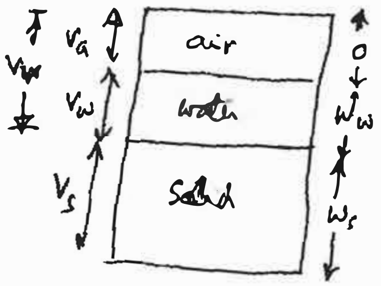
3D1 - Lent 2022

Q 1) a)



$$\gamma_b = \frac{\text{Total weight}}{\text{Total volume}} = \frac{W_w + W_s}{V_a + V_s + V_w} = \frac{W_w + W_s}{V_v + V_s}$$

$$W_w = \gamma_w V_w$$

$$W_s = G_s \gamma_w V_s$$

$$e = \frac{V_v}{V_s} \Rightarrow V_v = e V_s \quad ; \quad S_r = \frac{V_w}{V_v} \Rightarrow V_w = V_v S_r = e V_s S_r$$

$$\therefore \gamma_b = \gamma_w \frac{[V_w + G_s V_s]}{e V_s + V_s} = \gamma_w \frac{[e V_s S_r + G_s V_s]}{V_s (1+e)}$$

$$\therefore \gamma_b = \gamma_w \frac{[G_s + e S_r]}{1+e} \frac{\cancel{V_s}}{\cancel{V_s}}$$

[10%]

b) Plasticity Index is defined as the difference in liquid limit and plastic limit of a soil. $PI = \omega_L - \omega_p$

It indicates the amount of water to transition a soil from semi solid state to a liquid state. Soils with high PI will have high clay content.

If a soil has a low PI, then the amount of clay content will be small. Therefore such soils should behave more like sands. [10%]

c) Mass of the soil = 0.32 kg.

Volume of the soil = 220 − 100 = 120 ml.

$\therefore$ Density of soil grains $= \frac{0.32}{120 \times 10^{-3} \times 10^{-3}}$ kg/m³ = 2666.67 kg/m³

Density of water = 1000 kg/m³

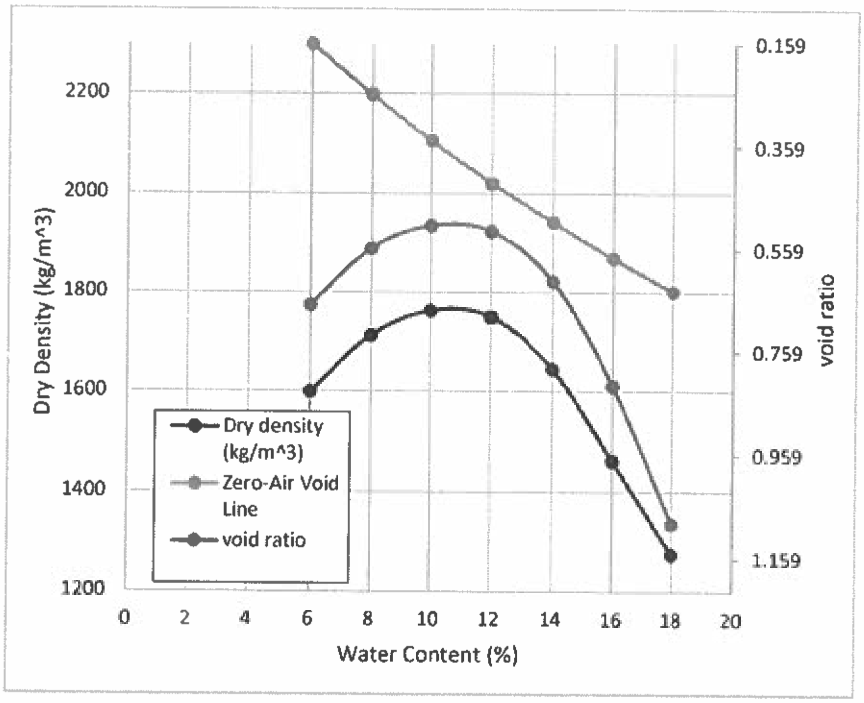
$$\therefore G_s = \frac{\rho_{solids}}{\rho_w} = \underline{\underline{2.67}}$$

[10%]

1 d) OMC and MDD:

