^2 / W^2}{\sqrt{1 - a/W}} \right)$$



$$K_I = \frac{2P}{\sqrt{2\pi x_o}}$$

$$K_{II} = \frac{2Q}{\sqrt{2\pi x_o}}$$

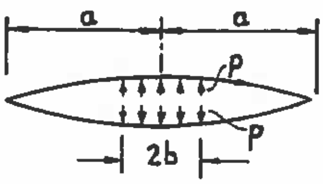
$$K_{III} = \frac{2T}{\sqrt{2\pi x_o}}$$



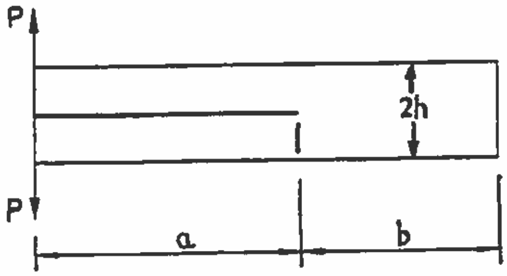
$$K_I = \frac{P_1}{\sqrt{\pi a}}\sqrt{\frac{a+x_0}{a-x_0}}$$

$$K_{II} = \frac{P_2}{\sqrt{\pi a}}\sqrt{\frac{a+x_0}{a-x_0}}$$

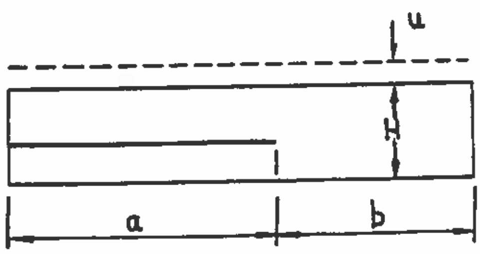
$$K_{III} = \frac{P_3}{\sqrt{\pi a}}\sqrt{\frac{a+x_0}{a-x_0}}$$



$$K_I = \frac{2pb}{\sqrt{\pi a}}\frac{a}{b}\arcsin\frac{b}{a}$$

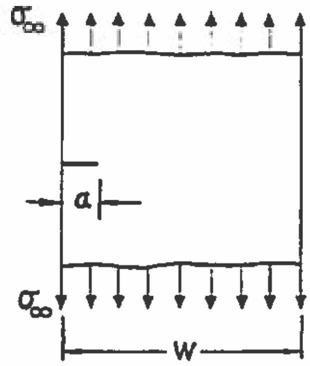


$$K_I = \frac{2\sqrt{3}}{h\sqrt{h}}\frac{Pa}{B} \qquad h \ll a \text{ and } h \ll b$$



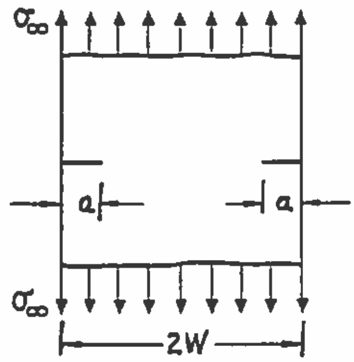
$$K_I = \sqrt{\frac{1}{2\alpha H}}\,Eu \qquad H \ll a \text{ and } H \ll b$$

$$\alpha = \begin{cases} 1-\nu^2 & \text{Plane stress} \\ 1-3\nu^2-2\nu^3 & \text{Plane strain} \end{cases}$$

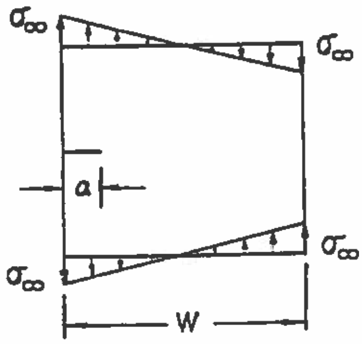


$a/W < 0.7$

$$K_I = \sigma_\infty \sqrt{\pi a}\left(1.12 - 0.23\frac{a}{W} + 10.6\frac{a^2}{W^2} - 21.7\frac{a^3}{W^3} + 30.4\frac{a^4}{W^4}\right)$$

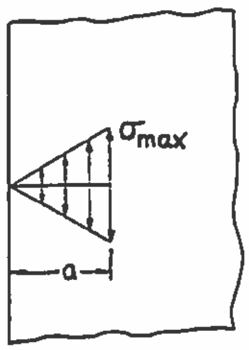


$$K_I = \sigma_\infty \sqrt{\pi a}\left(\frac{1.12 - 0.61a/W + 0.13a^3/W^3}{\sqrt{1-a/W}}\right)$$

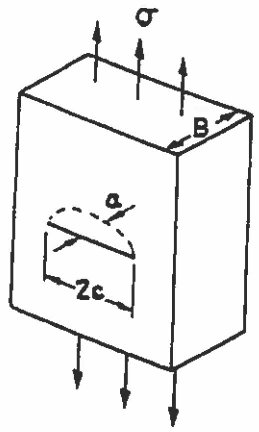


$a/W < 0.7$

$$K_I = \sigma_\infty \sqrt{\pi a}\left(1.12 - 1.39\frac{a}{W} + 7.3\frac{a^2}{W^2} - 13\frac{a^3}{W^3} + 14\frac{a^4}{W^4}\right)$$