H=	116.4	mm
D=	101.6	mm
Volume=	0.000944	m^3

Water Content %	Sample Mass (kg)	Bulk Density (kg/m^3)	void ratio *e*	Degree of Saturation Sr	Dry density (kg/m^3)	Zero-Air Void Line
6	1.6	1695.47	0.667	1.50	1599.50	2298.851
8	1.745	1849.12	0.557	1.67	1712.15	2197.802
10	1.83	1939.19	0.513	1.92	1762.90	2105.263
12	1.85	1960.39	0.524	2.36	1750.34	2020.202
14	1.77	1875.61	0.621	3.26	1645.27	1941.748
16	1.6	1695.47	0.824	4.95	1461.61	1869.159
18	1.42	1504.73	1.091	7.37	1275.19	1801.802



<u>Micro-mechanics of OMC & MDD</u>

Wet of optimum water content, air trapied between clumps of grains is easily expelled with low compactive effort, because pressure in the trapped water carries the impact of the hammer, which therefore induces less frictional resistance than might have been expected. Only isolated air bubbles are left (Sr ~ 90%, A ~ 3%) and these cannot be expelled by compaction. The inability of the pore-fluid to escape prevents further compaction.

At optimum water content, the presence of menisci in the d_{10} size voids make the grains stick together when hammered. The final fabric will have capillary suction which will be strong if the void size is small, and it may have trapped some larger voids and channels.

Dry of optimum water content, surface tension effects increase as the menisci retreat into the tightest spaces. The soil gets hard and difficult to compact, so density tends to reduce. It is also brittle, because the water droplet at a broken junction cannot find its way to another particle contact, so the initially large suction is lost and the desiccated clay crumbles. Ultimately dry powders "splash" all over the place when hit by hammers, and can not be compacted at all.

[40%]

1 e) A zero air voids line is the theoretical maximum dry density possible, if all the air voids are completely full of water. In practice, this is not possible to achieve, as it is very difficult to drive out all the air from the soil during compaction. Usually engineers refer to 90% or 95% compaction in the field with rest of the void space filled with air. The zero air void line for this fill material is shown on graph in part d).

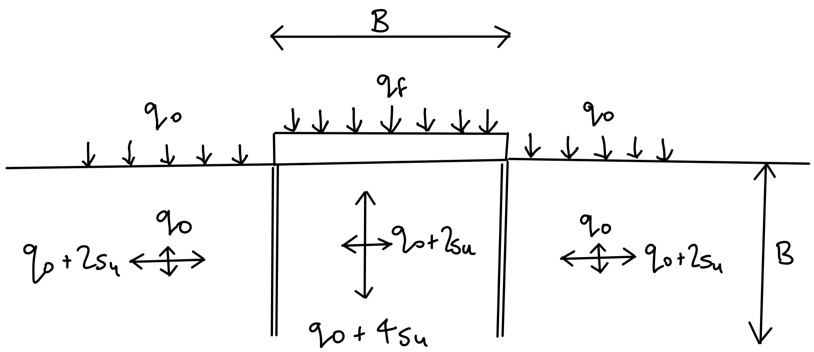
[10%]

1 f) In a modified Proctor test the compaction effort is higher compared to the standard Proctor test. There it is possible to achieve a higher MDD at a lower OMC compared to the standard test. In case of the embankment construction, you should speak to the contractor and see what type of compaction equipment they will be using. If a light roller equipment is being used (for small embankments) then the standard Proctor test would be relevant. For large embankments, then the equipment used will be heavy rollers or dynamic compaction equipment. In this case the modified Proctor test should be recommended.

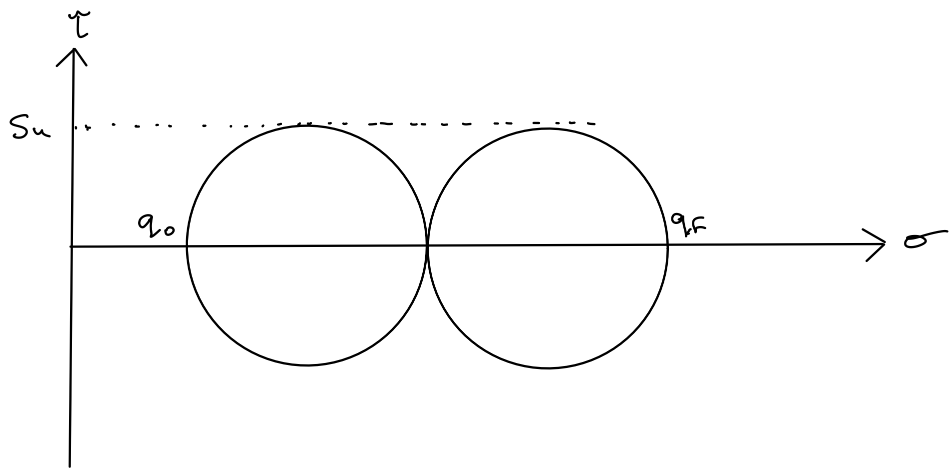
[20%]

A popular question tackled by nearly all students. Generally answered well apart from the fact that some confused dry density and bulk or dry unit weight in calculating the OMC and MDD.

2) a. Problem diagram:



Mohr circles of stress:

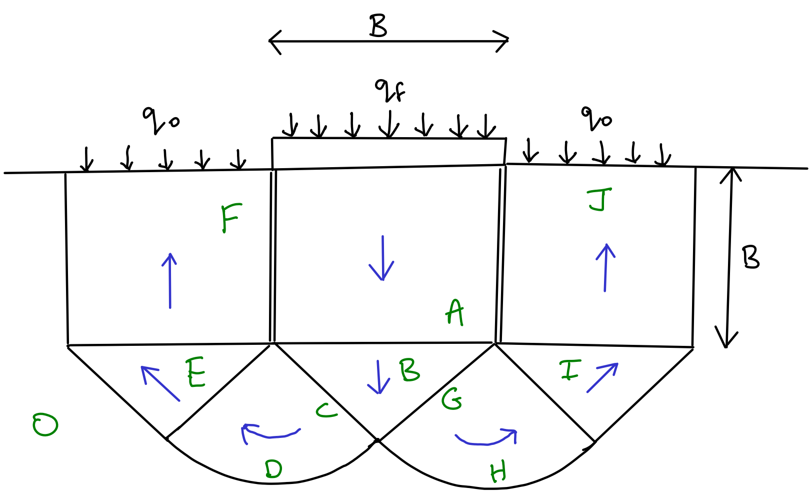


$\therefore q_f = q_0 + 4s_u$

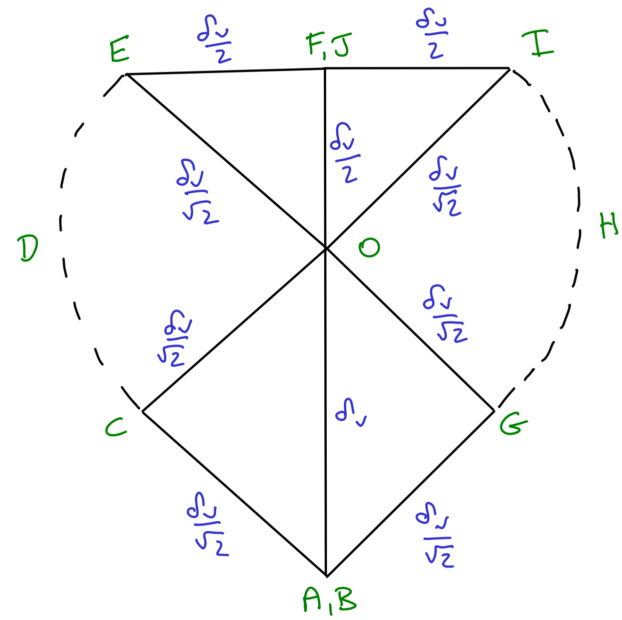
[25 %]

b.