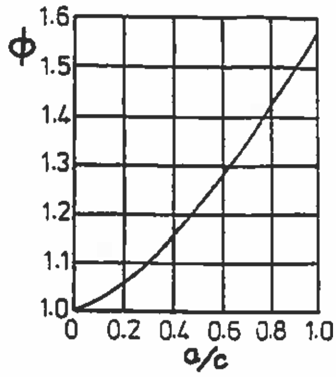


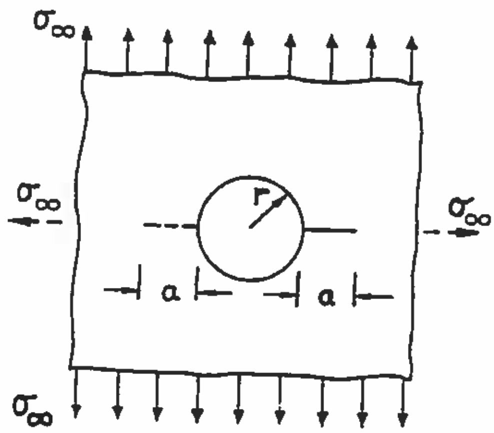
$$K_I = 0.683\, \sigma_{max} \sqrt{\pi a}$$



$$K_I = \frac{1.12}{\Phi}\sigma\sqrt{\pi a}$$

$$\Phi = \int_0^{\pi/2}\left(1-\frac{c^2-a^2}{c^2}\sin^2\theta\right)^{\frac{1}{2}} d\theta$$



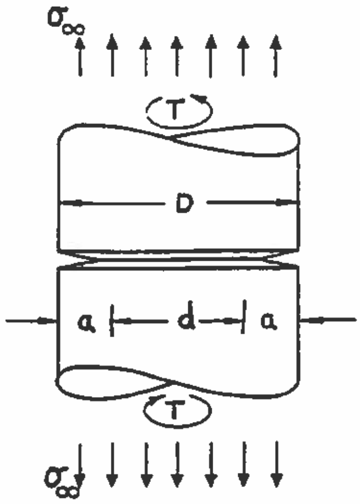


$$K_I = \sigma_\infty\sqrt{\pi a}\; F\left(\frac{a}{r}\right)$$

value of $F\,(a\,/\,r\,)^\dagger$

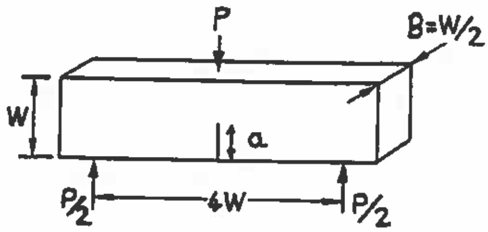
	One crack		Two cracks	
$\frac{a}{r}$	U	B	U	B
0.00	3.36	2.24	3.36	2.24
0.10	2.73	1.98	2.73	1.98
0.20	2.30	1.82	2.41	1.83
0.30	2.04	1.67	2.15	1.70
0.40	1.86	1.58	1.96	1.61
0.50	1.73	1.49	1.83	1.57
0.60	1.64	1.42	1.71	1.52
0.80	1.47	1.32	1.58	1.43
1.0	1.37	1.22	1.45	1.38
1.5	1.18	1.06	1.29	1.26
2.0	1.06	1.01	1.21	1.20
3.0	0.94	0.93	1.14	1.13
5.0	0.81	0.81	1.07	1.06
10.0	0.75	0.75	1.03	1.03
∞	0.707	0.707	1.00	1.00

$\dagger U$ = uniaxial σ_∞ $\quad B$ = biaxial σ_∞.

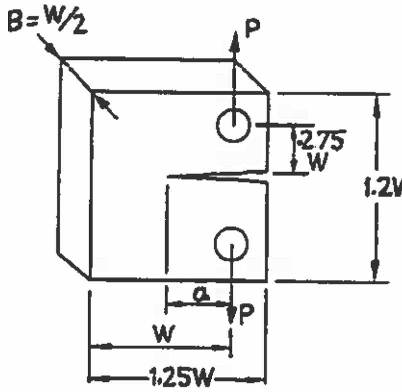


$$K_I = \sigma_\infty\sqrt{\pi a}\left(\frac{D}{d}+\frac{1}{2}+\frac{3}{8}\frac{d}{D}-0.36\frac{d^2}{D^2}+0.73\frac{d^3}{D^3}\right)\frac{1}{2}\sqrt{\frac{D}{d}}$$

$$K_{III} = \frac{16T}{\pi D^3}\sqrt{\pi a}\left(\frac{D^2}{d^2}+\frac{1}{2}\frac{D}{d}+\frac{3}{8}+\frac{5}{16}\frac{d}{D}+\frac{35}{128}\frac{d^2}{D^2}+0.21\frac{d^3}{D^3}\right)\frac{3}{8}\sqrt{\frac{D}{d}}$$



$$K_I = \frac{4P}{B}\sqrt{\frac{\pi}{W}}\left\{1.6\left(\frac{a}{W}\right)^{1/2}-2.6\left(\frac{a}{W}\right)^{3/2}+12.3\left(\frac{a}{W}\right)^{5/2}-21.2\left(\frac{a}{W}\right)^{7/2}+21.8\left(\frac{a}{W}\right)^{9/2}\right\}$$



$$K_I = \frac{P}{B}\sqrt{\frac{\pi}{W}}\left\{16.7\left(\frac{a}{W}\right)^{1/2}-104.7\left(\frac{a}{W}\right)^{3/2}+369.9\left(\frac{a}{W}\right)^{5/2}-573.8\left(\frac{a}{W}\right)^{7/2}+360.5\left(\frac{a}{W}\right)^{9/2}\right\}$$

NAF
March 2010