Problem diagram:



Displacement diagram:



$$q_f B \delta_v = \underbrace{2 q_o B \frac{\delta_v}{2}}_{\text{Surcharge}} + \underbrace{4 s_u \frac{B}{\sqrt{2}} \frac{\delta_v}{\sqrt{2}}}_{OE,\, OI,\, BC,\, BG} + \underbrace{4 s_u B \frac{\delta_v}{2}}_{OF,\, OJ,\, EF,\, IJ} + \underbrace{4 s_u \frac{B}{\sqrt{2}} \frac{\pi}{2} \frac{\delta_v}{\sqrt{2}}}_{\text{Slip fans}}$$

$$= q_o B \delta_v + 2 s_u B \delta_v + 2 s_u B \delta_v + s_u B \pi \delta_v$$

$$\therefore q_f = q_o + 2 s_u + 2 s_u + s_u \pi$$

$$= q_o + 4 s_u + s_u \pi$$

$$= q_o + (4 + \pi) s_u$$

[50 %]

c. Surcharge increases the bearing capacity in an additive manner for both the lower and upper bounds derived here.

[10 %]

d. There is a significant difference between the lower and upper bound collapse pressures of πs_u that is almost 80% of the lower bound collapse pressure. This is a large amount of uncertainty to carry in the design.

Finite Element Limit Analysis could be performed in order to numerically obtain closer bounds.

[15 %]

An unpopular question tackled by a small number of students. The lower bound part was generally answered correctly. The upper bound however was less well answered. Some students ignored the instruction to have a Prandtl-type mechanism at the base of the sheet pile walls, but showed that they knew how to calculate an upper bound nonetheless.

3) a. $H = 20\ m$; $B = 5\ m$; $L = 5\ m$; $E = 20\ MPa$

$$q = \frac{5000}{5 \times 5} = 200\ kPa.$$

$$w_{avg} = \mu_0 \mu_1 \frac{qB}{E}$$

$$\frac{D}{B} = \frac{0}{5} = 0 \quad \underline{\text{and}} \quad \frac{H}{B} = \frac{20}{5} = 4$$

from Charts $\mu_0 = 1$ $\underline{\text{and}}$ $\mu_1 = 0.6$

$$w_{avg} = 1 \times 0.6 \times \frac{200 \times 5}{20000} = 0.03\ m = 30\ mm$$

[20%]

b. $$c_v = 20\ m^2/year$$

$$= \frac{20}{365 \times 24 \times 60 \times 60}$$

$$= 6.34 \times 10^{-7}\ m^2/s$$

$$k = 1 \times 10^{-9}\ m/s$$

$$c_v = \frac{kE_o}{\gamma_w} \quad \therefore \quad E_o = \frac{c_v \gamma_w}{k}$$

$$= \frac{6.34 \times 10^{-7} \times 9.81}{1 \times 10^{-9}}$$

$$= 6221\ kPa$$

$$S_\infty = \frac{\Delta\sigma' L}{E_0} = \frac{200 \times 20}{6221} = 0.64 \text{ m}$$

$$= 642 \text{ mm}$$

Time to end of stage 1 consolidation:

$$t_1 = \frac{L^2}{12 c_v} = \frac{20^2}{12 \times 20} = 1.66 \text{ years}$$

$\therefore$ @ $t = 2$ years settlement is governed by stage 2 parabolic isochrone, hence:

$$S = \frac{u_i' L}{3E_0}\left[3 - 2\exp\left(-3c_v(t-t_1)/L^2\right)\right]$$

$$= \frac{200 \times 20}{3 \times 6221}\left[3 - 2\exp\left(-3 \times 20(2-1.66)/20^2\right)\right]$$

$$= 0.23 \text{ m} \qquad = 230 \text{ mm}$$

[40 %]

c. Total settlement @ $t = 2$ years

$$w_{2yr} = \underbrace{30}_{\text{Immediate}} + \underbrace{230}_{\text{Consolidation}} = \underbrace{260}_{\text{Total}} \text{ mm}$$

$$w_{tot} = 30 + 642 = 672 \text{ mm}$$

$$\% \text{ settlement} = \frac{100}{672} \times 260 = 38.7\,\%$$

[20%]

d. Consolidation could be accelerated by the creation of vertical drains or pre-loading during construction. The former shortens the drainage path and the latter increases the increment of effective stress such that the target effective stress is reached sooner.

[10%]

e. A one-dimensional consolidation solution has been assumed, however lateral drainage would also occur due to the soil layer depth being significantly greater than the foundation breadth or length. This means that the lateral drainage paths are shorter than the vertical ones, resulting in accelerated consolidation in reality. That means that the estimate for the percentage settlement complete at time t = 2 years in part b is likely to be an underestimate.

[10%]

A popular question tackled by most students. The most common and significant error was in the choice of elastic modulus in the elastic and consolitation settlement calculations - the two values E and E_0 were often conflated, resulting in strange settlement calculation. The latter part of the question was answered very well.

4) a. $\phi' = 35°$; $\gamma' = 20\ kN/m^3$; $B = 2\ m$; $L = 2\ m$; $\sigma_{v_0} = 5\ kPa$

Surcharge $\sigma_{v_0} = \gamma z = 20 \times 0.25$

$= 5\ kPa$

Bearing capacity factors from EC7:

$$N_q = \tan^2(\pi/4 + \phi'/2)\exp(\pi \tan\phi')$$

$$= 33.3$$

$$N_\gamma = 2(N_q - 1)\tan\phi'$$

$$= 2(33.3 - 1)\tan 35$$

$$= 45.23$$

Shape factors from EC7:

$$s_q = 1 + \frac{B}{L}\sin\phi'$$

$$= 1 + \frac{2}{2}\sin 35$$

$$= 1.57$$

$$s_\gamma = 1 - 0.3\frac{B}{L}$$

$$= 1 - 0.3 \times \frac{2}{2}$$

$$= 0.7$$

Bearing capacity:

$$q_f = s_q N_q \sigma'_{v_0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

$$= 1.57 \times 33.3 \times 5 + 0.7 \times 45.23 \times \frac{20 \times 2}{2}$$

$$= 261.4 + 633.2$$

$$= 894.72\ kPa$$

$\therefore V = q_f B L = 894.72 \times 2 \times 2 = 3578\ kN$

$= 3.6\ MN$

[40%]

b. $V_{pylon} = 200$ kN $\therefore$ $V_{foundation} = \frac{V_{pylon}}{4} = 50$ kN.

$$V_{foundation-M} = \frac{M}{e} = \frac{25 \times 20}{4 \times 0.5 \times 10} = 25 \text{ kN}$$

$$V_{foundation-Up} = V_{foundation} - V_{foundation-M} = 50 - 25 = 25 \text{ kN}$$

$$V_{foundation-down} = V_{foundation} + V_{foundation-M} = 50 + 25 = 75 \text{ kN}$$

If $M = 0$, Butterfield and Gottardi reduces to the following expression:

$$\left[\frac{H/V_{ult}}{th}\right]^2 = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

$$\frac{H/V_{ult}}{th} = \frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)$$

$$H/V_{ult} = \frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)th$$

$$H = V\left(1 - \frac{V}{V_{ult}}\right)th$$

$$H_{up} = 25\left(1 - \frac{25}{3578}\right)0.5 = 12.41 \text{ kN}$$

$$H_{down} = 75\left(1 - \frac{75}{3578}\right)0.5 = 36.71 \text{ kN}$$

$$FoS_{up} = \frac{12.41}{6.25} = 1.98 \quad \therefore \text{ SAFE}$$

$$FoS_{down} = \frac{36.71}{6.25} = 5.87 \quad \therefore \text{ VERY SAFE}$$

[40%]

c. If $\mu = \tan\delta = 0.3$ for concrete-soil interface:

$\frac{H}{V} < \mu$ else sliding failure occurs.

$$\frac{H_{up}}{V_{up}} = \frac{6.25}{25} = 0.25 < \mu \quad \therefore \text{ SAFE.}$$

$$\frac{H_{down}}{V_{down}} = \frac{6.25}{75} = 0.083 \ll \mu \quad \therefore \text{ VERY SAFE}$$

$$FoS_{up} = \frac{0.3}{0.25} = 1.2$$

$$FoS_{down} = \frac{0.3}{0.083} = 3.6$$

[15 %]

d. Factors of safety against sliding after accounting for interface friction are much lower, which highlights the need for a sliding failure check.

[5 %]

An unpopular question answered by only a handful of students. Part a was generally answered very well, whereas part b was less successfully tackled. Some students failed to notice that the connection betweent the foundations and pylon truss was prescribed as a frictionless ball - thereby precluding the generation of moments at the connection and simplifying the solution significantly.